To : Caroline Terquem

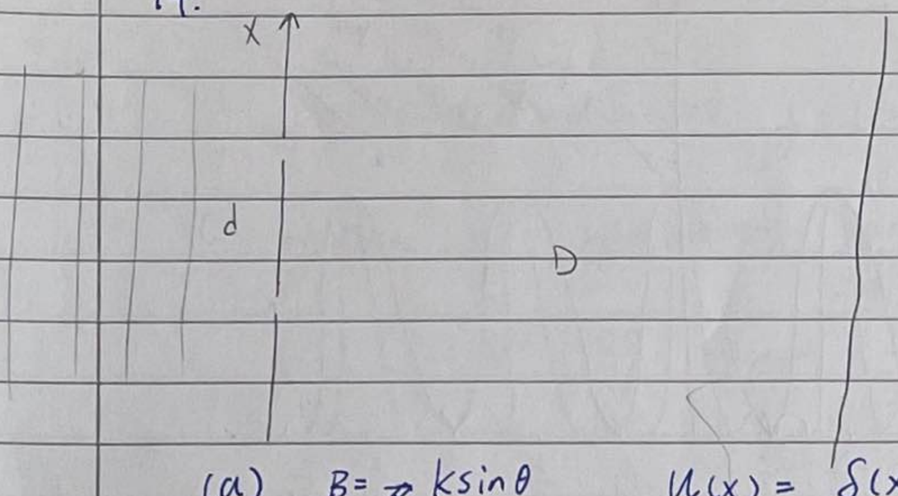
2

Optics (14. 15. 16. 18. 19) Ziyan Li

Very good!

Pb 19 – b, c has to be redone.

14.



(a) $\beta = \cancel{2} k\sin\theta \qquad u(x) = \delta(x-\frac{d}{2}) + \delta(x+\frac{d}{2})$

Fourier transform :

$$u(\beta) \propto \int_{-\infty}^{\infty} u(x) e^{i\beta x} dx = \int_{-\infty}^{\infty} \left(\delta(x-\tfrac{d}{2}) + \delta(x+\tfrac{d}{2})\right) e^{i\beta x} dx$$

$$= e^{i\beta d/2} + e^{-i\beta d/2} = 2\cos(\tfrac{1}{2}\beta d)$$

$$\therefore \quad u(\beta) = u_0 \cos(\tfrac{1}{2}\beta d)$$

$$\Rightarrow \quad I(\beta) = I_0 \cos^2(\tfrac{1}{2}\beta d) \checkmark$$

$$\tfrac{1}{2}\beta d = \tfrac{1}{2}(\tfrac{2\pi}{\lambda})\sin\theta \, d = \frac{\pi d}{\lambda}\sin\theta$$

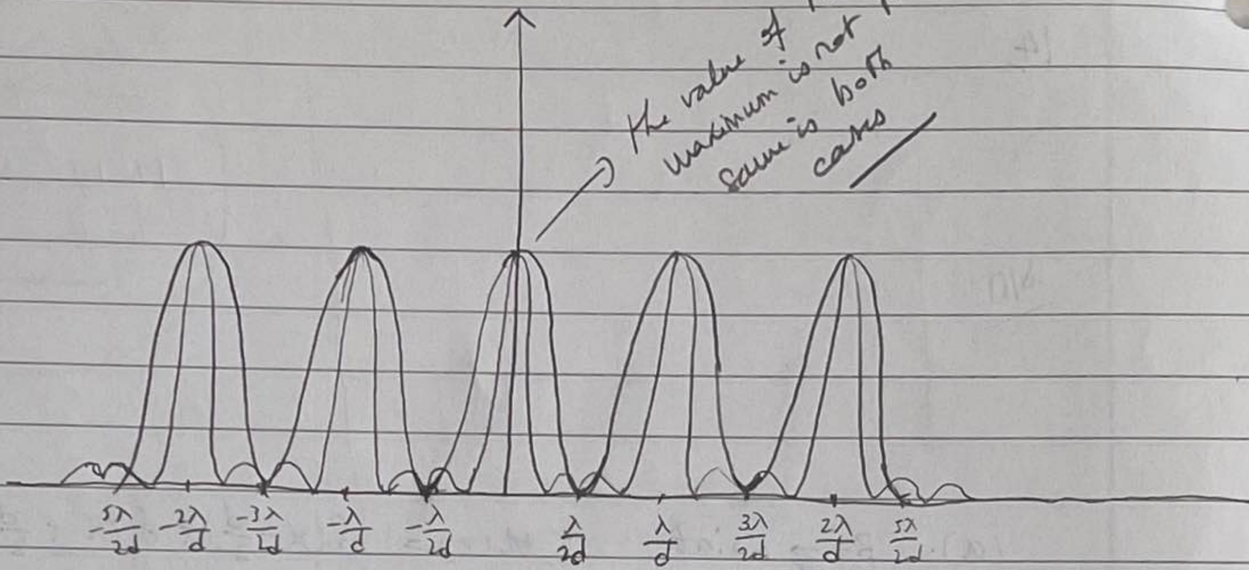
$\because D \gg d \quad \therefore \quad \theta$ is small $\quad \therefore \sin\theta \approx \theta = \tan\theta = \frac{x}{D}$

