To: Caroline Terquem

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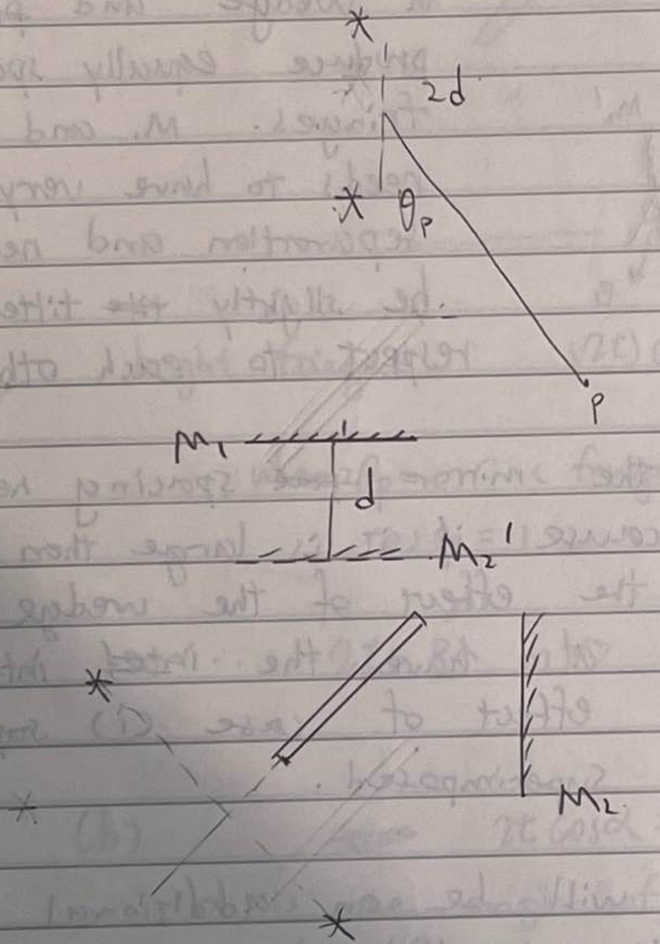
4

Optics (20, 21, 23, 24, 25, 27)

20.



Michelson interferometer illuminated by an extended monochromatic source produces localised fringes by the two images produced by one mirror and the image of another mirror, respectively.

(i)

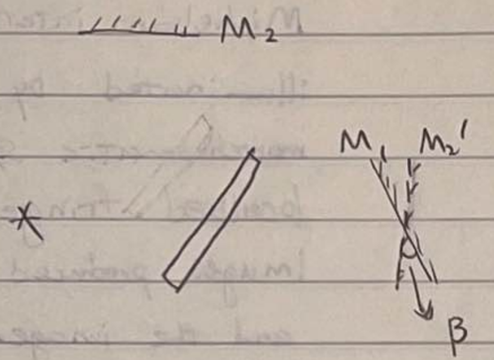
The configuration shown ~~on~~ on the left gives circular fringes.

The two mirrors have ~~difference~~ (one and the image of the other) have separation d. so the images of source have separation 2d. The images of the source produces cylindrically symmetrical circular fringes. localised at infinity. ~~The~~ M_1 and M_2' ~~has~~ needs to be parallel.

~~ma~~ ~~bright~~ bright fringes at

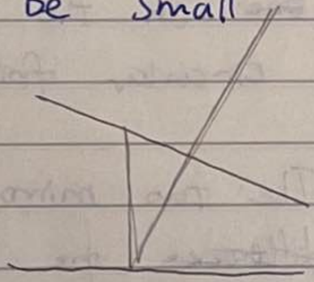
$$d\cos\theta_p = p\lambda$$

(ii)

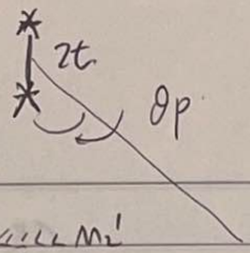


In this configuration M_1 ~~the~~ and M_2' act like a wedge and ~~produces~~ produce equally spaced fringes. M_1 and M_2' needs to have very small separation and needs to be slightly ~~tite~~ tilted with respect to each other.

