To: Caroline Terquem

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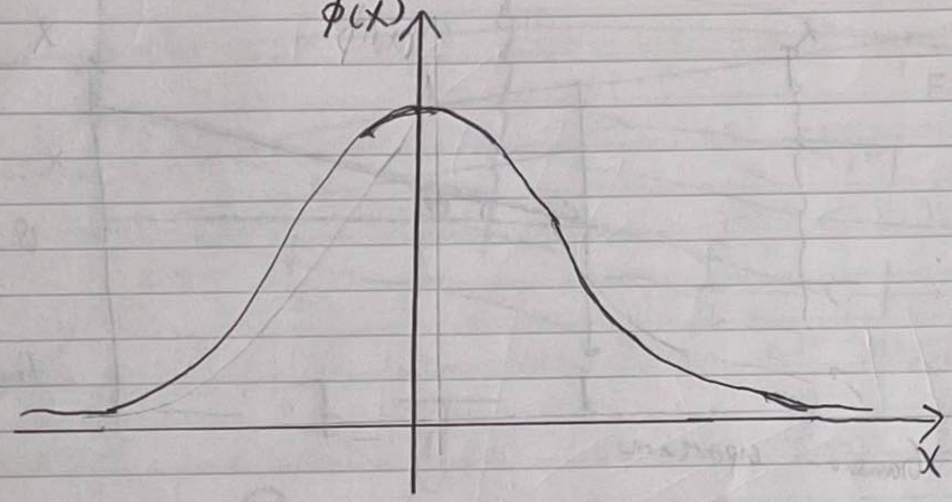
3

Optics (11, 12, 13, 17, 22, 23, 26, 28, 29)

11. Transmission function : $\phi(x) = \exp(-\frac{x^2}{2d})$



to get the amplitude distribution we take the Fourier transform of $\phi(x)$

$$u(\beta) \propto \int_{-\infty}^{\infty} \phi(x) e^{i\beta x}\, dx = \int_{-\infty}^{\infty} \exp(-\frac{x^2}{2d^2} + i\beta x)\, dx$$

$$= \int_{-\infty}^{\infty} \exp(-\alpha x^2 + i\beta x)\, dx = (\frac{\pi}{\alpha})^{\frac{1}{2}} \exp(-\frac{\beta^2}{4\alpha})$$

$$\underbrace{}_{\alpha = \frac{1}{2d^2}}$$

$$= \underbrace{\sqrt{2\pi d^2}}\, \exp(-\frac{\beta^2 d}{2})$$

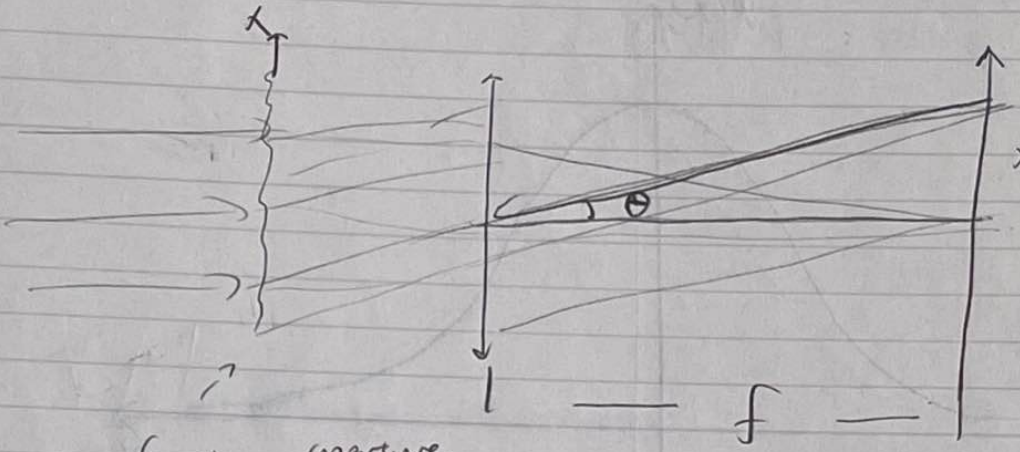
$$\frac{1}{4\alpha} = \frac{1}{4}(2d) = \frac{d}{2}$$

$$I(\beta) \propto |u(\beta)|^2 \propto \exp(-\beta^2 d^2)$$

$$\rightarrow \boxed{I(\beta) = I(0) \exp(-\beta^2 d^2)}$$

~~$(\beta = kd\sin\theta)$~~ $(\beta = k\sin\theta = \frac{2\pi}{\lambda}\sin\theta)$

$$\boxed{I(\theta) = I(0) \exp\left(-\frac{4\pi^2 d^2}{\lambda^2} \sin^2\theta\right)}$$



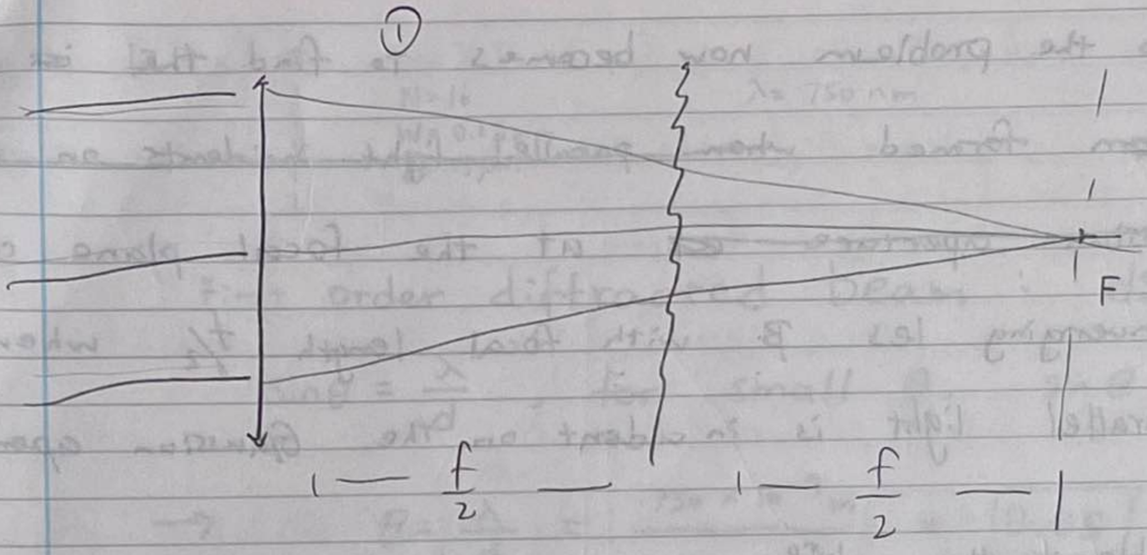
$\theta \approx \sin\theta \approx \tan\theta \approx \frac{x}{f}$

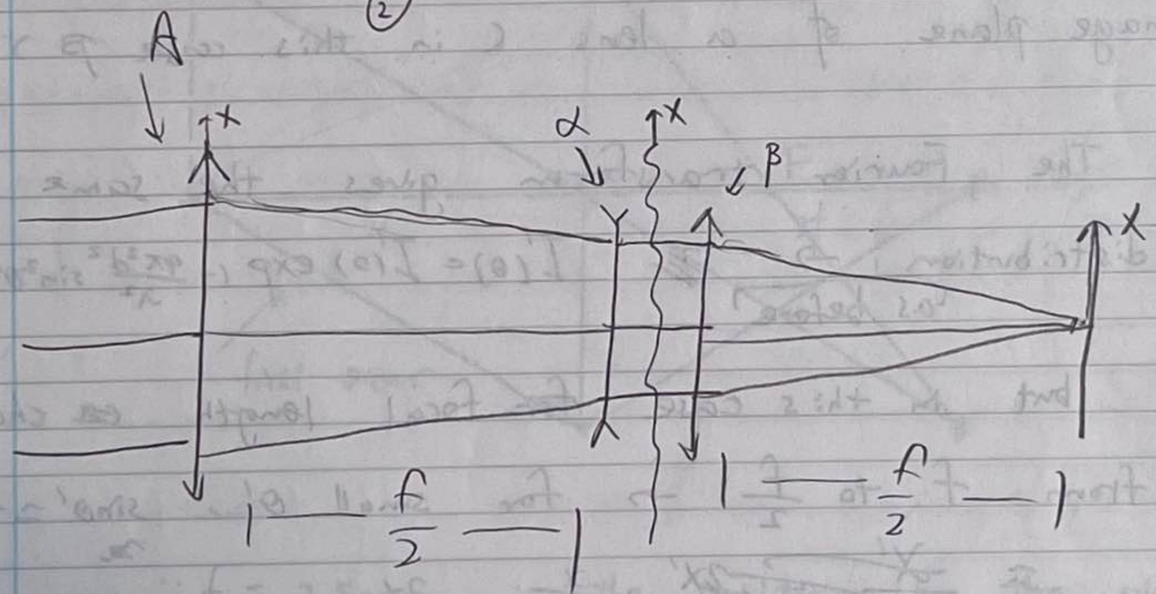
for small θ

For small θ, $\sin\theta \approx \tan\theta \approx \theta \approx \frac{X}{f}$ in the image plane

$$\therefore \quad \boxed{I(x) = I(0) \exp\left(-\frac{4\pi^2 d^2 X^2}{\lambda^2 f^2}\right)}$$

In practice, the use of Gaussian screen improves the image quality. Because the ~~inte~~ diffraction pattern produced by an incident Gaussian beam is also a ~~gau~~ Gaussian function, there is no successive maxima and minima, therefore no "wiggles". Hence image quality is improved (although in compensation for the loss of intensity)





The problem describes ①, and ② is the equivalent lens system of ①. α and β are ~~co~~ diverging and converging lenses, respectively, both with focal length $\frac{f}{2}$. This is because the problem requires that the Gaussian aperture to be placed half way between the lens $\hat{A}$ and the focal plane of ~~the lens~~ A

So the problem now becomes to find the pattern formed ~~when parallel light incidents on the Gaussian aperture~~ at the focal plane of converging les β with focal length $f/2$ when parallel light is incident on the Gaussian aperture.