$$\therefore \tfrac{1}{2}\beta d = \frac{\pi d\theta}{\lambda}$$

$\therefore I(\theta) = I_0 \cos^2(\frac{\pi d\theta}{\lambda})$ ✓ is the angular distribution.

Label the axes

The value of the maximum is not the same in both cases



maxima $\Rightarrow \quad \frac{\pi d\theta}{\lambda} = m\pi \quad \Rightarrow \quad \theta = \frac{m\lambda}{d} \quad (m = 0, \pm 1, \pm 2, \cdots)$

minima $\Rightarrow \quad \frac{\pi d\theta}{\lambda} = (m+\frac{1}{2})\pi \quad \Rightarrow \quad \theta = (m+\frac{1}{2})\frac{\lambda}{d} \quad (m = 0, \pm 1, \pm 2, \cdots)$

(b) With 4 slits, $u_0(x) = \sum_{j=0}^{3} \delta(x - jd)$

Fourier Transform:

$$u(\beta) \propto \int_{-\infty}^{\infty} u_0(x) e^{i\beta x} \, dx$$

$$= \int_{-\infty}^{\infty} \left(\sum_{j=0}^{3} \delta(x - jd) \right) e^{i\beta x} \, dx$$

$$= \sum_{j=0}^{3} e^{ij\beta d} = 1 + e^{i\beta d} + e^{2i\beta d} + e^{3i\beta d}$$

$$= \frac{1 - e^{4i\beta d}}{1 - e^{i\beta d}} = \frac{e^{+2i\beta d}(e^{-2i\beta d} - e^{2i\beta d})}{e^{i\beta d/2}(e^{-i\beta d/2} - e^{i\beta d/2})}$$

$$= e^{i\frac{3}{2}\beta d} \frac{-2\cancel{i}}{-2\cancel{i}} \frac{\sin(2\beta d)}{\sin(\frac{1}{2}\beta d)}$$

$$\Rightarrow I(\beta) = I(0)\ \frac{\sin^2(2\beta d)}{\sin^2(\frac{1}{2}\beta d)}$$ ✓

principle maxima $\Rightarrow \frac{1}{2}\beta d = m\pi \Rightarrow \theta = \frac{m\lambda}{d}$

minima $\Rightarrow$ ~~$2\beta d = (m+\frac{1}{2}$~~ $2\beta d = m\pi \Rightarrow$

$, \frac{1}{2}\beta d \neq m\pi$

$\theta = \frac{m\lambda}{4d}$ except $\frac{m}{4}$ is integer

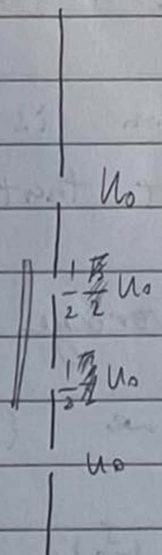
(c) As more slits are added, the principle maxima become more and more sharp, and more and more local maxima and local minima appear between the principle maxima.

