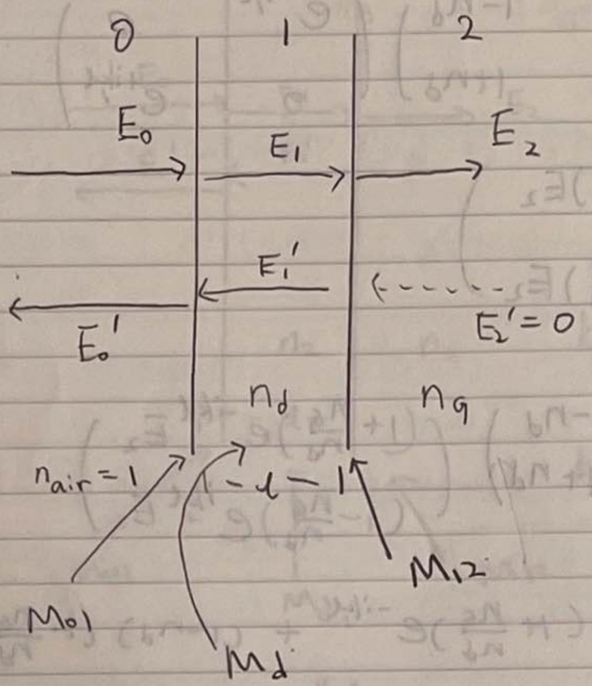
To: Caroline Terquem

5 Optics (30, 31, 32, 33, 34) Ziyan Li

Excellent!

30.



Use the matrix method:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = M_{01} M_d M_{12} \begin{pmatrix} E_2 \\ 0 \end{pmatrix}$$

$$M_{01} = \frac{1}{2}\begin{pmatrix} 1+n_d & 1-n_d \\ 1-n_d & 1+n_d \end{pmatrix}$$

$$M_d = \begin{pmatrix} e^{-ik_1 d} & 0 \\ 0 & e^{+ik_1 d} \end{pmatrix} \quad \left(k_1 = \frac{2\pi n_d}{\lambda}\right)$$

↑ wavelength in vacuum

$$M_{12} = \frac{1}{2}\begin{pmatrix} 1+\frac{n_G}{n_d} & 1-\frac{n_G}{n_d} \\ 1-\frac{n_G}{n_d} & 1+\frac{n_G}{n_d} \end{pmatrix}$$

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+n_d & 1-n_d \\ 1-n_d & 1+n_d \end{pmatrix}\begin{pmatrix} e^{-ik_1 d} & 0 \\ 0 & e^{+ik_1 d} \end{pmatrix}$$

$$\begin{pmatrix} 1+\frac{n_g}{n_d} & 1-\frac{n_G}{n_d} \\ 1-\frac{n_G}{n_d} & 1+\frac{n_G}{n_d} \end{pmatrix}\begin{pmatrix} E_2 \\ 0 \end{pmatrix}$$

$$= \cancel{\tfrac{1}{2}}\ \frac{1}{4}\begin{pmatrix} 1+n_d & 1-n_d \\ 1-n_d & 1+n_d \end{pmatrix}\begin{pmatrix} e^{-ik_1 l} & 0 \\ 0 & e^{+ik_1 l} \end{pmatrix}$$

$$\begin{pmatrix} (1+\frac{n_G}{n_d})E_2 \\ (1-\frac{n_G}{n_d})E_2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 1+n_d & 1-n_d \\ 1-n_d & 1+n_d \end{pmatrix}\begin{pmatrix} (1+\frac{n_G}{n_d})e^{-ik_1 l}E_2 \\ (1-\frac{n_G}{n_d})e^{ik_1 l}E_2 \end{pmatrix}$$

$$= \frac{E_2}{4}\begin{pmatrix} (1+n_d)(1+\frac{n_G}{n_d})e^{-ik_1 l} + (1-n_d)(1-\frac{n_G}{n_d})e^{ik_1 l} \\ (1-n_d)(1+\frac{n_G}{n_d})e^{-ik_1 l} + (1+n_d)(1-\frac{n_G}{n_d})e^{ik_1 l} \end{pmatrix}$$

$$\therefore r = \frac{E_0'}{E_0} = \frac{(1-n_d)(1+\frac{n_G}{n_d})e^{-ik_1 l} + (1+n_d)(1-\frac{n_G}{n_d})e^{ik_1 l}}{(1+n_d)(1+\frac{n_G}{n_d})e^{-ik_1 l} + (1-n_d)(1-\frac{n_G}{n_d})e^{ik_1 l}}$$

$$= \frac{(1-n_d+\frac{n_G}{n_d}-n_G)e^{-ik_1 l} + (1+n_d-\frac{n_G}{n_d}-n_G)e^{ik_1 l}}{(1+n_d+\frac{n_G}{n_d}+n_G)e^{-ik_1 l} + (1-n_d-\frac{n_G}{n_d}+n_G)e^{ik_1 l}}$$