Hence the diffraction is still Fraunhofer because it it equivalent to the diffraction ~~pattern~~ observed at the image plane of a lens (in this case β)

The Fourier transform gives the same angular distribution $\underbrace{,}_{\text{as before}}$ $\rightarrow$ $I'(\theta) = I'(0) \exp\left(-\frac{4\pi^2 d^2}{\lambda^2}\sin^2\theta'\right)$

but in this case ~~for~~ focal length changes from f to $\frac{f}{2}$ $\rightarrow$ for small θ', $\sin\theta' \simeq \tan\theta' \approx \theta'$

$$\theta \approx \cancel{\frac{x'}{(f/2)} = \frac{2x'}{f}} \quad \frac{x}{(f/2)} = \frac{2x}{f}$$

$$\therefore \cancel{I'(x') = I'(0)\exp\left(-\frac{16\pi^2 d^2 x'^2}{\lambda^2 f^2}\right)}$$

$$\therefore \boxed{I'(x) = I'(0)\exp\left(-\frac{16\pi^2 d^2 x^2}{\lambda^2 f^2}\right)}$$

$\therefore$ The new distribution has the same Gaussian shape as before but the standard deviation is reduced to ~~be~~ by half.

12.

$N = 16$

$W = 0.25\,mm$

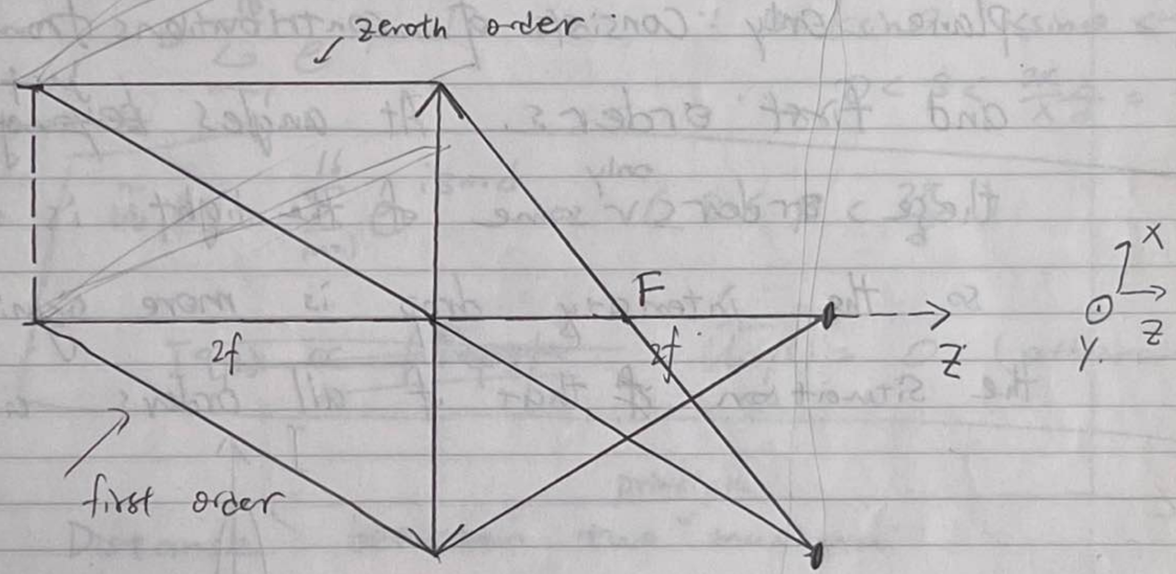
$d = 15\,\mu m$

$\lambda = 750\,nm$

First order diffracted beam : $d\sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{d}$, for small θ. $\sin\theta \simeq \theta$

$\rightarrow \quad \theta = \frac{\lambda}{d} = \frac{750 \times 10^{-9}\,m}{15 \times 10^{-6}\,m} = \boxed{0.05}$ rads.



$\because f = 2.5\,mm$ $\therefore$ the position in the plane of the mirror of first order diffracted beam is

$(2f)\cdot\theta = 5\,mm \times 0.05 = 0.25\,mm$ = radius of the lens

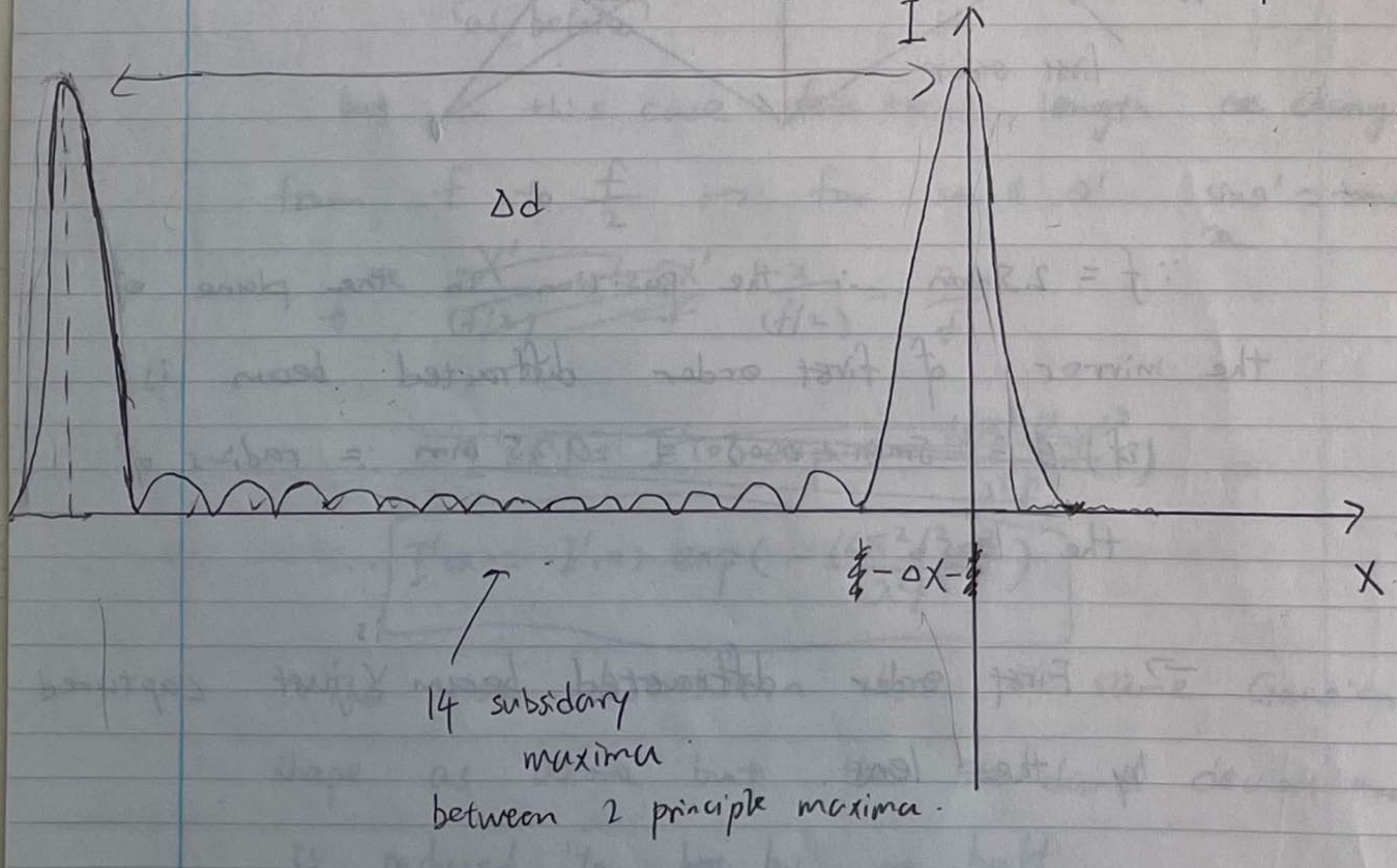
$\rightarrow$ First order diffracted beam is just captured by the lens.

In the plane at $z = 7.5\,mm$ (the focal pl

the amplitude distribution is the Fourier transfo

of the incident function set by the grating.

However, since only the zeroth and the first orders are picked up by the lens, the Fraunhofer diffraction pattern at the focal plane only consists of contributions from the zeroth and first orders. At angles ~~beyong~~ just beyond these orders only some of the light is picked up so the intensity drop is more significant than the situation ~~of~~ that if all orders are ~~pick~~ picked up



~~Fr~~ Ignoring the effect of partially picked up light,

incident function : $u(x) = \sum_{m=1}^{16} \delta(x-md)$

Fourier transform : $\cancel{\tilde{u}(\beta) \propto \int_{-\infty}^{\infty}}$

$$\tilde{u}(\beta) \propto \int_{-\infty}^{\infty} \sum_{m=1}^{16} \delta(x-md) e^{i\beta x}\, dx.$$

$$= \sum_{m=1}^{16} e^{i\beta m d}.$$

$\because \beta = k\sin\theta,\ 0 \leq \sin\theta \leq \frac{\lambda}{d}$

$\therefore 0 < \beta < \frac{2\pi}{\lambda}\frac{\lambda}{d} = \frac{2\pi}{d}$

$$\rightarrow \boxed{\begin{array}{ll} \tilde{u}(\beta) = \sum_{m=1}^{16} e^{i\beta m d} & \left(0 < \beta < \frac{2\pi}{d}\right) \\ \cancel{I(\beta) \propto |\tilde{u}(\beta)|^2} & \tilde{u}(\beta) = 0 \quad (\text{otherwise}) \end{array}}$$

Distance between two principle maxima

$$\Delta d = \frac{f\lambda}{d} = f\theta = 2.5\,\text{mm} \times 0.05 = \boxed{0.125\,\text{mm}}$$

Distance between ~~the~~ one principle maxima and ~~one~~ its adjacent minima.

$$\Delta x = \frac{f\lambda}{Nd} = \frac{f\theta}{N} = \frac{0.125}{16}\,\text{mm} = \boxed{7.81 \times 10^{-3}\,\text{mm}}$$

In ~~For~~ the plane at $Z = 10\,\text{mm}$ (the image plane), the amplitude distribution is the Fourier transform of the amplitude distribution at the focal plane.