$$I(\beta) = I_0 \frac{\sin^2(\frac{N}{2}\beta d)}{\sin^2(\frac{1}{2}\beta d)}$$ ✓

principle maxima at $\theta = \frac{m\lambda}{d}$

~~prim~~ minima at $\theta = \frac{m\lambda}{Nd}$ ($\frac{m}{N} \neq$ integer)

(d)



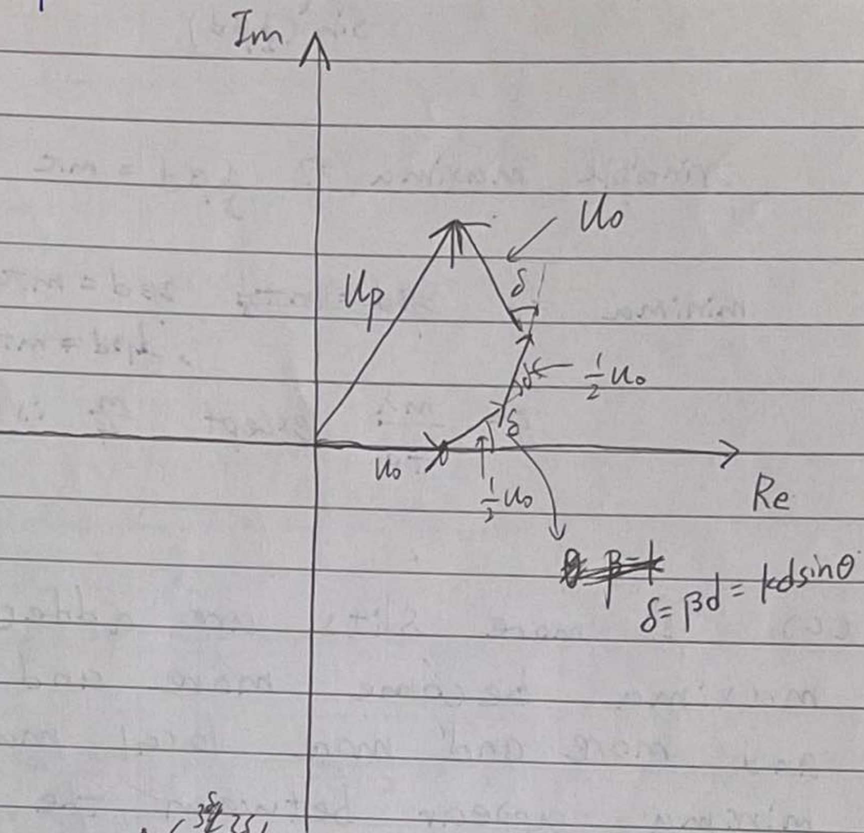
Intensity transmitted = 25% = $\frac{1}{4}$

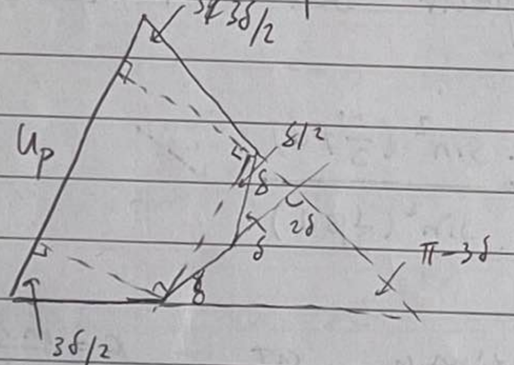
Amplitude transmitted = $\sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore u(x) = \delta(x) + \frac{1}{2}\delta(x-d) + \frac{1}{2}\delta(x-2d)$

$+ \delta(x-3d)$

Using phasors :





$$u_p = 2u_0\cos\left(\frac{3\delta}{2}\right) + u_0\cos\left(\frac{\delta}{2}\right)$$

$$\Rightarrow \quad I(\delta) = I(0)\left(\cos\left(\frac{3\delta}{2}\right) + \frac{1}{2}\cos\left(\frac{\delta}{2}\right)\right)^2$$