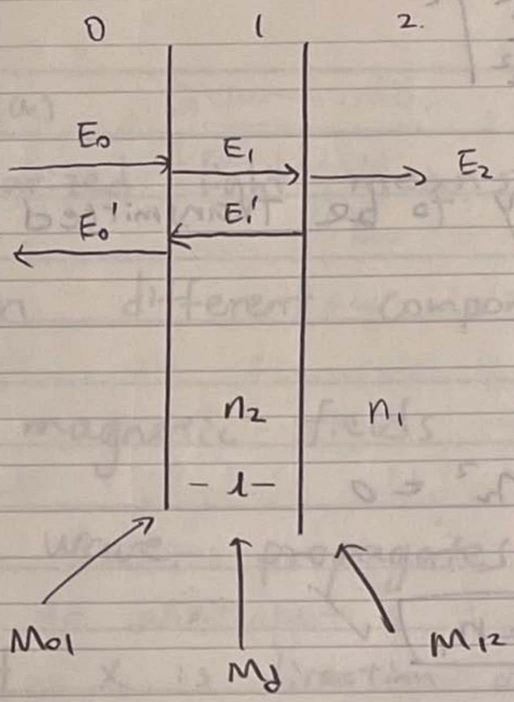
$$= \frac{(e^{ik_1 l}+e^{-ik_1 l}) - (\frac{n_G}{n_d})(e^{ik_1 l}-e^{-ik_1 l}) + n_d(e^{ik_1 l}-e^{-ik_1 l}) - n_G(e^{ik_1 l}+e^{-ik_1 l})}{(e^{ik_1 l}+e^{-ik_1 l}) - (\frac{n_G}{n_d})(e^{ik_1 l}-e^{-ik_1 l}) - n_d(e^{ik_1 l}-e^{-ik_1 l}) + n_G(e^{ik_1 l}+e^{-ik_1 l})}$$

$$\times \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$= \frac{\cos(k_1 l) - i(n_G/n_d)\sin(k_1 l) + i n_d \sin(k_1 l) - n_G\cos(k_1 l)}{\cos(k_1 l) - i(n_G/n_d)\sin(k_1 l) - i n_d\sin(k_1 l) + n_G\cos(k_1 l)}$$

✓

31.



$$l = \frac{\lambda}{4n_2}$$

$$k_1 = \frac{2\pi n_2}{\lambda}$$

$$k_1 l = \frac{2\pi n_2}{\lambda} \cdot \frac{\lambda}{4n_2} = \frac{\pi}{2}$$

$$\therefore \quad M_d = \begin{pmatrix} e^{-ik_1 l} & 0 \\ 0 & e^{+ik_1 l} \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = M_{01} M_d M_{12} \begin{pmatrix} E_2 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1+n_2 & 1-n_2 \\ 1-n_2 & 1+n_2 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1+\frac{n_1}{n_2} & 1-\frac{n_1}{n_2} \\ 1-\frac{n_1}{n_2} & 1+\frac{n_1}{n_2} \end{pmatrix}\begin{pmatrix} E_2 \\ 0 \end{pmatrix}$$

$$= \frac{E_2}{4}\begin{pmatrix} 1+n_2 & 1-n_2 \\ 1-n_2 & 1+n_2 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1+\frac{n_1}{n_2} \\ 1-\frac{n_1}{n_2} \end{pmatrix}$$

$$= \frac{E_2}{4}\begin{pmatrix} 1+n_2 & 1-n_2 \\ 1-n_2 & 1+n_2 \end{pmatrix}\begin{pmatrix} -i(1+\frac{n_1}{n_2}) \\ i(1-\frac{n_1}{n_2}) \end{pmatrix}$$

$$= \frac{E_2}{4}\begin{pmatrix} -i(1+n_2)(1+\frac{n_1}{n_2}) + i(1-n_2)(1-\frac{n_1}{n_2}) \\ -i(1-n_2)(1+\frac{n_1}{n_2}) + i(1+n_2)(1-\frac{n_1}{n_2}) \end{pmatrix}$$

$$\therefore \quad r = \frac{E_0'}{E_0} = \frac{-\cancel{i}(\cancel{1} - n_2 + \frac{n_1}{n_2} - \cancel{n_1}) + \cancel{i}(\cancel{1} + n_2 - \frac{n_1}{n_2} - \cancel{n_1})}{-\cancel{i}(\cancel{1} + n_2 + \frac{n_1}{n_2} + \cancel{n_1}) + \cancel{i}(\cancel{1} - n_2 - \frac{n_1}{n_2} + \cancel{n_1})}$$

$$= \frac{n_2 - \frac{n_1}{n_2}}{-n_2 - \frac{n_1}{n_2}} = \frac{n_1 - n_2^2}{n_1 + n_2^2}$$

$$R = |r|^2 = \left| \frac{n_1 - n_2^2}{n_1 + n_2^2} \right|^2$$

For all energy to be transmitted,

$$R = 0$$

$$\rightarrow \quad n_1 - n_2^2 = 0$$

$$\rightarrow \quad \boxed{n_2^2 = n_1} \checkmark$$

32. (a)

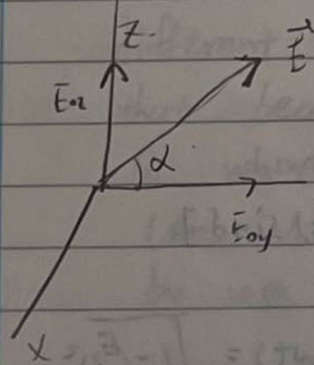
Polarized light means that the phase between different components of the electric and magnetic fields remain constant as the wave propagates.