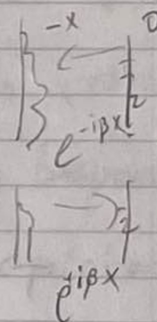
$$\therefore \quad \tilde{\tilde{u}}(x) \propto \int_{-\infty}^{\infty} \tilde{u}(\beta) e^{i\beta x}\, d\beta$$

same β because $u = v = 2f$ $\therefore m = \frac{v}{u} =$

$$= \int_0^{\frac{2\pi}{d}} \left(\sum_{m=1}^{16} e^{i\beta m d} \right) e^{i\beta x}\, d\beta$$

"+" sign again because inverse of negative transform is positive

$$= \int_0^{\frac{2\pi}{d}} \left(\sum_{m=1}^{16} e^{i\beta (x+md)} \right) d\beta$$



$$= \sum_{m=1}^{16} \int_0^{2\pi/d} e^{i\beta(x+md)}\, d\beta$$

$$= \sum_{m=1}^{16} \left[\frac{e^{i\beta(x+md)}}{i(x+md)} \right]_{\beta=0}^{\beta=\frac{2\pi}{d}}$$

$$= \sum_{m=1}^{16} \frac{e^{i\frac{2\pi}{d}x} e^{2\pi m i} - 1}{i(x+md)}$$

$$\cancel{= \sum_{m=1}^{\infty} \sum_{m=1}^{16} e^{i\beta x} - 1}$$

$$= \sum_{m=1}^{16} \frac{e^{\frac{2\pi i x}{d}} - 1}{i(x+md)}$$

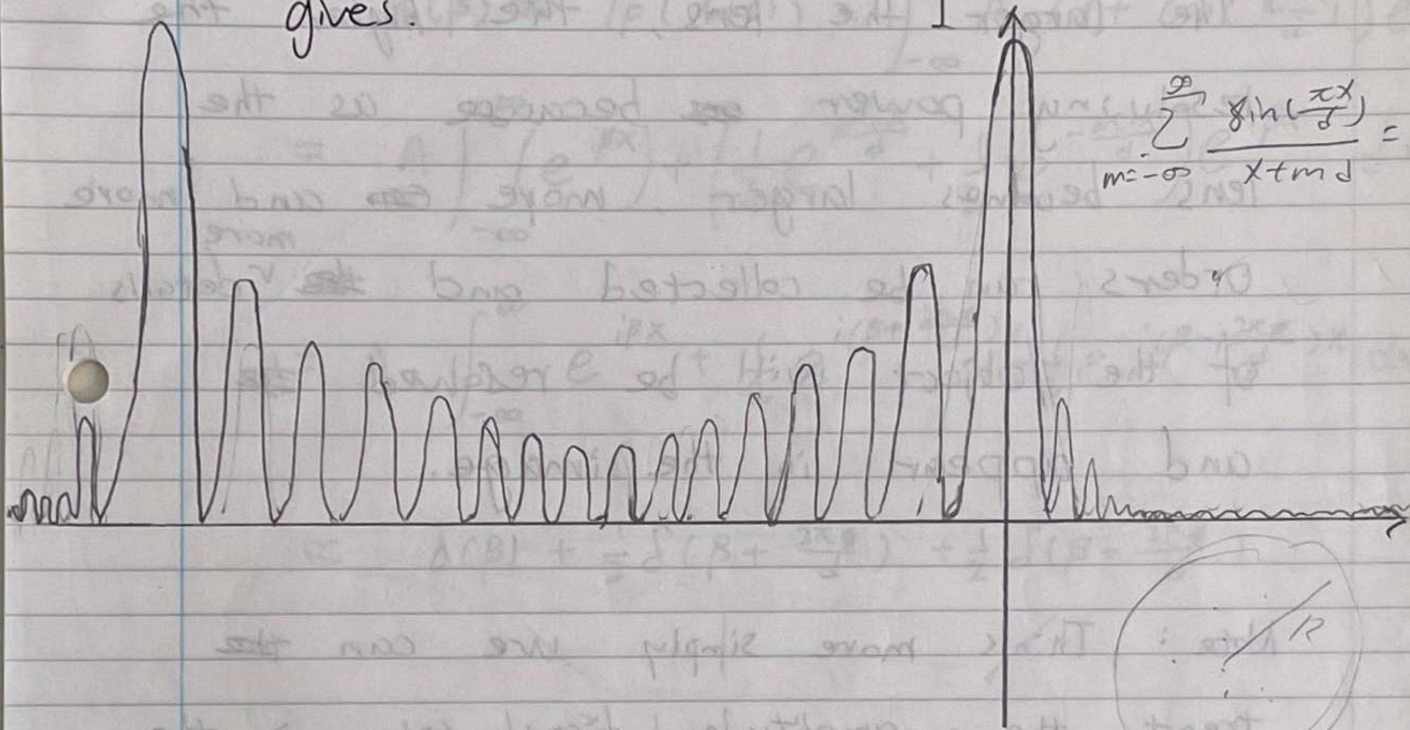
$$= \frac{e^{\frac{i\pi x}{d}}}{i} \sum_{m=1}^{16} \frac{e^{\frac{i\pi x}{d}} - e^{-\frac{i\pi x}{d}}}{x+md}$$

$$\propto \approx 2 \sum_{m=1}^{16} \frac{\sin\left(\frac{\pi x}{d}\right)}{x+md}$$

(equal in modulus)

$$\rightarrow \tilde{\tilde{u}}(x) \propto \sum_{m=-4}^{16} \frac{\sin(\frac{\pi x}{d})}{x+md}, \quad \tilde{\tilde{I}}(x) \propto |\hat{u}(x)|^2$$

Use computer to ~~plots~~ plots this function gives.



$$\sum_{m=-\infty}^{\infty} \frac{\sin(\frac{\pi x}{d})}{x+md} = \frac{\pi}{d}\cos(\frac{\pi x}{d})$$

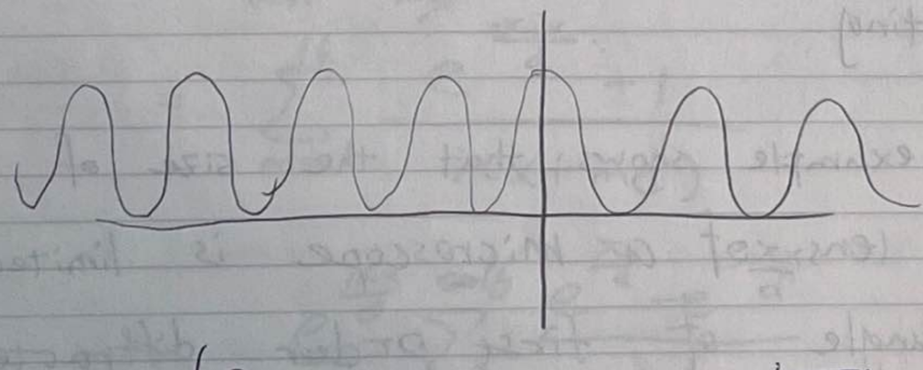
The distance between the peaks is $\frac{d}{2}$ (half the period of $\sin(\frac{\pi x}{d})$).

~~that~~ This shows that the resulting pattern ~~it~~ is the image of the grating.

→ This example shows that the size of the Object lens of a microscope is limited by the angle of first order diffracted light. If the lens is too small such that only one order is collected by the lens, then there is no diffraction and we cannot see the image. ~~The larger~~ i.e. the object cannot be resolved.

The larger the lens, the higher the resolving power ~~as~~ because as the lens becomes larger, more ~~an~~ and more orders can be collected and ~~the~~ more details of the object will be resolved ~~be~~ and appear in the image.

Note: Think more simply we can ~~the~~ treat the amplitude distribution in the Fourier plane as two ~~ligh~~ point sources (Delta functions) and the resulting pattern in the image plane will be a double-slit pattern. — a cosine function



(But in this case we ~~would~~ would treat $N \to \infty$ istead of 16.)

13.