This intensity distribution is very similar to the original one except that ~~the~~

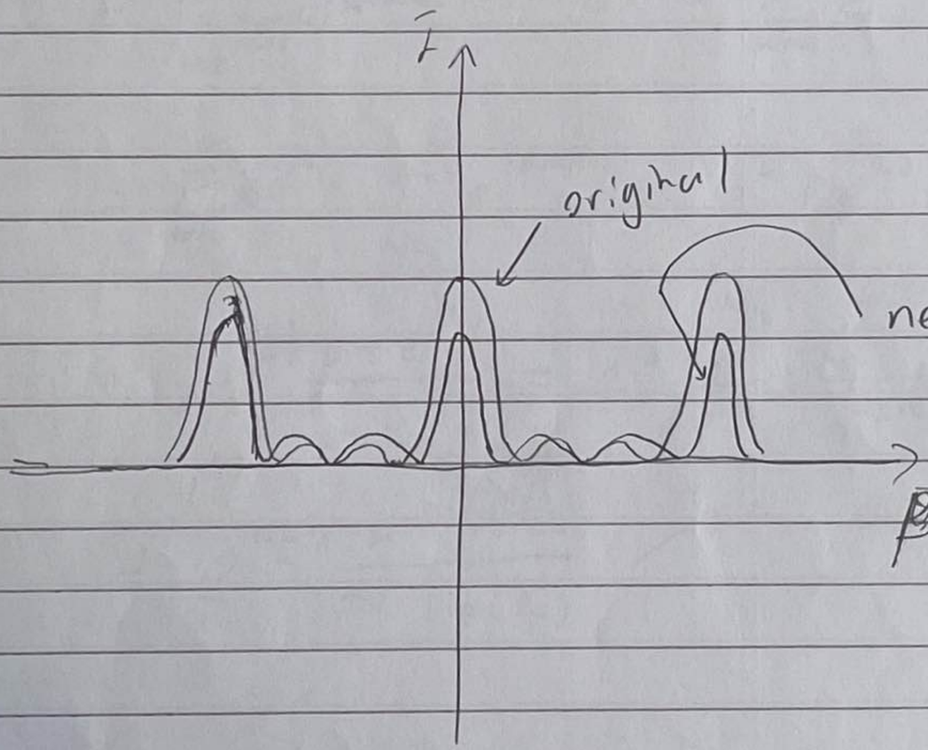
It has to be compared with the values obtained in (a) and (b)

① the overall average intensity falls to $\frac{5}{8}$ ~~to~~ ~~$\frac{9}{16}$~~ of original value ($\frac{5}{8}$ comes from

$1+1=2$, $1+\frac{1}{4}=\frac{5}{4}$, $(\frac{5}{4})/2 = \frac{5}{8}$)

② The principle maxima become narrower then before. ✓

③ The positions of local maxima and minima shifts. Each shifts towards the closest priheiple maxima. Only Principle maxima and the minima right in the middle between two priheiple maxima remain seationary.



15. (a) incident intensity function $u(x)$ is

$$u(x) = \sum_{m=1}^{N} \delta(x-md) \quad , \quad \delta = \frac{2\pi}{\lambda} d\sin\theta = kd\sin\theta = \beta d.$$

Fourier Transform:

$$u(\beta) \propto \int_{-\infty}^{\infty} u(x) e^{i\beta x} dx$$

$$= \int_{-\infty}^{\infty} \sum_{m=1}^{N} \delta(x-md) e^{i\beta x} dx = \sum_{m=1}^{N} e^{i\beta md}$$

$$= \cancel{\sum_{m=1}^{N} e^{i\beta d}} \sum_{m=1}^{N} e^{i\delta m} = e^{i\delta} + e^{2i\delta} + \cdots + e^{Ni\delta}$$

$$= e^{i\delta} \frac{1-e^{Ni\delta}}{1-e^{i\delta}} = e^{i\delta} \frac{e^{Ni\delta/2}}{e^{i\delta/2}} \frac{e^{-Ni\delta/2} - e^{Ni\delta/2}}{e^{-i\delta/2} - e^{i\delta/2}}$$

$$= e^{\frac{N+1}{2} i\delta} \frac{\cancel{-2i} \sin(\frac{N\delta}{2})}{\cancel{-2i} \sin(\frac{\delta}{2})}$$ ✓