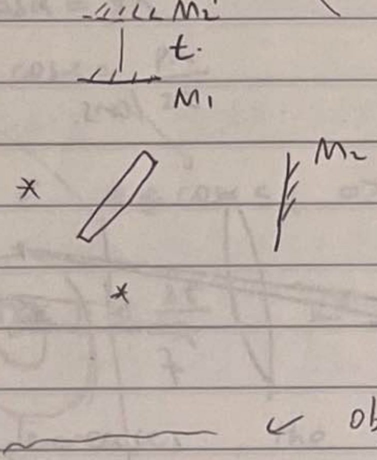
In this case the mirror ~~space~~ spacing needs to be small, because if it is large then beside the effect of the wedge we also have the ~~intef~~ interference effect of case (i) ~~superimposed~~ superimposed.



Hence there will be an additional $\cos^2\theta$ variation in intensity and the interference pattern will change from ~~stat~~ straight lines to hyperbolic ~~lines~~. curves



21. (a)



← observation plane

bright fringes : $(2t)\cos\theta_p = p\lambda$

when $\theta_p = 0$ (fringe on the axis)

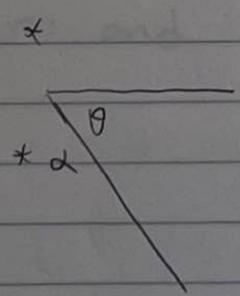
$\cos\theta_p = 1$

$\therefore \quad 2t = p_0\lambda \quad \Rightarrow \quad \boxed{p_0 = \frac{2t}{\lambda}}$

(b) ~~2t~~ $2t\cos\alpha = p\lambda$

α is the angle of inclination, p is the order of diffraction, p is an integer

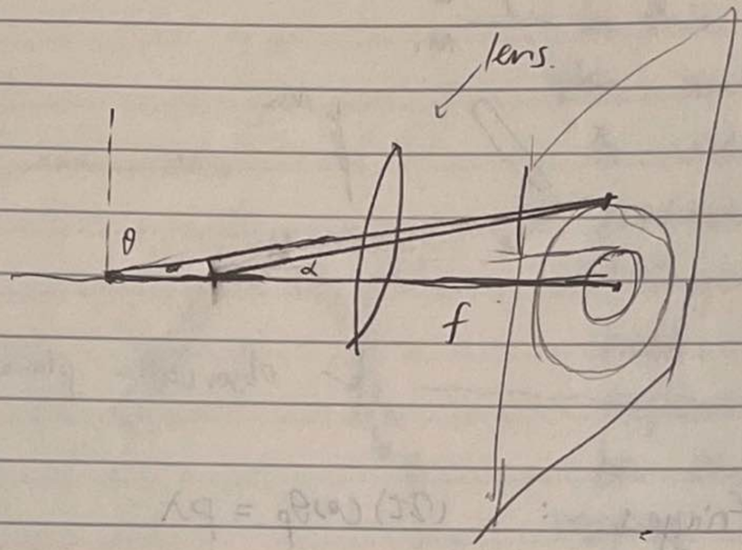
(c)



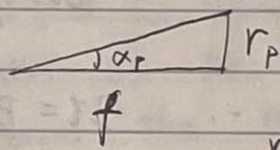
The 0th order is ~~~~ when light coming from the two images have no phase difference. This happens when $\theta = \cancel{90^\circ}, 0^\circ$, $\alpha = 90^\circ$
When θ increases, α decreases, the order of interference increases as the phase difference gets larger.