If x is direction of travel, then

$$\underline{E_y} = E_{0y} \cos(kx - \omega t)\, \hat{y}$$

$$\underline{E_z} = E_{0z} \cos(kx - \omega t + \delta)\, \hat{z}$$

The light is polarized if δ is constant.



① When $\delta = 0$

$$\tan\alpha = \frac{E_z}{E_y} = \frac{E_{0z}}{E_{0y}} = \text{constant}$$

$\rightarrow$ α is fixed

$\rightarrow$ linearly polarised.

② When $\delta = \pm\frac{\pi}{2}$, $E_{0y} = E_{0z} = E_0$

$$E_z = E_{0z} \cos(kx - \omega t \pm \tfrac{\pi}{2}) = \mp E_{0z} \sin(kx - \omega t)$$

$\rightarrow \tan\alpha_{\pm} = \frac{E_z}{E_y} = \frac{\mp \sin(kx-\omega t) E_0}{\cos(kx-\omega t) E_0} = \mp \tan(kx-\omega t)$

$\rightarrow \quad \alpha_{\pm} = \mp (kx-\omega t)$

take $x=0 \rightarrow \quad \alpha_{\pm} = \pm \omega t$

$\rightarrow$ tip of $\vec{E}$ vector rotates with angular frequency ω. , magnitude of $\vec{E}$ remains constant.

$\rightarrow$ circularly polarised

$+$ sign $\rightarrow$ left polarised.

$-$ sign $\rightarrow$ right polarised.

(3) for general E_{0y}, E_{0z}, and δ

$$E_y = E_{0y} \cos(kx-\omega t)$$
$$E_z = E_{0z} \cos(kx-\omega t+\delta)$$

$$E_z = E_{0z} [\cos(kx-\omega t)\cos\delta - \sin(kx-\omega t)\sin\delta].$$

$$\cos(kx-\omega t) = \frac{E_y}{E_{0y}}, \quad \sin(kx-\omega t) = \sqrt{1-\left(\frac{E_y}{E_{0y}}\right)^2}$$

$$\rightarrow E_z = E_{0z}\left[\frac{E_y}{E_{0y}}\cos\delta - \sqrt{1-\left(\frac{E_y}{E_{0y}}\right)^2}\sin(\delta)\right]$$

$$\rightarrow \left(\frac{E_y}{E_{0y}}\cos\delta - \frac{E_z}{E_{0z}}\right)^2 = \left(1-\left(\frac{E_y}{E_{0y}}\right)^2\right)\sin^2\delta$$

$$\rightarrow \left(\frac{E_y}{E_{0y}}\right)^2\cos^2\delta - 2\frac{E_y}{E_{0y}}\frac{E_z}{E_{0z}}\cos\delta + \frac{E_z}{E_{0z}}^2 = \sin^2\delta - \frac{E_y}{E_{0y}}\sin^2\delta$$

$$\rightarrow \quad \frac{E_y^2}{E_{0y}^2} + \frac{E_z^2}{E_{0z}^2} - 2\frac{E_y}{E_{0y}}\frac{E_z}{E_{0z}}\cos\delta = \sin^2\delta$$

→ an equation for an ellipse

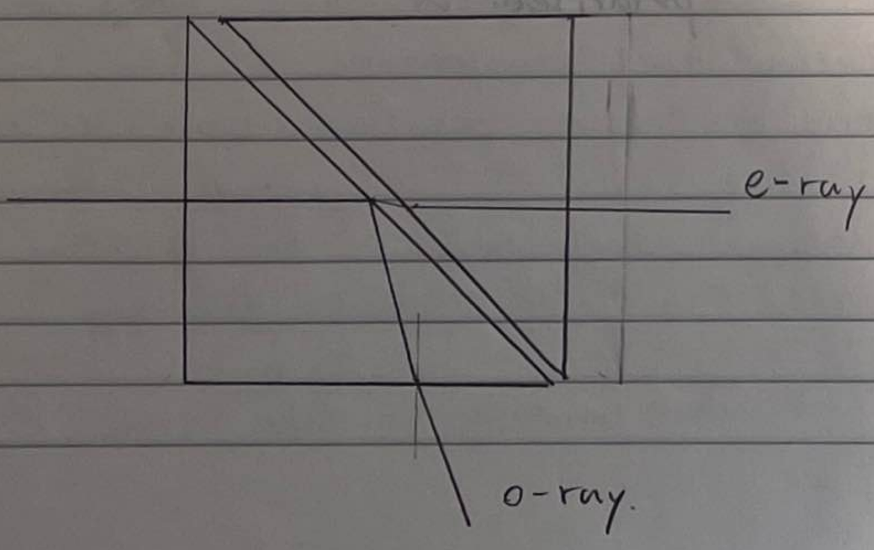
→ elliptical polarisation.