$I(\beta) \propto |u(\beta)|^2 \Rightarrow$ ~~$I(\beta)$~~ $I(\delta) \propto |u(\delta)|^2$

$$\Rightarrow \quad I(\delta) = \frac{\sin^2(\frac{N\delta}{2})}{\sin^2(\frac{\delta}{2})} \quad \text{where } \delta = \frac{2\pi}{\lambda} d\sin\theta$$ ✓

minima occurs when $\frac{N\delta}{2}$ = integer $\times \pi$ but $\frac{\delta}{2} \neq$ integer $\times \pi$

$d\sin\theta_p = p\lambda$ for the pth principle maximum
(~~$\delta = 2\pi \times$ integer~~ $\delta_p = 2\pi p$)

For the minima adjacent to this principle maximum, $\cancel{\Delta\delta = \frac{\lambda}{Nd}}$ let $\delta' = \delta_p + \Delta\delta_p = 2\pi p + \Delta\delta_p$

$$(Np+1)\pi = \frac{N\delta}{2} = \frac{N}{2}(2\pi p + \Delta\delta_p) = Np\pi + \frac{N\Delta\delta_p}{2}$$

$$\therefore \frac{N\Delta\delta_p}{2} = \pi \quad \therefore \Delta\delta_p = \frac{2\pi}{N}$$

$$\because \delta_p = \frac{2\pi}{\lambda} d\sin\theta_p \quad \therefore \Delta\delta_p = \frac{2\pi}{\lambda} d\cos\theta_p \Delta\theta_p$$

$$\therefore \cancel{\Delta\theta_p = \frac{\lambda\Delta\delta_p}{2\pi d}} \quad \Delta\theta_p = \frac{\lambda\Delta\delta_p}{2\pi d\cos\theta}$$

$$\because \Delta\delta = \frac{2\pi}{N} \Rightarrow \Delta\theta_p = \frac{\lambda}{Nd\cos\theta}$$

(b) For pth order.

$$\frac{\delta}{2} = \pi p \Rightarrow \delta = 2\pi p$$

$$\Rightarrow \frac{2\pi}{\lambda} d\sin\theta_p = 2\pi p \Rightarrow d\sin\theta_p = p\lambda$$

$$\Rightarrow d\cos\theta_p \, \partial\theta_p = p\,\partial\lambda$$

$$\therefore \frac{\partial\theta_p}{\partial\lambda} = \frac{p}{d\cos\theta}$$

~~(c)~~ $$\cancel{\frac{\Delta\theta_p}{\Delta\lambda} = \frac{1}{\Delta\lambda}\left(\frac{\lambda}{Nd\cos\theta}\right) = \frac{\lambda}{\Delta\lambda}\left(\frac{1}{Nd\cos\theta}\right)}$$

$$\cancel{\Rightarrow \frac{p}{d\cos\theta} = \frac{\lambda}{\Delta\lambda}\frac{1}{Nd\cos\theta} \Rightarrow \Delta\lambda}$$

(c) Apply rayleigh creterion :

The ~~cos~~ principle maximum of p^{th} order ~~one~~ ~~on~~ of one wavelength must be outside of the first adjacent minimum of the other wavelength for the two wave-length to be resolved