When $\alpha = 0^\circ$, $\theta = 90^\circ$. p_0 is the maximum order. Hence all other orders are smaller than p

(d)



$2t\cos\alpha_p = p\lambda \qquad 2t\cos\alpha_{p+1} = (p+1)\lambda$



$\alpha_p = \frac{r_p}{f}$

$\therefore \alpha_p = \frac{r_p}{f} \qquad \cos\alpha_p \approx 1 - \frac{\alpha_p^2}{2}$

$\therefore 1 - \frac{\alpha_p^2}{2} = \frac{p\lambda}{2t}, \quad 1 - \frac{\alpha_{p+1}^2}{2} = \frac{(p+1)\lambda}{2t}$

$\therefore \frac{1}{2}(\alpha_p^2 - \alpha_{p+1}^2) = \frac{\lambda}{2t}$

$\therefore \alpha_p^2 - \alpha_{p+1}^2 = \frac{\lambda}{t}$

$\therefore \frac{r_p^2}{f^2} - \frac{r_{p+1}^2}{f^2} = \frac{\lambda}{t}$

$\Rightarrow \quad r_p^2 - r_{p+1}^2 = \frac{f^2\lambda}{t}$

(e) $2t\cos\alpha = p\lambda$

$\therefore \cos\alpha = \dfrac{p\lambda}{2t}$

$\because \cos\alpha \le 1 \Rightarrow \dfrac{p\lambda}{2t} \le 1$

$\therefore p \le \dfrac{2t}{\lambda} \qquad \therefore P_{max} = \dfrac{2t}{\lambda}$

$\therefore$ As t decreases, the maximum order P_{max} also decreases. Since the highest order of interference is that on the axis, when ~~we~~ we decrease t we see ~~that the highest order~~ less and less fringes and the highest order always disappear from ~~the one~~ the axis.

When ~~we~~ t is 0 there is no interference fringe. The observation plane receives constant illumination

When we increases t from the other direction the interference pattern re-emerges and more and more fringes appear

23. $\cos k_1x \cos k_2x = \frac{1}{2}[\cos[(k_1+k_2)x] + \cos[(k_1-k_2)x]]$

$\sin k_1x \sin k_2x = \frac{1}{2}[-\cos[(k_1+k_2)x] + \cos[(k_1-k_2)x]]$.

$\therefore \quad I(x) = 3I_0 + 3I_0 \cos k_1x \cos k_2x - I_0 \sin k_1x \sin k_2x$

$= 3I_0 + \frac{3}{2}I_0 \cos[(k_1+k_2)x] + \frac{3}{2}I_0 \cos[(k_1-k_2)x]$

$+ \frac{1}{2}I_0 \cos[(k_1+k_2)x] - \frac{1}{2}I_0 \cos[(k_1-k_2)x]$

$\cancel{= 2I_0(1+\cos[(k_1+k_2)x]) +}$

$= 2I_0(1+\cos[(k_1+k_2)x]) + I_0(1+\cos[(k_1-k_2)x])$

$\rightarrow$ sum of 2 patterns

(a) $\bar{\nu}_1 = \frac{k_1+k_2}{2\pi}$

$\bar{\nu}_2 = \frac{k_1-k_2}{2\pi}$

$\therefore \bar{\nu} = \frac{\bar{\nu}_1+\bar{\nu}_2}{2} = \frac{k_1}{2\pi} = \boxed{2.29\times10^6 \text{ m}^{-1}}$

(b) $\Delta\bar{\nu} = \bar{\nu}_1 - \bar{\nu}_2 = \frac{2k_2}{2\pi} = \frac{k_2}{\pi} = \boxed{1.11\times10^5 \text{ m}^{-1}}$