$u(x) = A[1 + \cos(2\pi x/d)]$

the resulting pattern :

$$\tilde{u}(\beta) \propto T_F(u(x)) = \int_{-\infty}^{\infty} A\,(1+\cos(\tfrac{2\pi x}{d}))\,e^{i\beta x}\,dx$$

$$= A\int_{-\infty}^{\infty} (e^{i\beta x}) + \left(\frac{1}{2}e^{\frac{2\pi i x}{d}} + \frac{1}{2}e^{-\frac{2\pi i x}{d}}\right)e^{i\beta x}\,dx.$$

$$= A\int_{-\infty}^{\infty} e^{i\beta x} + \frac{1}{2}e^{i(\beta+\frac{2\pi}{d})x} + \frac{1}{2}e^{i(\beta-\frac{2\pi}{d})x}\,dx$$

$$\propto \delta(\beta) + \frac{1}{2}\delta(\beta+\tfrac{2\pi}{d}) + \frac{1}{2}\delta(\beta-\tfrac{2\pi}{d})$$

3 beams emerging

$\beta = k\sin\theta = \frac{2\pi}{\lambda}\sin\theta \approx \frac{2\pi\theta}{\lambda}$

0^{th} order : $\beta = 0 \rightarrow \boxed{\theta_0 = 0}$

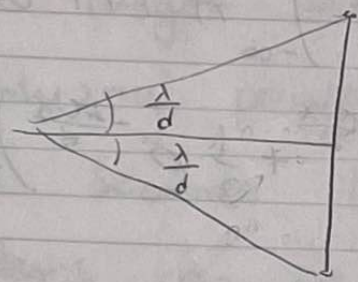
1^{st} order : $\beta = \frac{2\pi}{d} \rightarrow \frac{2\pi}{\lambda}\theta_1 = \frac{2\pi}{d}$

$$\rightarrow \boxed{\theta_1 = \frac{\lambda}{d}}$$

-1^{st} order : $\beta = -\frac{2\pi}{d} \rightarrow \frac{2\pi}{\lambda}\theta_{-1} = -\frac{2\pi}{d}$

$$\rightarrow \boxed{\theta_{-1} = -\frac{\lambda}{d}}$$

For the grating spacing to be seen in the image, ~~at least two~~ all three diffraction orders need to be collected by the lens.



$$\cancel{\sin\theta_{max} = \frac{\lambda}{d_{min}}}$$

$$\rightarrow \cancel{d_{max}}\ \cancel{d_{min} = \frac{\lambda}{\sin\theta_{max}} = \frac{465\,nm}{\sin 0.5}}$$

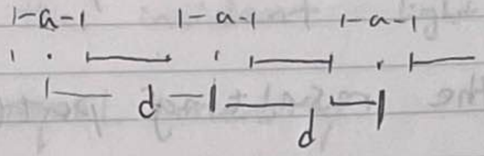
$$\cancel{= \boxed{970\,nm}}$$

$$\rightarrow \frac{2\lambda}{d_{min}} = \theta_{max} = 0.5\ rad$$

$$\rightarrow d_{min} = \cancel{\theta_{max}}\ \frac{2\lambda}{\theta_{max}}$$

$$= \frac{2 \times 465\,nm}{0.5} = \boxed{1.9\,\mu m}$$

17. $\cancel{\nu = \frac{1}{d} = 20/\text{mm}}$



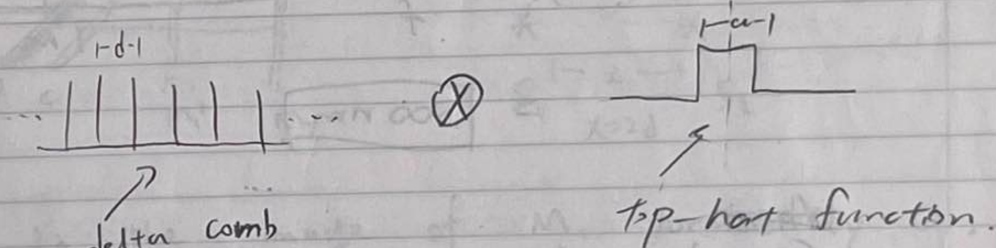
$\rightarrow d = 2a$

$\rightarrow$ spatial frequency : 20 slits / mm

$\therefore \; 20 \times 2a = 1\text{ mm}$

$\therefore \; a = \frac{1}{40}\text{ mm} = 25\ \mu m$, $d = 50\ \mu m$.

Treat this grating as an infinite grating with finite width for each slit.

$u(x) =$ 

$$\therefore T_F(u(x)) = T_F(\text{delta comb}_d) \times T_F(\text{top hat}_a)$$

$$u(x) = \sum_{m=-\infty}^{\infty} \delta(x - md) \otimes \text{rect}(x, a)$$

$$\boxed{\text{rect}(x,a) = \begin{cases} 1 & |x| < a \\ 0, & \text{otherwise} \end{cases}}$$

$\cancel{\hat{u}(\beta) \propto T_F(u(x)) = T}$

$$\hat{u}(\beta) \propto T_F(u(x)) = T_F\left(\sum_{m=-\infty}^{\infty} \delta(x - md)\right) \times T_F(\text{rect}(x,a))$$

$$= \left(\sum_{m=-\infty}^{\infty} \delta\left(\beta - \frac{2\pi m}{d}\right)\right) \times \text{sinc}\left(\frac{\beta a}{2}\right)$$

$$\beta = k\sin\theta \approx \frac{2\pi}{\lambda}\theta \approx \frac{2\pi}{\lambda}\frac{x}{f}$$

$\therefore$ $\cancel{\hat{u}(\beta)}$ the resulting pattern :

$$\tilde{u}(x) \propto \left(\sum_{m=-\infty}^{\infty} \delta\left(2\pi\left(\frac{x}{\lambda f} - \frac{m}{d}\right)\right)\right) \times \text{sinc}\left(\frac{\pi a x}{\cancel{2}\lambda f}\right)$$

$$= \left(\sum_{m=-\infty}^{\infty} \delta\left(2\pi\left(\frac{x}{\lambda f} - \frac{m}{2a}\right)\right)\right) \times \text{sinc}\left(\frac{\pi a x}{\cancel{2}\lambda f}\right)$$

First peak $m=1$: $\frac{x}{\lambda f} = \frac{1}{2a}$

$$\rightarrow \quad x_1 = \frac{\lambda f}{2a}$$

$$\rightarrow \quad \lambda = \frac{2 a x_1}{f} = \frac{2 \times 25 \times 10^{-6}\,m \times 10 \times 10^{-3}\,m}{1\,m}$$

$$= \boxed{500\,nm}$$

~~When~~ Second order $m=2$: $x_2 = \frac{2\lambda f}{2a} = \frac{\lambda f}{a}$.

~~sinc~~ but $\text{sinc}\left(\frac{\pi a x_2}{\cancel{2}\lambda f}\right) = \cancel{\text{sinc}\left(\frac{\pi}{2}\right)}$

$$= \text{sinc}(\pi) = 0$$

$\rightarrow \hat{u}(\beta) = 0 \rightarrow$ this order is missing

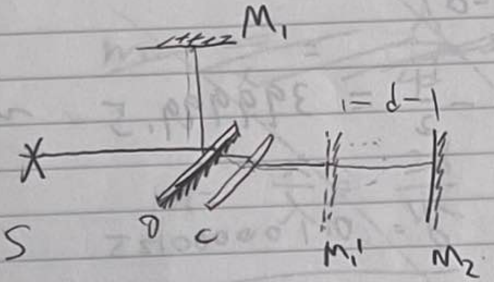
Third order $m=3$: $x_3 = \frac{3\lambda f}{2a}$

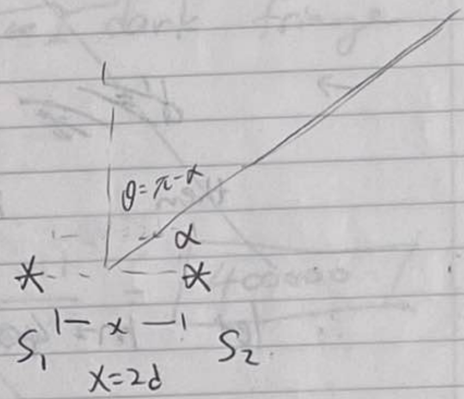
$\text{sinc}\left(\frac{\pi a x_3}{\lambda f}\right) \neq 0 \rightarrow$ order exists.

position : $x_3 = \frac{3\lambda f}{2a} = 3x_1 = \boxed{30\,mm}$

22. ~~22.~~ → The element O acts as a beam splitter that splits the incident light beam into two beams of equal intensity.

→ The ~~elec~~ element C ~~acts as a~~ is used to compensate for the optical ~~pass~~ path difference between the two beams ~~ca~~ caused by the thickness of the glass of the ~~elec~~ element O.



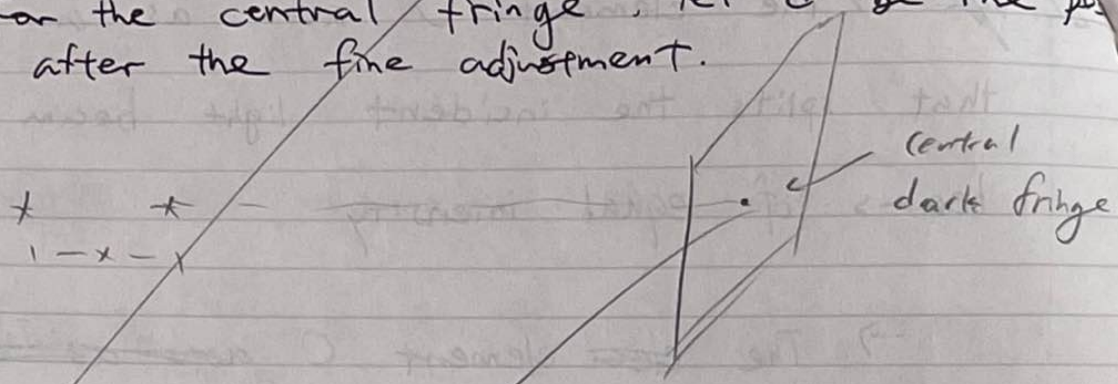


M_1' is the image of M_1 formed by O, ~~S~~ S_1 and S_2 are the images of the source S formed by M_1' and M_2 respectively

→ Interference pattern is formed by the two sources S_1 and S_2

→ The pattern can be observed ~~using~~ at the focal plane of a lens or at a screen that is in large distance from the inteferometer.

→ For the central fringe, let d' be the pos of M_2 after the fine adjustment.