$\therefore$ minimum $\Delta\lambda$ to be resolved ~~must~~ ($\Delta\lambda_{INST}$) must give a change in angle equal to $\Delta\theta_p$ calculated in (a)

yes

$\therefore$ $\cancel{\Delta\lambda = \frac{\partial\theta_p}{\partial\lambda} \Delta\lambda\, \Delta\theta_p}$

$$\Delta\theta_p = \frac{\partial\theta_p}{\partial\lambda}\Delta\lambda$$

$$\therefore \Delta\lambda = \frac{\Delta\theta_p}{\partial\theta_p/\partial\lambda} = \frac{\lambda/Nd\cos\theta}{p/d\cos\theta} = \boxed{\frac{\lambda}{Np}}$$

(d) Chromatic resolving power

$$R = \frac{\lambda}{\Delta\lambda_{INST}} = \boxed{Np}$$

If the grating is used in a medium of high refractive index ($n > 1$)

β changes from $k\sin\theta$ to $\beta = nk\sin\theta$

$\therefore$ ~~$I(\beta)$~~ $I(\delta) = \dfrac{\sin^2(\frac{N\delta}{2})}{\sin^2(\frac{\delta}{2})}$ where $\delta = \beta d$

~~β~~ $\therefore \delta = \dfrac{2\pi n}{\lambda} d\sin\theta$, $\Delta\delta_p = \dfrac{2\pi}{N}$

$\Rightarrow \Delta\theta_p = \dfrac{\lambda}{nNd\cos\theta}$

For p^{th} order : $\delta_p = 2\pi p$, $\dfrac{2\pi n}{\lambda} d\sin\theta_p = 2\pi p$.

$\Rightarrow n d\sin\theta_p = p\lambda$

$\Rightarrow \dfrac{\partial\theta_p}{\partial\lambda} = \dfrac{p}{nd\cos\theta}$

$\Rightarrow \Delta\lambda = \dfrac{\Delta\theta_p}{\partial\theta_p/\partial\lambda} = \dfrac{\lambda/nNd\cos\theta}{p/nd\cos\theta} = \dfrac{\lambda}{pN}$.

$\therefore R = \dfrac{\lambda}{\Delta\lambda} = Np$ is unchanged. yes

but you can use higher order of diffraction, up to $p = \dfrac{nd}{\lambda}$

16. ~~~~ For $d >> a$

$$u(x) = \delta(x - \tfrac{d}{2}) + \delta(x + \tfrac{d}{2})$$

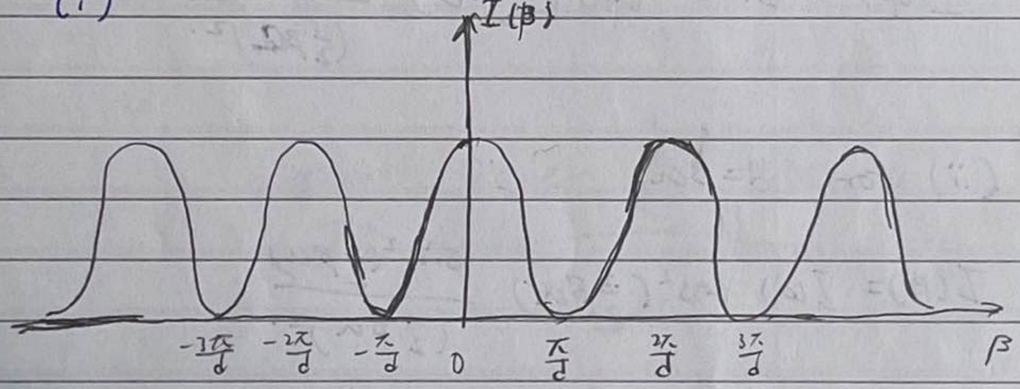
$$u(\beta) = \int_{-\infty}^{\infty} [\delta(x - \tfrac{d}{2}) + \delta(x + \tfrac{d}{2})] e^{i\beta x} dx$$

$$= e^{-i\beta d/2} + e^{i\beta d/2} = 2\cos(\tfrac{1}{2}\beta d)$$

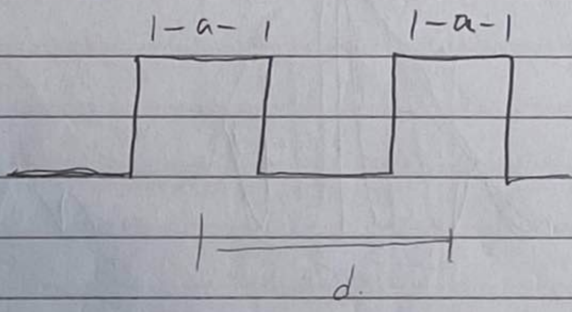
$$\therefore I(\beta) = I(0) \cos^2(\tfrac{1}{2}\beta d)$$

$$I(\theta) = I(0) \cos^2(\tfrac{1}{2} kd \sin\theta)$$

(i)