The uniaxial birefringent can change the relative phase by

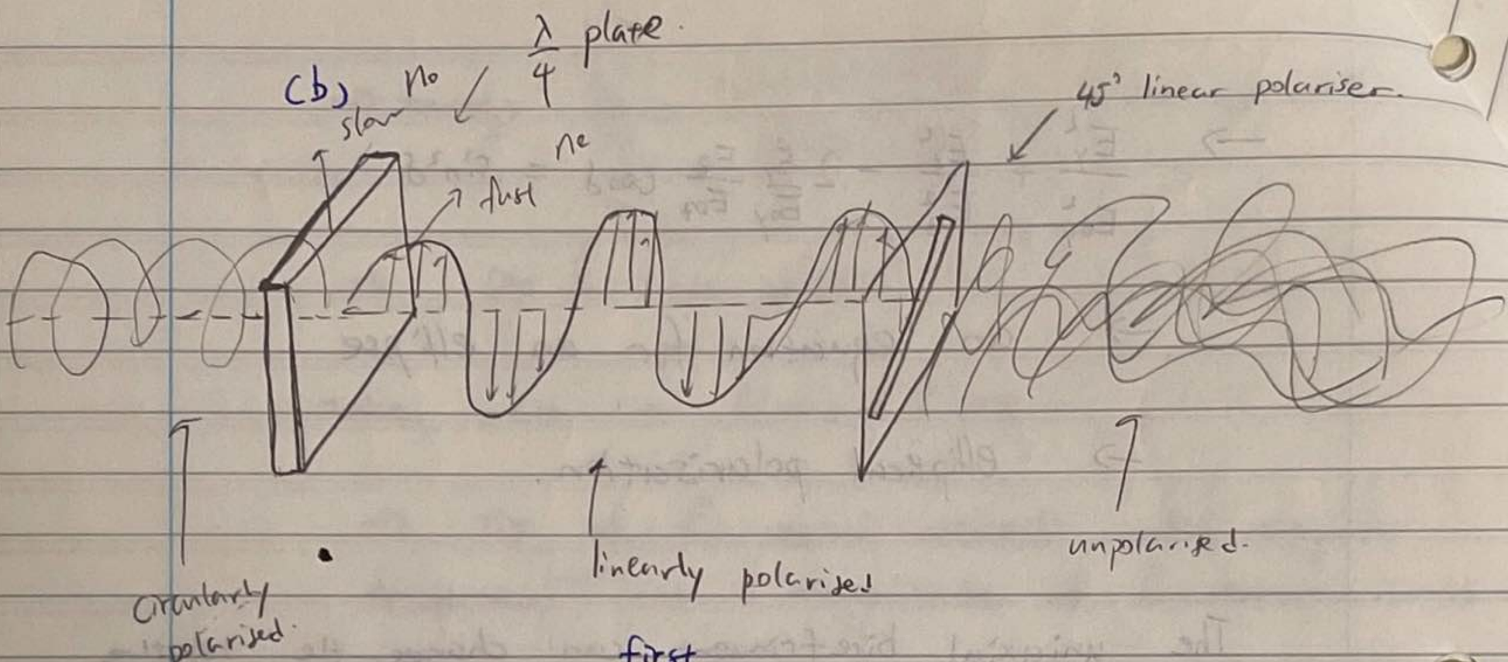
$$\delta = \frac{2\pi}{\lambda}|n_o - n_e|l$$

Consider the operation of a birefringent prism.

o-rays and e-rays have different refractive indices so different angle of refraction and different critical angles θ_c. Prism may be cut so that beam strikes angled face at incidence angle θ_i where $\theta_i > \theta_c$ for o-ray and $\theta_i < \theta_c$ for e-ray. (or vice versa). Deviation may be compensated by use of a second prism.



(b)