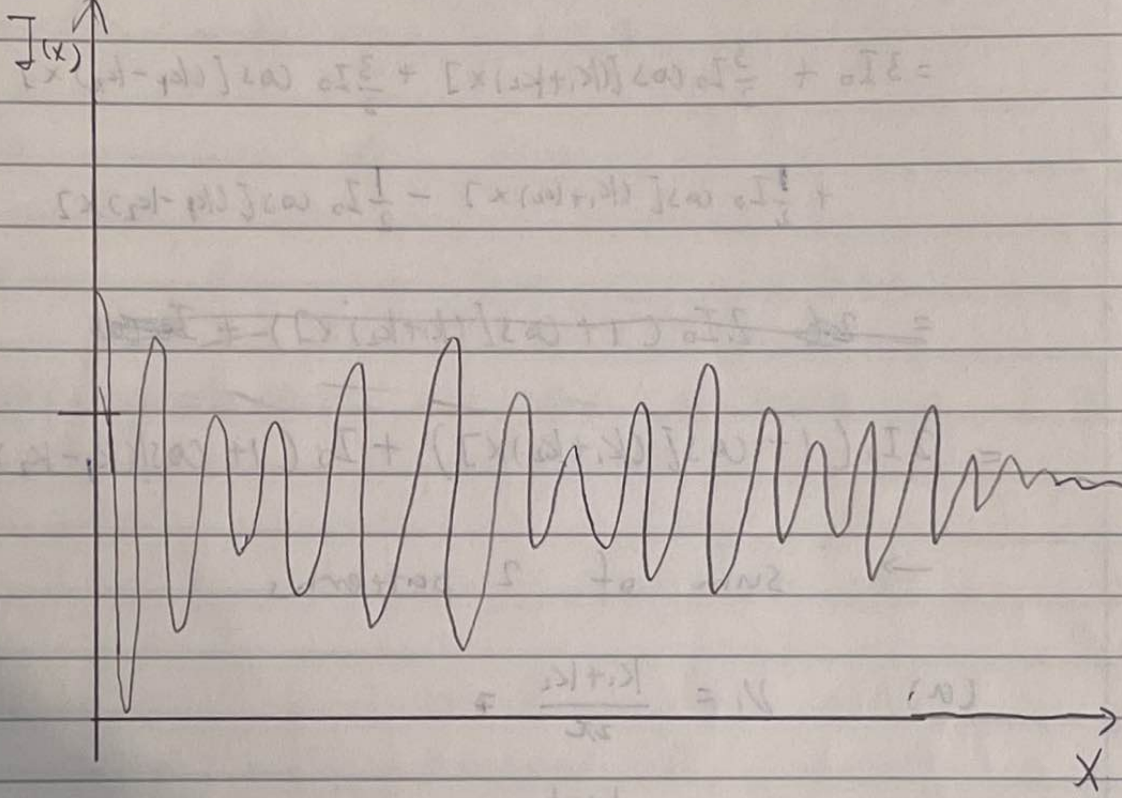
(c) relative intensity is

$= \frac{I_{\nu_1}}{I_{\nu_2}} = \frac{2I_0}{I_0} = \boxed{2}$

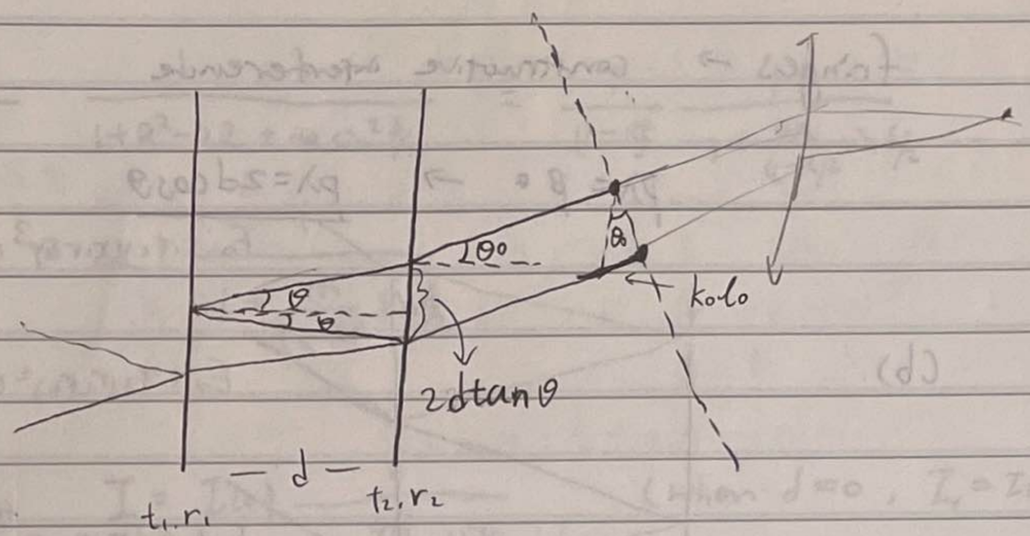
Consider the term

$$f(x) \propto \exp(-k_3^3 x^2)$$

$$k_3 = 5.02\ m^{-1} \ll k_2, k_1$$



24.