Path difference $= x = (m+\frac{1}{2})\lambda$ (m = integer)

→ $2d' = (m+\frac{1}{2})\lambda$

→ $d' = \frac{\lambda}{2}(m+\frac{1}{2})$ $\quad (d' \sim 0.1\,m,\ m \in \mathbb{Z})$ $\lambda = 500\,nm$

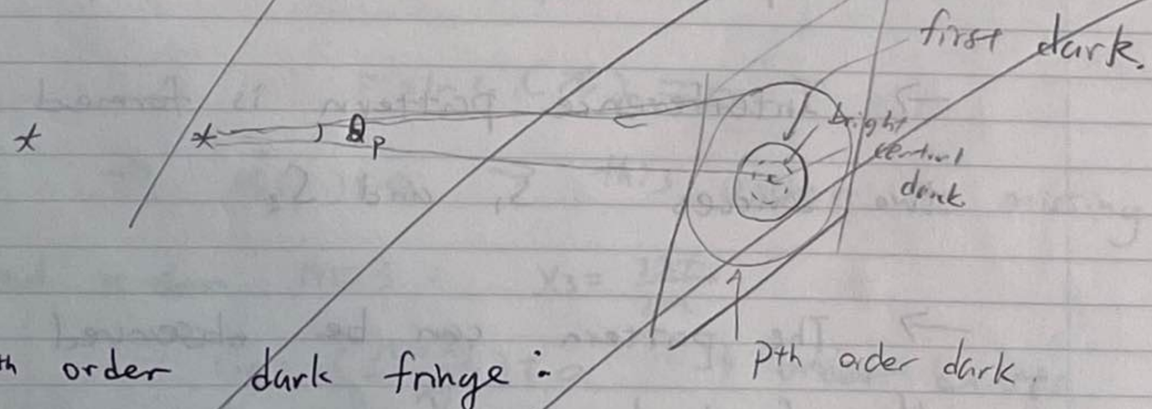
→ $\cancel{d' = \frac{\lambda}{2}}$ if $d' = d$

then $m \approx \frac{2d}{\lambda} - \frac{1}{2} = 399999.5 \sim 400000$

∴ let $m = 400000$, $d' = 0.100000125$

~~→ The order is 400000~~

→ The order is $\boxed{400000}$



Pth order dark fringe:

$$d'\cos\theta_p = \frac{\lambda}{2}\left(400000 + \frac{1}{2} - p\right) \quad ①$$

$$d' = \frac{\lambda}{2}\left(400000 + \frac{1}{2}\right) \quad ②$$

OR NOT !

There is an extra phase shift of π between the two beams because one ~~bou~~ bounces at the beam splitter from low index of refraction to high index of ~~re~~ refraction ~~and~~ whereas the other beam goes the other way around.

$\rightarrow$ $\cancel{d\cos\theta_p = m\lambda}$

$x = m\lambda$ is $\underbrace{\cancel{a}\ \text{I}}_{\text{the central}}$ dark fringe

$\cancel{m = \frac{\lambda}{x} = \frac{\lambda}{2d}}$

$$m = \frac{x}{\lambda} = \frac{2d}{\lambda} = \frac{2 \times 0.1}{500 \times 10^{-9}} = \boxed{400000}$$

Pth order dark fringe:

$$d\cos\theta_p = \frac{\lambda}{2}(400000 - P) \quad ①$$

$$d = \frac{\lambda}{2}(400000) \quad ②$$

$② - ① \rightarrow \quad 1 - \cos\theta_p = \frac{P\lambda}{2d}$

for small θ, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\rightarrow \quad 1 - (1 - \frac{\theta_p^2}{2}) = \frac{P\lambda}{2d}$$

$$\rightarrow \quad \theta_p^2 = \frac{P\lambda}{d} \quad \rightarrow \quad \boxed{\theta_p = \left(\frac{P\lambda}{d}\right)^{\frac{1}{2}}}$$

the width of wavelength is $\Delta\lambda = 1nm$

wavenumber difference is

$$\Delta \nu_L = \left| \frac{d\nu}{d\lambda} \right| \Delta\lambda = \frac{\Delta\lambda}{\lambda^2}$$

$\because \Delta\nu_L \approx \frac{1}{X_0}$ ← X_0 is the minimum path difference that brings the visibility to 0

$$\rightarrow X_0 = \frac{\lambda^2}{\Delta\lambda} = 2.5 \times 10^{-4} \, m$$

$(\lambda = 500 \, nm, \quad \Delta\lambda = 1 \, nm)$

$\because \quad X = 2d = 0.2 \, m$

$\therefore \quad X \gg X_0$

$\therefore$ Visibility of fringes $= 0$

→ Fringes do not exist.

Note : How to prove $\Delta\nu_L = \frac{1}{X_0}$

~~之後~~ $X_0 = m\lambda$

$X_0 = (m-1)(\lambda + \Delta\lambda)$

$\rightarrow$ $X_0 = m\lambda = m\lambda - \lambda + m\Delta\lambda - \Delta\lambda$

$(m-1)\Delta\lambda = \lambda$

$\frac{\lambda}{\Delta\lambda} \approx m$.

$\Delta\nu_L = \frac{\Delta\lambda}{\lambda^2}$

$\frac{1}{\Delta\nu_L} = \frac{\lambda^2}{\Delta\lambda} = m\lambda = X_0$

$\rightarrow$ $\Delta\nu_L = \frac{1}{X_0}$

order = 399202. $\leq$ 400000

differ by 783 ...

23. $I(x) = 3I_0 + 3I_0 \cos(k_1 x)\cos(k_2 x) - I_0 \sin(k_1 x)\sin(k_2 x)$

$= 3I_0 + 3I_0 \left\{ \frac{1}{2}\left[\cos[(k_1+k_2)x] + \cos[(k_1-k_2)x] \right] \right\}$

$- I_0 \left\{ \frac{1}{2}\left[\cos[(k_1-k_2)x] - \cos[(k_1+k_2)x] \right] \right\}$

$= 3I_0 + \frac{3}{2} I_0 \cos[(k_1+k_2)x] + \frac{3}{2} I_0 \cos[(k_1-k_2)x]$

$+ \frac{1}{2} I_0 \cos[(k_1+k_2)x] - \frac{1}{2} I_0 \cos[(k_1-k_2)x]$

$= 2I_0 (1 + \cos[(k_1+k_2)x]) + I_0 (1 + \cos[(k_1-k_2)x])$

$\rightarrow (k_1+k_2)x = 2\pi \bar{\nu}_1 (2x)$

$\rightarrow \bar{\nu}_1 = \frac{k_1+k_2}{4\pi} = \underline{1.17 \times 10^6 \ m^{-1}}$

$\rightarrow (k_1-k_2)x = 2\pi \bar{\nu}_2 (2x)$

$\rightarrow \bar{\nu}_2 = \frac{k_1-k_2}{4\pi} = \underline{1.12 \times 10^6 \ m^{-1}}$

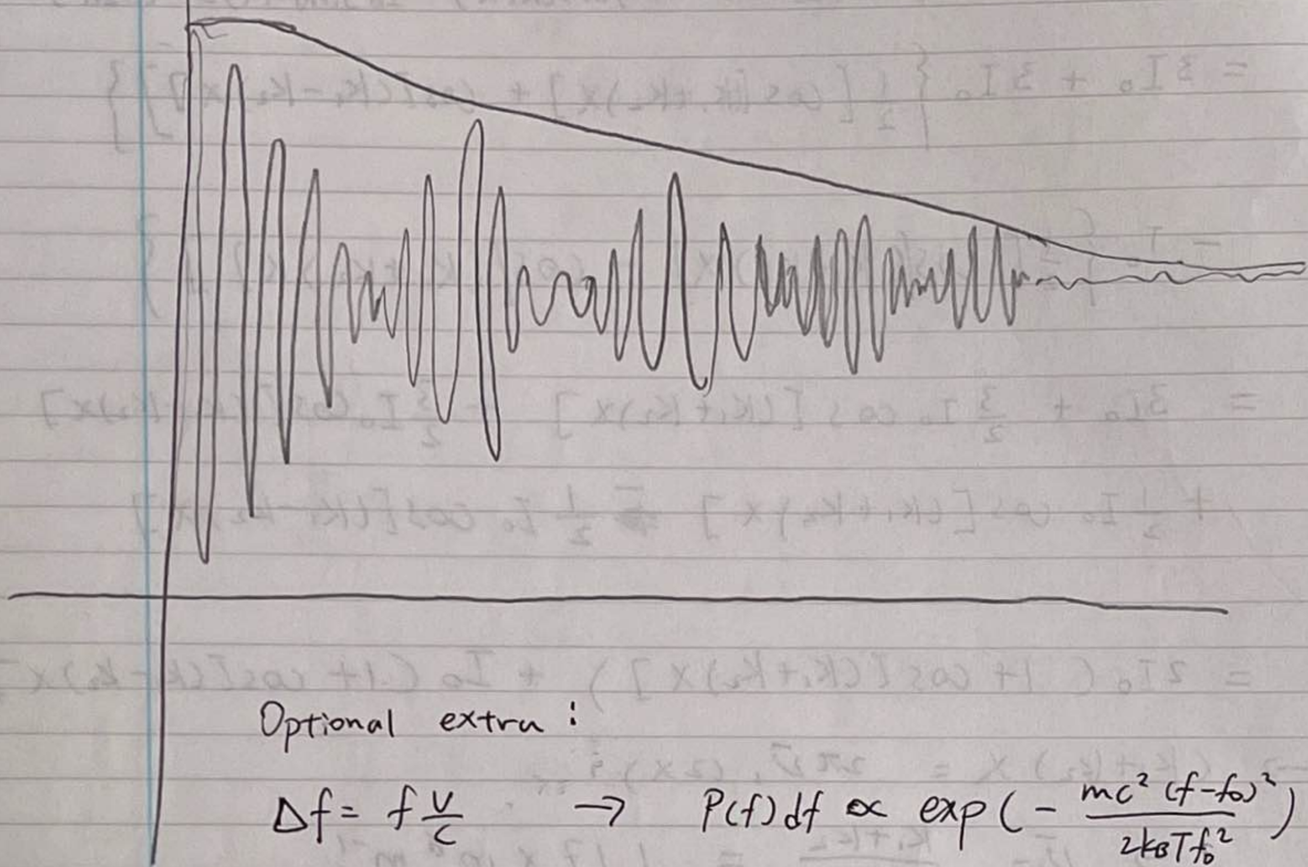
(a) $\bar{\nu} = \frac{\bar{\nu}_1 + \bar{\nu}_2}{2} = \frac{1}{4\pi} \cdot \frac{1}{2} \cdot (2k_1)$