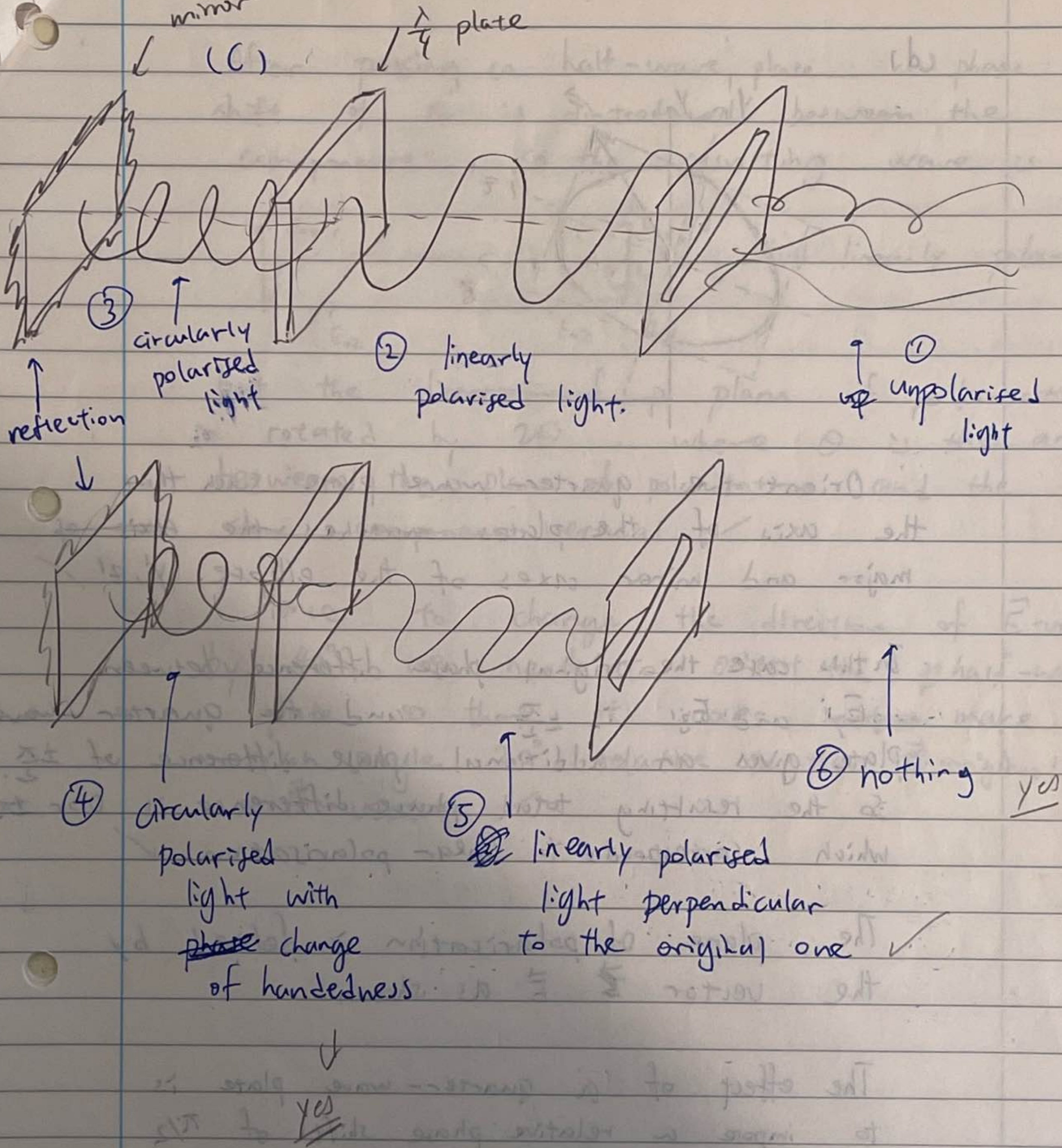


Unpolarised first passes through a ~~linearly~~ linear polariser that is ~~45~~ 45° with respect to the axis of the quarter-wave plate. and become a linearly polarised wave with $\alpha = 45°$, ~~the~~ this ensures that

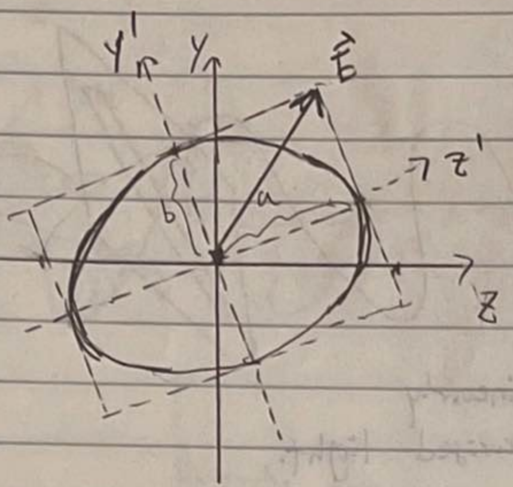
$E_{oy} = E_{oz}$

Then the wave passes through the quater-wave plate so the components acquire a relative phase of $\pm \frac{\pi}{2}$.

→ The resulting wave is circularly polarised.


mirror
(C)
$\frac{\lambda}{4}$ plate
③ circularly polarised light
reflection
② linearly polarised light.
~~up~~ ① unpolarised light
④ circularly polarised light with ~~phase~~ change of handedness
yes
⑤ linearly polarised light perpendicular to the original one
⑥ nothing
yes

(d)



Orient the quarter-wave plate such that the axis of the plates matches the ~~axis of~~ major and minor axes of the ellipse. y', z' ✓

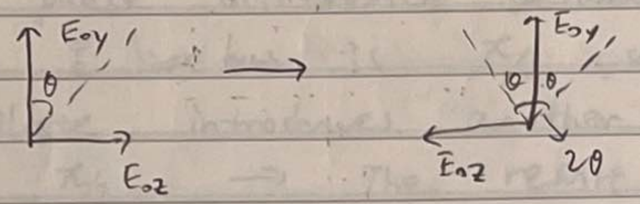
In this case the original phase difference between $E_{y'}$ and $E_{z'}$ is $\pm\frac{\pi}{2}$, and the quarter-wave plate gives an additional phase difference of $\pm\frac{\pi}{2}$. So the resulting total phase difference is 0 or $\pm\pi$ which corresponds to linear polarisation. ✓

The plane of polarisation is defined by the vector ~~$\vec{E}$~~ $\vec{E}$ as shown.

The effect of a quarter-wave plate is to impose a relative phase shift of $\pi/2$ between the components of the wave, so when a linearly polarised wave passes through, as long as its components ~~are meq~~ have unequal magnitudes, the resulting wave would be ~~ell~~ elliptically polarised.