~~The optical path difference is~~ $\cancel{\frac{2d}{\cos\theta}}$

The optical path ~~phase~~ difference between two beams is

$\cancel{\Delta\phi = \frac{2kd}{\cos\theta} - k_0 l_0}$ $\quad \beta = \frac{2nd}{\cos\theta} - l_0$

$l_0 = 2d\tan\theta\sin\theta_0$, $n\sin\theta = \sin\theta_0$

$\cancel{k = \frac{2\pi n}{\lambda}, \quad k_0 = \frac{2\pi}{\lambda}}$

$\cancel{\Delta\phi}$ $\beta = \frac{2nd}{\cos\theta} - 2nd\tan\theta\sin\theta$

$= 2nd\left(\frac{1}{\cos\theta} - \tan\theta\sin\theta\right)$

$= 2nd\left(\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}\right)$

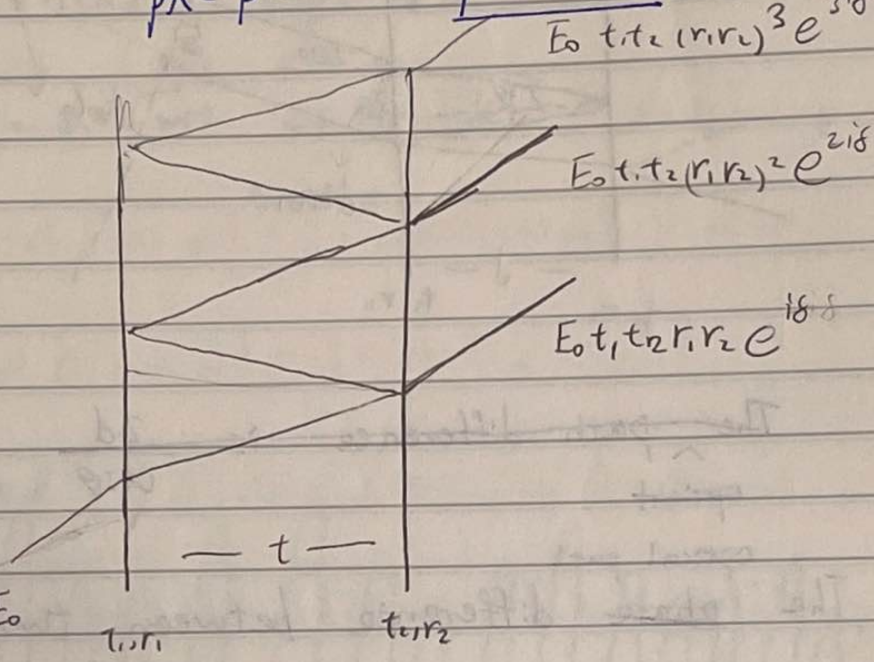
$= 2nd\left(\frac{1-\sin^2\theta}{\cos\theta}\right) = 2nd\left(\frac{\cos^2\theta}{\cos\theta}\right) = \cancel{2nd\sin\theta}$ $2nd\cos\theta$

For $n=1$ we have $\cancel{\beta = 2ds}$ $\beta = 2d\cos\theta$

fringes $\rightarrow$ constructive interference

$$p\lambda = \beta \rightarrow \underline{p\lambda = 2d\cos\theta}$$

(b)



$$\delta = k\beta = \frac{2\pi}{\lambda} 2nd\cos\theta \underbrace{=}_{d=t} \frac{2\pi}{\lambda} 2nt\cos\theta$$

$$E_t = E_0 t_1 t_2 + E_0 t_1 t_2 (r_1 r_2) e^{i\delta} + E_0 t_1 t_2 \left((r_1 r_2) e^{i\delta} \right)^2 + \dots$$

$$= \cancel{E_0 t_1 t_2 e^i} = E_0 t_1 t_2 \left(\frac{1}{1 - r_1 r_2 e^{i\delta}} \right)$$

$$I_t \propto |E_t|^2 = E_t E_t^* = E_0^2 t_1^2 t_2^2 \frac{1}{(1 - r_1 r_2 \cos\delta)^2 + \sin^2\delta\, r_1 r_2}$$