$= \frac{k_1}{4\pi} = \boxed{1.15 \times 10^6 \ m^{-1}}$

(b) $\Delta\bar{\nu} = |\bar{\nu}_1 - \bar{\nu}_2| = \frac{k_2}{2\pi} = \boxed{5.5 \times 10^4 \ m^{-1}}$

(c) relative intensity $= \frac{I_{\bar{\nu}_1}}{I_{\bar{\nu}_2}}$

$= \frac{2I_0}{I_0} = \boxed{2}$



Optional extra:

$$\Delta f = f\frac{v}{c} \quad \to \quad P(f)\,df \propto \exp\left(-\frac{mc^2 (f-f_0)^2}{2k_BTf_0^2}\right) df$$

$$\because \quad f = 2\pi\bar{\nu} \quad \therefore \quad P(\bar{\nu}) \propto \exp\left(-\frac{mc^2 \Delta\nu^2}{2k_BT\nu^2}\right)$$

$(\Delta\bar{\nu} = \bar{\nu} - \bar{\nu}_0)$ (see Wikipedia for derivation)

$\therefore$ FWHM: $\Delta\bar{\nu} = \sqrt{\frac{8k_BT\ln 2}{Mc^2}}\,\bar{\nu}$

take M in atomic mass unit ($M_{Cs} = 133$)

$$\frac{\Delta\bar{\nu}}{\bar{\nu}} = 7.16\times10^{-7}\left(\frac{T}{M}\right)^{\frac{1}{2}}$$ due to doppler shift

To get the power spectrum, fourier cosine transform the function $f(x) \propto \exp(-k_3^2x^2)$

$$S_{\text{doppler}}(\bar{\nu}) \propto \int_0^\infty \exp(-k_3^2x^2)\cos(4\pi\bar{\nu}x)\,dx$$

$$\propto \int_{-\infty}^{\infty} \exp(-k_3^2x^2)\exp(4\pi i\bar{\nu}x)\,dx$$

$$\left(\text{using} \int_{-\infty}^{\infty} \exp(-\alpha x^2 + i\beta x)\,dx = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}}\exp\left(-\frac{\beta^2}{4\alpha}\right)\right)$$

$S(\bar{\nu})_{doppler} \propto \exp\left(-\frac{(4\pi\Delta\bar{\nu})^2}{4K_3^2}\right)$ ($\bar{\nu}$ becomes $\Delta\bar{\nu}$ because we take into account the cosine funcitons, which after transform becomes delta functions)

$$S(\bar{\nu})_{doppler} \propto \exp\left(-\frac{4\pi^2\Delta\bar{\nu}^2}{K_3^2}\right)$$

this distribution should be exactly the same as that calculated from doppler boardening

Full width Half Maximum?

$$\ln\left(\frac{1}{2}\right) = -\frac{4\pi^2\Delta\bar{\nu}_{\pm}^2}{K_3^2}$$

$$\rightarrow \Delta\tilde{\bar{\nu}} = \Delta\nu_+ - \Delta\nu_-$$

$$= 2 \times \left(\frac{K_3\sqrt{\ln 2}}{2\pi}\right)$$

$$= \frac{K_3\sqrt{\ln 2}}{\pi} = 1.33 \text{ m}^{-1}$$

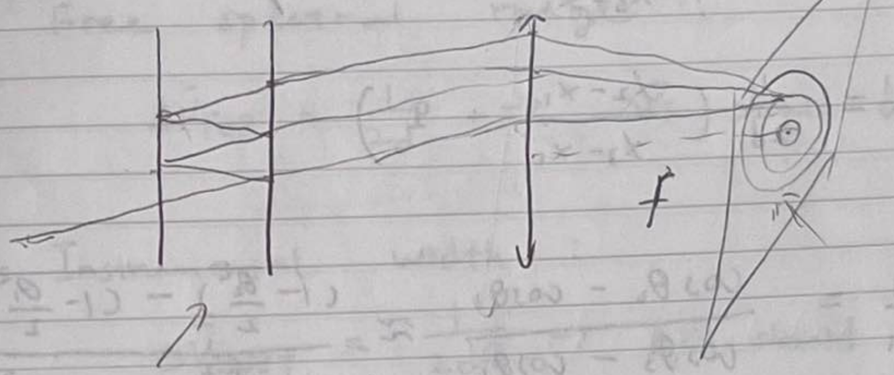
take $\bar{\nu} = \frac{\bar{\nu}_1 + \bar{\nu}_2}{2} = 1.15 \times 10^6 \text{ m}^{-1}$

then $\cancel{\frac{\Delta\bar{\nu}}{\bar{\nu}}}$ $T = M \times \cancel{7.16}\left[\left(\frac{\Delta\tilde{\bar{\nu}}}{\bar{\nu}}\right)/(7.16\times10^{-7})\right]^2$

$$\rightarrow T = 133 \times \left(\frac{1.33/1.15\times10^{6}}{7.16\times10^{-7}}\right)^2$$

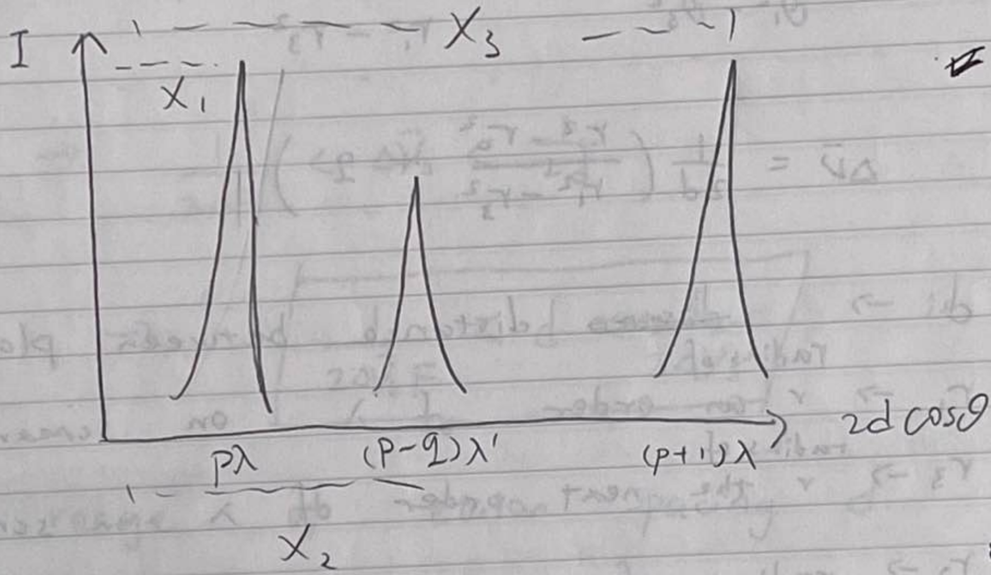
$$= \boxed{347\text{ K}}$$

26. (a)



Fabry - Perot etalon

$r = f\theta$, $p\lambda = 2d\cos\theta \approx 2d$ for pth order



Assume that the $(p-q)$ th order of λ' is between the p th and $p+1$ th ~~and~~ order of λ

then :

$$\frac{X_2 - X_1}{X_3 - X_1} = \frac{(p-q)\lambda' - p\lambda}{(p+1)\lambda - p\lambda} = \frac{p(\lambda'-\lambda) - q\lambda'}{\lambda}$$

$$\longrightarrow \frac{X_2 - X_1}{X_3 - X_1} + q\frac{\lambda'}{\lambda} = \frac{p(\lambda'-\lambda)}{\lambda}$$

$$= p\lambda' \frac{\lambda'-\lambda}{\lambda'\lambda} \approx 2d\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 2d(\bar{\nu} - \bar{\nu}')$$

$$= 2d\,\Delta\bar{\nu}$$

$$q\frac{\lambda'}{\lambda} \approx q \quad \rightarrow q \text{ is an integer}$$

$$\rightarrow \quad \Delta\bar{\nu} = \frac{1}{2d}\left(\frac{x_2 - x_1}{x_3 - x_1} + \text{integer}\right)$$

$$= \frac{1}{2d}\left(\frac{x_2 - x_1}{x_3 - x_1} + q\right)$$

$$\frac{x_2 - x_1}{x_3 - x_1} = \frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_3 - \cos\theta_1} \approx \frac{(1 - \frac{\theta_2^2}{2}) - (1 - \frac{\theta_1^2}{2})}{(1 - \frac{\theta_3^2}{2}) - (1 - \frac{\theta_1^2}{2})}$$

$$= \frac{\theta_1^2 - \theta_2^2}{\theta_1^2 - \theta_3^2} \approx \frac{r_1^2 - r_2^2}{r_1^2 - r_3^2}$$

$$\rightarrow \quad \Delta\bar{\nu} = \frac{1}{2d}\left(\frac{r_1^2 - r_2^2}{r_1^2 - r_3^2} + q\right)$$

$d \rightarrow$ ~~distance~~ distance between plates.