→ The converse is also true. ✓

After passing a half-wave plate, a phase shift of π is introduced between the components. So the resulting wave is

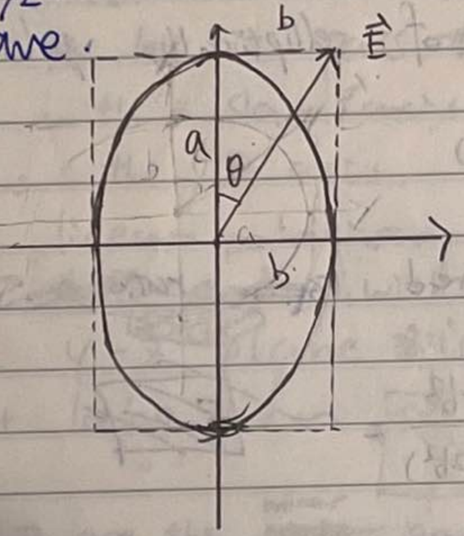


still linearly polarised

but the ~~direction of p~~ plane of polarisation is rotated by 2θ, where θ is the angle between the plane of polarisation and the ~~slow component.~~ vertical ✓

Hence to change the direction of $\vec{E}$-vector by 40° we need to orient the half-wave plate so that its axes ~~makes~~ make an angle 20° relative to the original $\vec{E}$ - vector. ✓

33. Orient the quarter-wave plate so that its axes ~~match~~ matches the major and minor axes of the ellipse. In this case the original phase difference between components of ~~wave~~ $\vec{E}$ ~~is but~~ is $\pi/2$ and the quarter-wave plate introduces another ~~phas~~ phase ~~d~~ shift of $\pi/2$. → The result is a linearly polarised wave.



The angle θ is given by

$$\theta = \arctan\left(\frac{b}{a}\right) \checkmark$$

From the two experiments: we know that

$$I = I_p + I_u$$

polarised, unpolarised.

Intensity proportional to square of Electric field

$$\rightarrow I_p(a) = I_0 a^2 , \quad I_p(b) = I_0 b^2$$

unpolarised light passes through ~~linearly~~ linear polariser → intensity reduce by half

$$2 = \frac{I_{max}}{I_{min}} = \frac{I_0 a^2 + I_u/2}{I_0 b^2 + I_u/2} \checkmark, \quad \frac{b}{a} = \tan\theta = \tan(33.21^\circ)$$

$$= 0.655.$$

$$\rightarrow \quad 2I_0 b^2 + \frac{\cancel{8}I_u}{\cancel{8}} = I_0 a^2 + \frac{I_u}{2}$$

$$\rightarrow \quad \frac{1}{2} I_u = I_0 (a^2 - 2b^2)$$

$$\rightarrow \quad \frac{I_0}{I_u} = \frac{1}{2(a^2 - 2b^2)}$$

Average intensity of elliptically polarised wave

is $I_E = I_0 (a^2 + b^2)$

$\rightarrow$ the required ~~ratio~~ ratio is

$$\frac{I_E}{I_0} = \frac{a^2 + b^2}{2(a^2 - 2b^2)} \sim \cancel{\boxed{5.0}}$$

$$= \frac{1 + 0.655^2}{2(1 - 2 \times 0.655^2)} \simeq \boxed{5.0} \checkmark$$

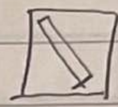
Consider a right ~~polarise~~ elliptically polarised wave

$\delta = -\frac{\pi}{2}$

$$E_y = E_{0y} \cos(kx - \omega t)$$
$$E_z = E_{0z} \sin(kx - \omega t)$$

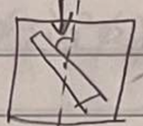
① ~~put a~~ let it pass through a quarter-wave plate, ~~with~~ that shifts E_y by $\frac{\pi}{2}$ relative to E_z ($\hat{y}$ is the slow-axis)

then
$$E_y' = -E_{0y} \sin(kx - \omega t)$$
$$E_z' = E_{0z} \sin(kx - \omega t)$$

Now ~~ps~~ put a linear polariser like 

→ we can see maximum intensity.