$$= (E_0 t_1 t_2)^2 \frac{1}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos\delta}$$

let $R = r_1 r_2$ where R is the reflection coefficient

then $I_t \propto \dfrac{1}{1 + R^2 - 2R\cos\delta} = \dfrac{1}{1 + R^2 - 2R(1 - 2\sin^2\frac{\delta}{2})}$

$$= \frac{1}{1+R^2-2R+4R\sin^2 \delta/2} = \frac{1}{(1-R)^2}\frac{1}{1+\frac{4R}{(1-R)^2}\sin\delta/2}$$

$$\propto \frac{1}{1+\frac{4R}{(1-R)^2}\sin\delta/2}$$

$$\therefore \quad I_t = I(0)\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\delta/2} \qquad (\text{when } \delta=0,\ I_t = I(0))$$

The Finesse is $F = \frac{\pi\sqrt{R}}{1-R}$

$$\therefore \frac{4F^2}{\pi^2} = \frac{4\pi^2 R^2}{(1-R)^2\pi^2} = \frac{4R^2}{(1-R)^2}$$

$$\Rightarrow \quad I(\phi) = I_0\left[1+\left(\frac{4F^2}{\pi^2}\right)\sin^2\left(\frac{\delta}{2}\right)\right]^{-1}$$

for $\delta = \left(\frac{2\pi}{\lambda}\right)2nt\cos\theta$

(C) $\boxed{F = \frac{\pi\sqrt{R}}{1-R}}$

maxima of transmitted intensity $2nd\cos\theta = p\lambda$

Find Full width half maximum

$\frac{I(\phi)}{I(0)} = \frac{1}{2}$ $\quad\therefore\quad$ $\frac{4F^2}{\pi^2}\sin^2\left(\frac{\delta}{2}\right) = 1$

$\Rightarrow$ $\cancel{\sin^2}$ $\sin\frac{\delta}{2} = \pm\frac{\pi}{2F}$ $\qquad$ let $\delta = 2\pi p + \varepsilon$

then $\pm\pi/2F = \sin\left(\frac{\delta}{2}\right) = (-1)^p\sin(\varepsilon/2)$

$\Rightarrow \quad \varepsilon = 2\sin^{-1}\left(\frac{\pm\pi}{2F}\right)$

For small ε, $\quad \varepsilon = 2\left(\pm\frac{\pi}{2F}\right) = \pm\frac{\pi}{F}$

$\text{FWHM} = 2|\varepsilon| = |\varepsilon - (-\varepsilon)| = \frac{2\pi}{F}$

$\therefore$ on the δ-axis.

$$\frac{\text{separation of peaks}}{\text{FWHM of one peak}} = \frac{2\pi}{2\pi/F} = F$$

$\rightarrow$ F measures the sharpness of the fringes.

25. (a)

As shown in 24. (c)

$$\varepsilon = \pm\frac{\pi}{2F} \quad \therefore \quad \delta = 2\pi p + \varepsilon = 2\pi p \pm \frac{\pi}{2F}$$

$$= 2\pi\left(p \pm \frac{\pi}{2F}\right)$$

(b) In the limiting case

$$(p+1)\lambda = p\lambda_F \qquad \nu = \frac{1}{\lambda} \qquad \nu_F = \frac{1}{\lambda_F}$$

$$\Rightarrow \cancel{\frac{p}{\nu_F} = \frac{p+1}{\nu_F}} \quad \frac{p}{\nu_F} = \frac{p+1}{\nu}$$

$$\Rightarrow \quad p\nu = (p+1)\nu_F$$

$$\Rightarrow \quad p(\nu - \nu_F) = \nu_F = \frac{1}{\lambda_F}$$

$$\delta\nu_{FSR} = \nu - \nu_F = \frac{1}{p\lambda_F} = \frac{1}{2nt\cos\theta} \approx \boxed{\frac{1}{2nt}}$$

(c) $\Delta\delta_{inst} = \dfrac{2\pi}{F}$ (calculated in 24 (c))