$r_1 \rightarrow$ radius of r an order of λ on screen

$r_3 \rightarrow$ radius of r the next order of λ on screen

$r_2 \rightarrow$ radius of an order of λ' between r_1 and r_3

which circle belongs to which wavelength can be seen by fitting the ring pattern to the Airy function

$\rightarrow$ If the wavelength difference $\Delta\bar{\nu}$ is within the free spectral range $\frac{1}{2d}$, then

q can only be 0 or -1

We can use another Fabry-Perot etalon with a different d to resolve the ambiguity.

(b)

Free spectral range:

$$\Delta\bar{\nu}_{FSR} = \frac{1}{2d} \rightarrow \Delta\bar{\nu}_s \text{ should be within this}$$

Instrumental width:

$$\Delta\bar{\nu}_{INST} = \frac{1}{2dF} \rightarrow \Delta\bar{\nu}_s \text{ should be greater than this}$$

(F is the Finesse)

$$\rightarrow \frac{1}{2dF} < \Delta\bar{\nu}_s < \frac{1}{2d}$$

$$\rightarrow \boxed{\frac{1}{2\Delta\bar{\nu}_s F} < d < \frac{1}{2\Delta\bar{\nu}_s}}$$ is the permitted range of etalon spacing d.

$\because$ $\Delta\nu_s$ is within $\frac{1}{2d} = \Delta\bar{\nu}_{FSR}$

$\therefore$ g can only be 0 or -1.

If 0 and -1 yield the same value than there is no ambiguity

↳ absolute value for $\Delta\nu_s$

$$\rightarrow \frac{x_2 - x_1}{x_3 - x_1} = \frac{1}{2}$$

$$\rightarrow \frac{1}{2} = 2d\,\Delta\nu_s \rightarrow \boxed{d = \frac{1}{4\Delta\bar{\nu}_s}}$$

is the suitable value for d I suggest

For sodium D-lines $\lambda = 589$ nm $\Delta\lambda = 0.6$ nm

$$\Delta\bar{\nu}_s = \frac{\Delta\lambda}{\lambda^2} = 1.73\times10^3\ m^{-1}$$

$$\rightarrow \quad d = \frac{1}{4\Delta\bar{\nu}_s} \sim \boxed{1\times10^{-4}\ m}$$

(c) $R \sim 1$ let $R = 1-\varepsilon$

$$F = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{1-\varepsilon}}{\varepsilon} \approx \frac{\pi(1-2\varepsilon)}{\varepsilon} \approx \frac{\pi}{\varepsilon}$$

$$= \frac{\pi}{1-R}$$

F is in fact monotonically increasing with R

$$\because \quad d \geq \frac{1}{2\Delta\bar{\nu}_s F} \rightarrow 2\Delta\bar{\nu}_s F d \geq 1$$

$\therefore$ Minimum $R \rightarrow$ min $F \rightarrow$ ~~max~~ $\cancel{\frac{1}{const}}$

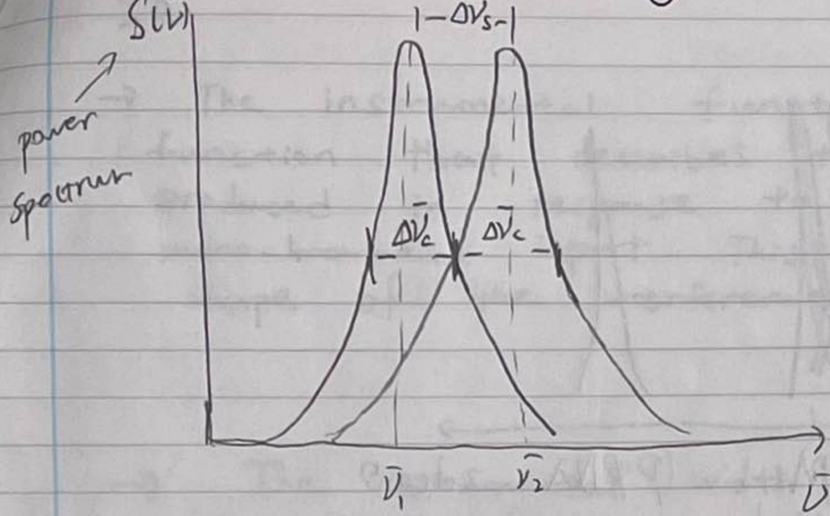
$\rightarrow 2\Delta\bar{\nu}_s F d = 1$

$$\therefore \frac{2\Delta\bar{\nu}_s d\pi}{1-R} = 1$$

$$\rightarrow \quad 1-R = \cancel{2\Delta\bar{\nu}_s}\ 2\pi d\Delta\bar{\nu}_s$$

$$\rightarrow \quad \boxed{R \approx 1 - 2\pi d\Delta\bar{\nu}_s}$$

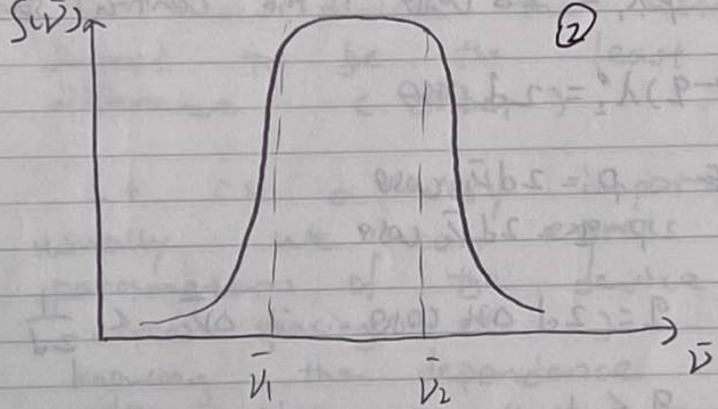
1

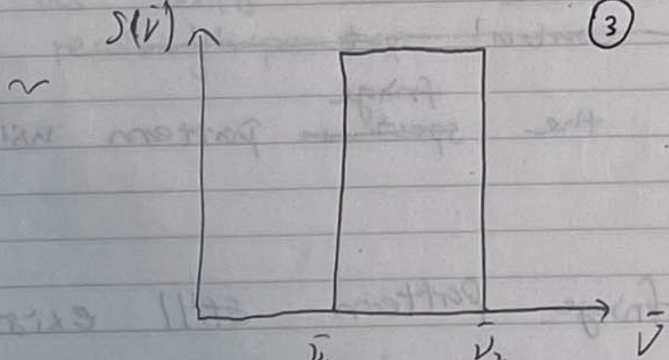


$$\Delta \bar{\nu}_c = \Delta \bar{\nu}_s$$

$$\Delta \bar{\nu}_s = |\bar{\nu}_2 - \bar{\nu}_1|$$

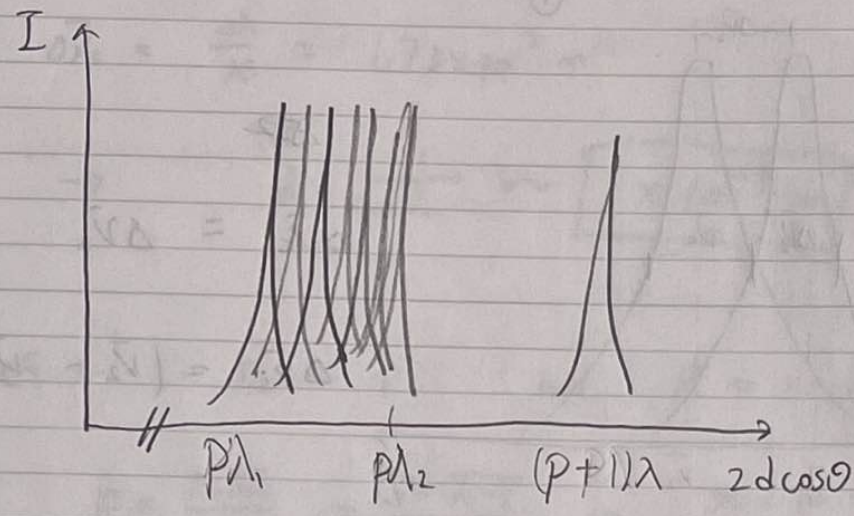
$\Delta \bar{\nu}_c$ due to boardening





1 becomes 2, which is similar to 3

We know $\Delta \bar{\nu}_s = |\bar{\nu}_2 - \bar{\nu}_1| < \Delta \bar{\nu}_{FSR} = \frac{1}{2d}$



consider $\quad p\lambda_1 = 2d\cos\theta$ (the central spot. $\theta = 0$)

$$(p-q)\lambda_2 = 2d\cos\theta$$

$\rightarrow$ $\quad p = 2d\bar{\nu}_1\cos\theta$

$$p - q = 2d\bar{\nu}_2\cos\theta$$

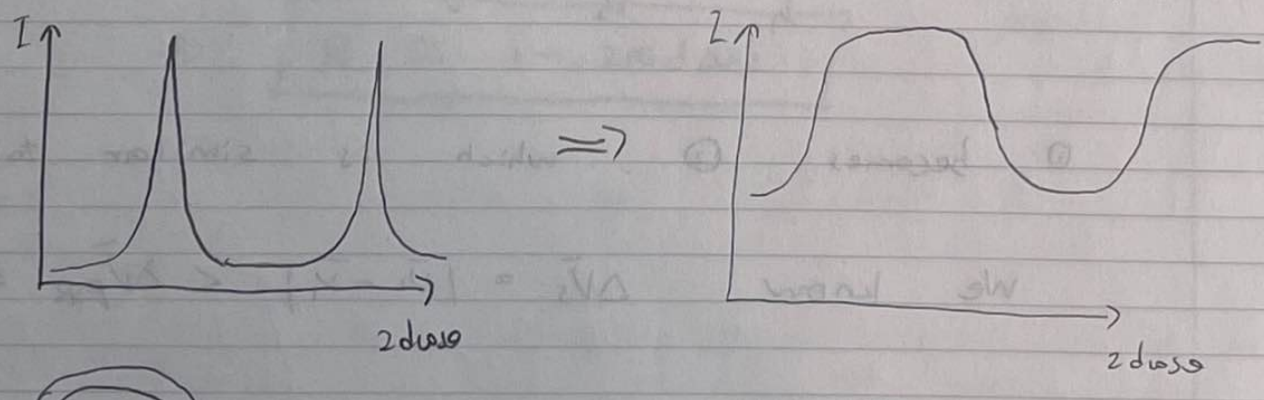
$$q = 2d\,\Delta\bar{\nu}_s\cos\theta \quad \because \Delta\bar{\nu}_s < \frac{1}{2d}$$