~~put a linear polariser lik~~

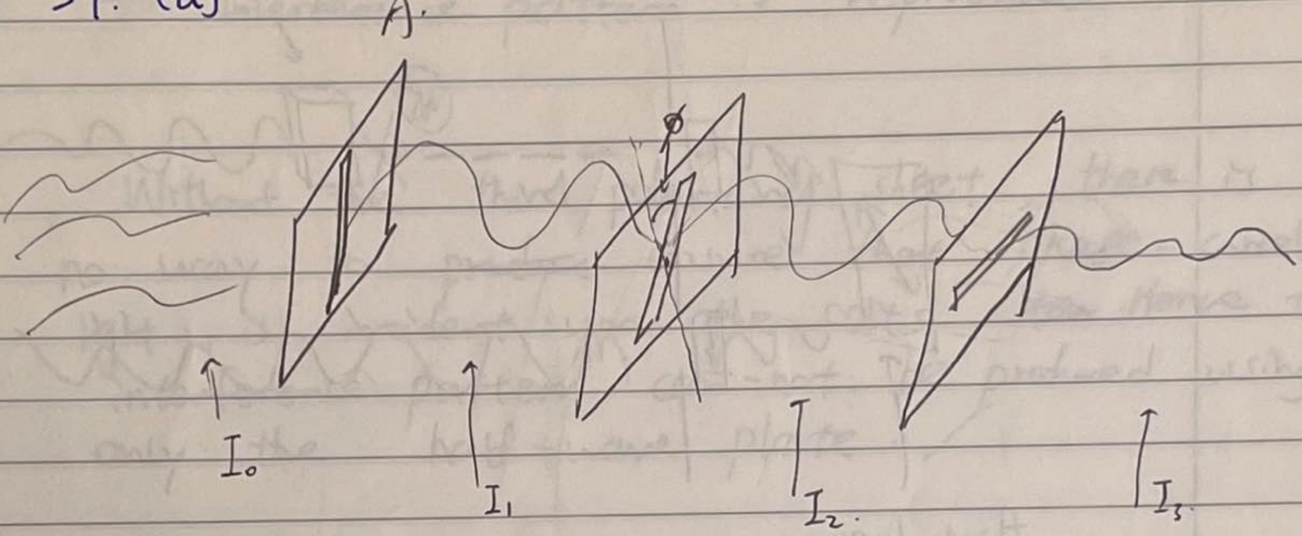
∴ To determine the handedness of a wave, first ~~it~~ let it pass through a quarter-wave plate with slow-axis the y axis and fast axis the z axis, then let the wave pass a linear polariser that has a cut slanted ~~at~~ with negative gradient in the y-z plane 33.21° (Basically a linear polariser that looks like )

at 33.21°, (with the z axis)

(y and z are the ~~major~~ minor and ~~minor~~ major axes of the ellipse)

If we obtain maximum transmitted intensity ~~we have~~ the original wave is right-handed, If we ~~see~~ obtain total extinction then the original wave is left-handed. ✓

34. (a)



$$I_1 = \frac{1}{2} I_0 \ , \quad I_2 = I_1 \cos^2\phi \ , \quad I_3 = I_2 \cos^2\left(\frac{\pi}{2} - \phi\right) = I_2 \sin^2\phi$$

$$\therefore \quad I_3 = I_0 \left(\frac{1}{2} \cos^2\phi \sin^2\phi\right)$$

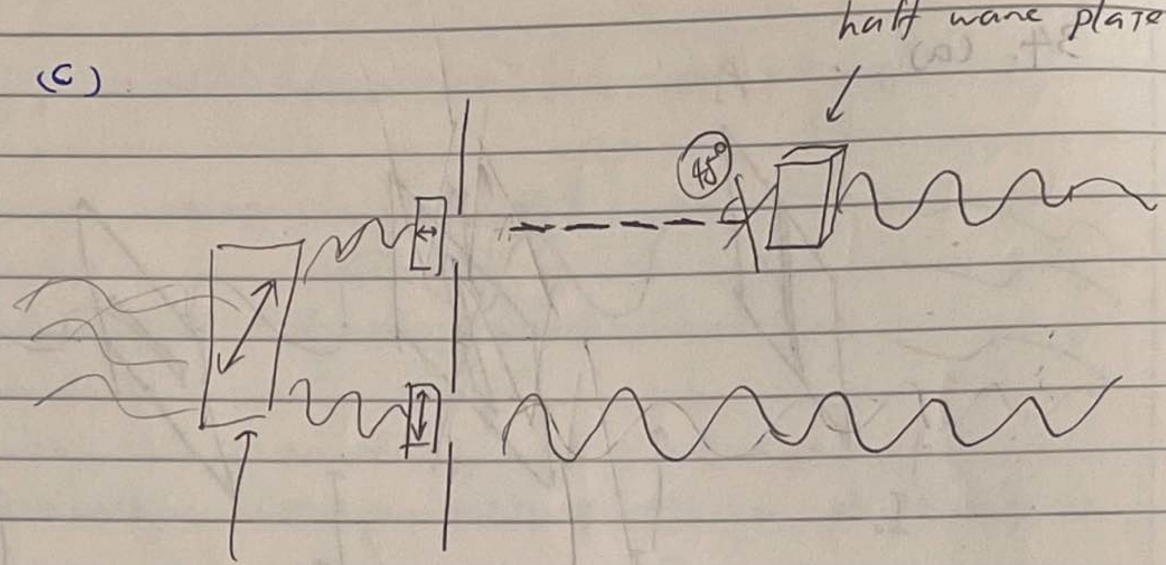
$$\therefore \quad \frac{I_3}{I_0} = \frac{1}{2} \cos^2\phi \sin^2\phi = \cancel{\tfrac{1}{4}}\ \frac{1}{8}\left(4 \cos^2\phi \sin^2\phi\right)$$

$$= \boxed{\frac{1}{8} \sin^2(2\phi)}$$

(b) ~~No interference is observed.~~

Light ~~come~~ coming out of the horizontal polariser is horizontally polarised, and that from the vertical ~~on~~ one is vertically polarised. Orthogonally polarised waves do not interfere, so there is no interference and hence no fringes.

(c)



We place the third polarising sheet in front of the double slits and the half wave plate ~~behind~~ ~~behind~~ behind one of the slits.