~~δ~~ For on-axis fringes, $\cos\theta = 1$

$$\delta = 2kd = 2\pi\nu 2dn$$

$$\Rightarrow \quad \Delta\delta_{inst} = \cancel{2\pi\nu} \; 2\pi \cdot 2dn\Delta\nu_{inst} = \frac{2\pi}{F}$$

$$\therefore \quad \Delta\nu_{inst} = \frac{1}{2nd} \times \frac{1}{F} = \boxed{\frac{1}{2nt} \times \frac{1}{F}}$$

$d = t$

27 $\bar{\nu}-\bar{\nu}' = \frac{1}{2d} \times \left(\frac{r_1^2 - r_2^2}{r_1^2 - r_3^2} + \text{integer} \right)$

choose $r_1 = 1.82\,mm$ $r_2 = 3.30\,mm$ $r_3 = 4.84\,mm$

knowing $d = 25\,mm$

$$\bar{\nu}-\bar{\nu}' = 7.535\,m^{-1} + \text{integer} \times \frac{1}{2 \times 0.025}\,m^{-1}$$

$$= (7.535 + 20 \times \text{integer})\,m^{-1}$$

$\because \Delta\nu_s = |\bar{\nu}-\nu'| \le 20\,m^{-1}$

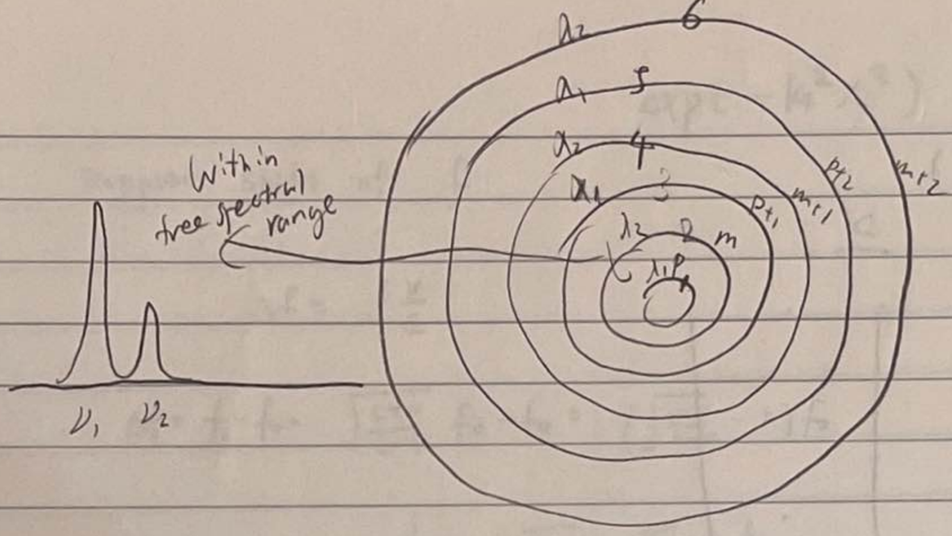
$\therefore (7.535 + 20 \times \text{integer}) \le 20\,m^{-1}$

$\therefore$ integer can be 0 or -1

$= 0 \Rightarrow \Delta\nu_s = 7.535\,m^{-1}$

$= -1 \Rightarrow \Delta\nu_s = 12.465\,m^{-1}$

To resolve the ambiguity, use another étalon with a different d, so ~~that~~ it will yield a different possible set of $\Delta\nu_s$. Vary ~~the~~ d many times, only one value ~~of~~ of $\Delta\nu_s$ will be possible for all the choice of d. We can then conclude that that $\Delta\nu_s$ is the actual $\Delta\nu_s$ we are trying to find.