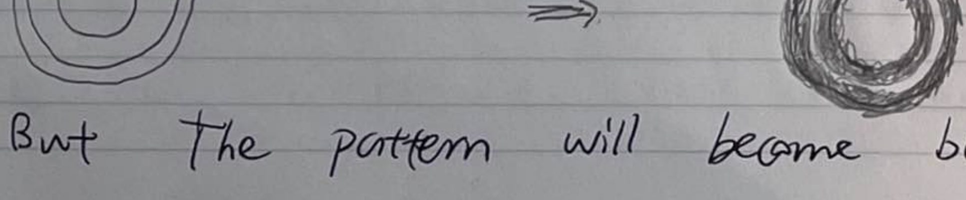
$$\therefore \quad q < 1$$

$\therefore$ ~~Even the central spot will~~

$\rightarrow$ No where in the ~~spectrum~~ fringe pattern will have order overlap

$\rightarrow$ The fringe pattern still exists.





But The pattern will become blurred

28.

→ The instrumental function is the mathematical function that describes the shape of the spectrum produced in response to a delta function or monochromatic input. This function describes the shape of the interference pattern i.e. a spectral line.

→ The instrumental width is the width of this instrumental function defined in some arbitrary way

In most devices the instrumental width is defined to be the least resolvable wavenumber difference ($\Delta\bar{\nu}_{INST}$)

It is a very important property because usually we can express it in terms of the parameters of the device (~~width~~ # of slits, etalon spacing , etc ...) , so that ~~___~~ ~~___~~ knowing the dependence of the instrumental ~~width~~ width on those parameters will ~~__~~ help us build more accurate devices with higher resolving power.

29. $Np \geq \frac{\lambda}{\Delta\lambda}$ (resolution of grating

→ assume $p \approx 1$, $N \geq \frac{\lambda}{\Delta\lambda} = \frac{600\,nm}{0.04\,nm} = 15000$

Spacing $d = \frac{1}{500\,mm^{-1}} = 2 \times 10^{-6}\,m$

min width of grating → $W = Nd = 15000 \times 2 \times 10^{-6}\,m$

$= \boxed{0.03\,m}$

~~W ≤ Nd~~

→ For maximum order $d\underbrace{\sin\theta}_{\sim 1} = \cancel{max}\, p\lambda$

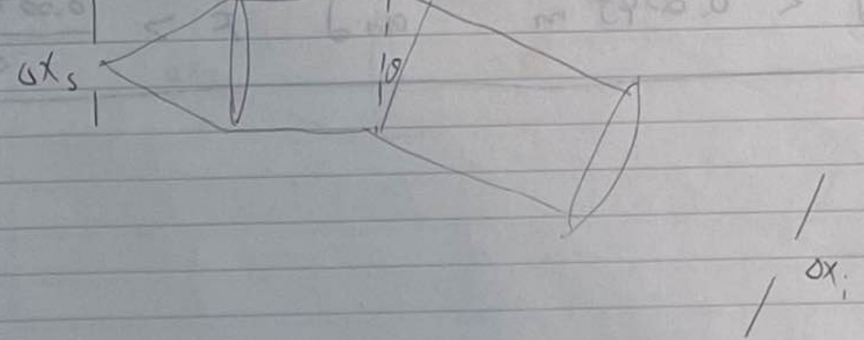
$\therefore p = \frac{d}{\lambda}$

$Np \geq \frac{\lambda}{\Delta\lambda} \rightarrow Nd \geq \frac{\lambda^2}{\Delta\lambda}$

∴ min width of grating

$W = \frac{\lambda^2}{\Delta\lambda} = \boxed{0.009\,m}$

→ Maximum slit width for the grating spectrograph.



$\Delta x_s = \frac{f_1\lambda}{Nd} = \frac{f_1\lambda}{W} = \frac{(500 \times 10^{-3})(600 \times 10^{-9})}{0.03}$

$= \boxed{1 \times 10^{-5}\,m}$

→ For Michelson interferometer if $x = x_{max}$

$$\Delta\bar{\nu} = \frac{1}{x} \rightarrow \frac{\Delta\lambda}{\lambda^2} = \frac{1}{x}$$

$$\rightarrow \quad x = \frac{\lambda^2}{\Delta\lambda} = \boxed{0.009\ m}$$

$$d = \frac{x}{2} = \boxed{4.5\ \cancel{mm}\ m}$$

→ For a Fabry - Perot interferometer

$$\Delta\bar{\nu}_s = \frac{\Delta\lambda}{\lambda^2} = 111\ m^{-1}$$

Q 26. (b) gives $\dfrac{1}{2\Delta\bar{\nu}_s F} < d < \dfrac{1}{2\Delta\bar{\nu}_s}$

$$\rightarrow \quad d < \frac{1}{2 \times 111 m^{-1}} = 0.0045\ m$$

$$F > \cancel{\frac{1}{2\Delta\bar{\nu}_s} d}\ \frac{1}{2\Delta\bar{\nu}_s d} = \frac{0.0045 m}{d}$$

∴ Fabry - Perot interferometer is a good choice if the etalon ~~space~~ spacing d and Finesse F ~~satisfies~~ satisfy

$$d < 0.0045\ m \quad \text{and} \quad F > \frac{0.0045 m}{d}$$

29. $\lambda = 600\,nm$ $\Delta\lambda = 0.04\,nm$

$\frac{1}{d} = 500$ lines / mm^{-1} $\rightarrow$ $d = \frac{1}{500 \times 1000} m = 2\times10^{-6} m$

minimum number of slits (N) needed is

$\frac{\lambda}{\Delta\lambda} = mN$, where m is the maximum order of interference

$\because$ $d\sin\theta = m\lambda$ and $\sin\theta \leq 1$

~~$\therefore m = [\frac{\lambda}{d}]$ $m = \lfloor \frac{\lambda}{d} \rfloor = \lfloor \frac{\ }{}$ -6~~

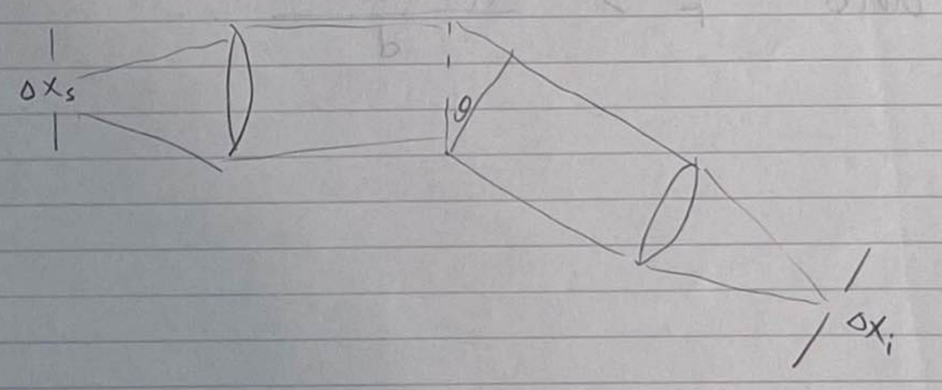
$\rightarrow$ $m = \lfloor \frac{d}{\lambda} \rfloor = \lfloor \frac{2\times10^{-6}m}{600\,nm} \rfloor = \lfloor 3.33 \rfloor = 3$

(" $\lfloor x \rfloor$ " = maximum integer below x)

$\rightarrow$ $3N = \frac{\lambda}{\Delta\lambda}$

$\rightarrow$ $w = Nd = \frac{\lambda d}{3\Delta\lambda} = \frac{(600nm)(2\times10^{-6}m)}{(0.04nm)(3)} = \boxed{0.01m}$

is the minimum width of the grating



$\rightarrow$ Maximum slit width for the grating spectrograph :

$\Delta x_s = \frac{f_1\lambda}{Nd} = \frac{f_1\lambda}{w} = \frac{(500\times10^{-3})(600\times10^{-9})}{0.01} m$

$= \boxed{3.0\times10^{-5}\,m}$

→ For Michelson interferometer if $x = x_{max}$

$$\Delta\bar{\nu} = \frac{1}{x} \rightarrow \frac{\Delta\lambda}{\lambda^2} = \frac{1}{x} = \frac{1}{2d}$$

$$\rightarrow d = \frac{\lambda^2}{2\Delta\lambda} = \boxed{4.5 \times 10^{-3}\ m}$$ is the distance necessary to scan.

→ For a Fabry - Perot interferometer,

$$\Delta\bar{\nu}_s = \frac{\Delta\lambda}{\lambda^2} = 111\ m^{-1}$$

Q26 (b) gives $\frac{1}{2\Delta\bar{\nu}_s F} < d < \frac{1}{2\Delta\bar{\nu}_s}$

$$\rightarrow d < \frac{1}{2 \times 111 m^{-1}} = 4.5 \times 10^{-3}\ m$$

$$F > \frac{1}{2\Delta\bar{\nu}_s d} = \frac{0.0045 m}{d}$$

∴ Febry - Perot interferometer is a good Choice if the etalon spacing d and Finesse F satisfy $d < 4.5 \times 10^{-3}\ m$ and $F > \frac{4.5 \times 10^{-3} m}{d}$.