The third polarising sheet is oriented 45° with respect to the slits so that the ~~st~~ light ~~pa~~ that is incident on the slits have angle 45° ~~to~~ with respect to both horizontal and vertical polarisers of the two slits. This ensures that the lights passes through the two slits have equal ~~intent~~ intensity (both dropped by factor $(\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$). Also, ~~a~~ placing a polariser in front of the slits ensures that phase-correlated (nearly coherent) light is incident upon the slits.

yes

The half wave plate is placed 45° with respect to the ~~dir~~ direction of ~~po~~ axis of the polarising sheet in front of it. It rotates the polarisation of incident wave by $2 \times 45° = 90°$. so that the wave has the same polarisation as the wave passing through the other slit.

yes

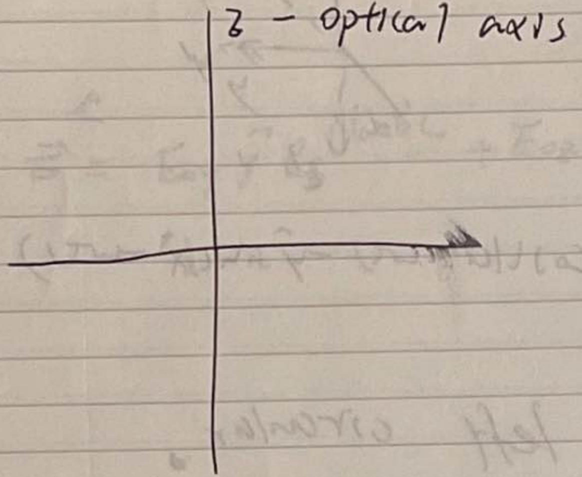
→ ~~inte~~

→ Interference pattern is reproduced.

Without the third polarising sheet, there is no way to ~~produce~~ ensure that phase-correlated light is incident upon the slits. ~~Hen~~ Hence the interference pattern can-not be produced using only the half-wave plate. ✓

$E_0 (\hat{z} \cos(kz - \omega t) + \hat{y} \cos(kz - \omega t + \delta))$

$\vec{E}_1 = E_0 (\hat{z} + \hat{y}) \cos(kx - \omega t)$



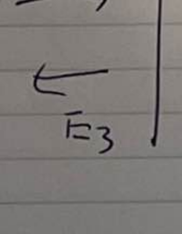
$\vec{E}_2 = E_0 (\hat{z} e^{ikn_e L} + \hat{y} e^{ikn_o L})$

$$E_0 (\hat{z} + \hat{y}\, e^{\underbrace{ik(n_o - n_e)}_{\pm\frac{\pi}{2}}})\, e^{i(kx + kn_e L - \omega t)}$$

$$= E_0 (\hat{z} - i y)\, e^{i(kx - \omega t + kn_e L)}$$

$Re(E_2) = E_0 (\hat{z} \cos(kx - \omega t) + \hat{y} \sin(kx - \omega t))$

circularly polarised wave. (right circular)

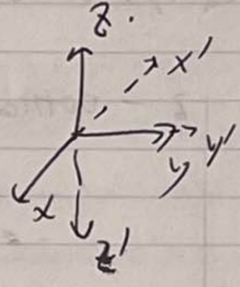


$$\vec{E}_3 = E_0(-\hat{z}\cos(-kx-\omega t) - \hat{y}\sin(-kx-\omega t))$$

x', y', z'

$=\;\;\;=\;\;\;=$

$-x\;\;\; y\;\;\; -z$



$$E_3 = E_0 \times (\hat{z}'\cos(kx'-\omega t) - \hat{y}\sin(kx'-\omega t))$$

left circular.

Opposite handedness

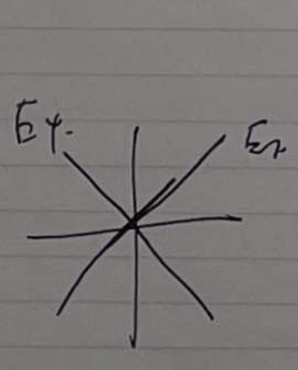
$$\vec{E}_3 = E_0\left(-\hat{z}\cos(kx'-\omega t) - \hat{y}\sin(kx'-\omega t)\right)$$

$$\vec{E}_3 = E_0(-\hat{z} + \hat{y}e^{i\frac{\pi}{2}})e^{i(kx'-\omega t)}$$

$$= E_0(-\hat{z} + \hat{y}e^{i\frac{\pi}{2}}e^{-i\frac{\pi}{2}})e^{i(kx'-\omega t)}$$

$$\vec{E}_4 = E_0(-\hat{z} + \hat{y}\underbrace{e^{i\frac{\pi}{2}}e^{-i\frac{\pi}{2}}}_{1})e^{i(kx'-\omega t)}$$

$$= E_0(-\hat{z} + \hat{y})e^{i(kx'-\omega t)}$$



$\vec{E} = E_{0y}\,\hat{y}\cos(kx - \omega t) + E_{0z}\,\hat{z}\sin(kx - \omega t)$

→ ~~into~~ entering the $\lambda/4$ plate

$\vec{E} = \left(E_{0y}\,\hat{y} + E_{0z}\,\hat{z}\,e^{-i\frac{\pi}{2}}\right)e^{i(kx-\omega t)}$

$\vec{E}^{t} = E_{0y}\,\hat{y}\,e^{ikn_0L} + E_{0z}\,\hat{z}\,e^{-i\frac{\pi}{2}} \sim \ldots$