$$r_p^2 - r_{p+1}^2 = \frac{\lambda_1}{d} f_1$$

$$\underline{r_p^2 = \left(2 - \frac{p\lambda}{d}\right) f^2} \qquad \cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$p\lambda = \delta = 2d\cos\theta \quad \propto 1 - \frac{\theta^2}{2}$$

$$f\theta \equiv r$$

$$\frac{\delta_2 - \delta_1}{\delta_3 - \delta_1} = \frac{\theta_1^2 - \theta_2^2}{\theta_1^2 - \theta_3^2}$$

$$= \frac{r_1^2 - r_2^2}{r_1^2 - r_3^2}$$

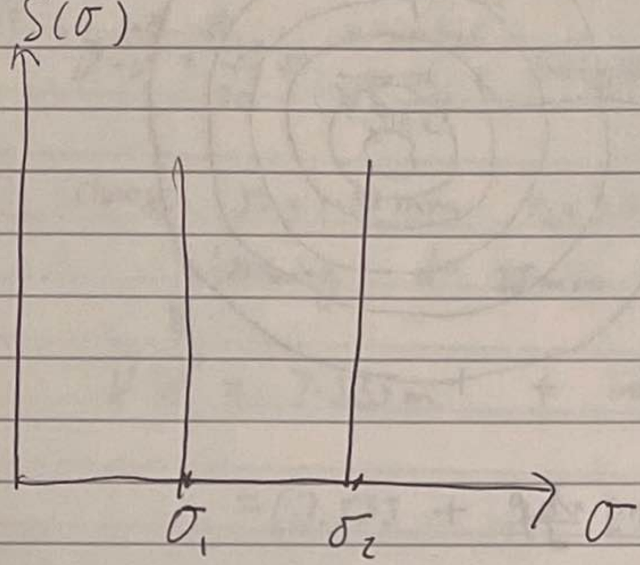
$$= \frac{p\lambda_2 - p\lambda_1}{(1+p)\lambda_1 - p\lambda_1} = \frac{p\lambda_2\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)}{d} = 2d\Delta\bar{\nu}$$

$2d$

$\cos\theta \sim 1$

1, 2 ring are of same order

2, 3 ring are of same order



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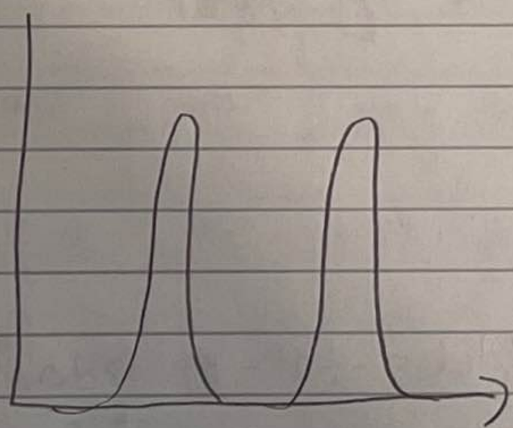

$\exp(-k_3^2 x^2)$

Doppler shift of f

c

$$\Delta f = f\frac{v}{c}$$

$$\Delta f = f_1 - f_0 = \sqrt{\frac{c+v}{c-v}}\, f_0 - f_0 = \left(\sqrt{\frac{c+v}{c-v}} - 1\right) f_0$$

$$= \frac{1}{\sqrt{c-v}}\left(\sqrt{c+v} - \sqrt{c-v}\right) f_0$$

$$= \frac{c}{\sqrt{c-v}}\left(\sqrt{1+\frac{v}{c}} - \sqrt{1-\frac{v}{c}}\right) f_0$$

$$= \frac{c}{\sqrt{c-v}}\left(1+\frac{v}{2c} - \left(1-\frac{v}{2c}\right)\right) f_0$$

$$\approx (1)\left(\frac{v}{c}\right) f_0 \sim \frac{v}{c} f_0$$

$\boxed{\Delta\nu}$

$$\Delta f \sim \frac{v}{c} f$$

$$\langle v \rangle = \sqrt{\frac{3k_B T}{m}}$$

$c = f\lambda$

$\frac{c}{f} = \lambda$

$\bar{\nu} =$

$k = 2\pi\bar{\nu}$

$P_\nu d\nu$

$p(\nu)\, d\nu$

$$\bar{\nu} = \frac{1}{\lambda} = \cancel{f} \cdot \frac{f}{c}$$

$$\therefore \bar{\nu} \propto f$$

$$\Delta\nu \propto \Delta f$$

$$\Delta f = \frac{v}{c} f$$

$$\Delta\nu \sim \frac{v}{c}\nu$$