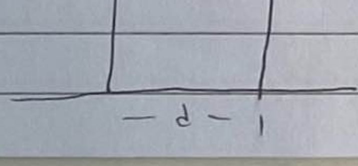


For $d \sim a$

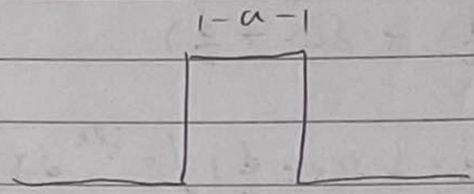
$u(x) =$ 

$= V(x) \otimes W(x)$ where

$V(x) = \delta(x - \tfrac{d}{2}) + \cancel{xx}\ \delta(x + \tfrac{d}{2})$

$=$ 

and $w(x)$ = top hat function =



$\therefore\ U(\beta) = \mathcal{T}_F(u(x)) = \mathcal{T}_F(\cancel{\ }\ v(x) \otimes w(x))$

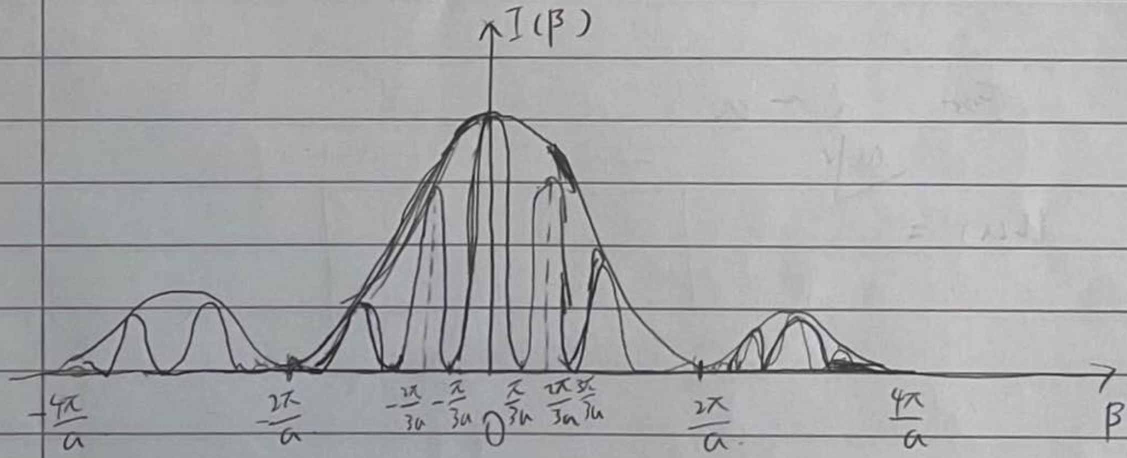
$= \mathcal{T}_F(v(x)) \cdot \mathcal{T}_F(w(x))$ (Convolution theorem)

$= \cos(\frac{1}{2}\beta d)\ \mathrm{sinc}(\frac{1}{2}\beta a)$

$I(\beta) = I(0)\ \cos^2(\frac{1}{2}\beta d)\ \dfrac{\sin^2(\frac{1}{2}\beta a)}{(\frac{1}{2}\beta a)^2}.$ ✓

(ii) for $d = 3a$

$I(\beta) = I(0)\ \cos^2(\frac{3}{2}\beta a)\ \dfrac{\sin^2(\frac{1}{2}\beta a)}{(\frac{1}{2}\beta a)^2}.$

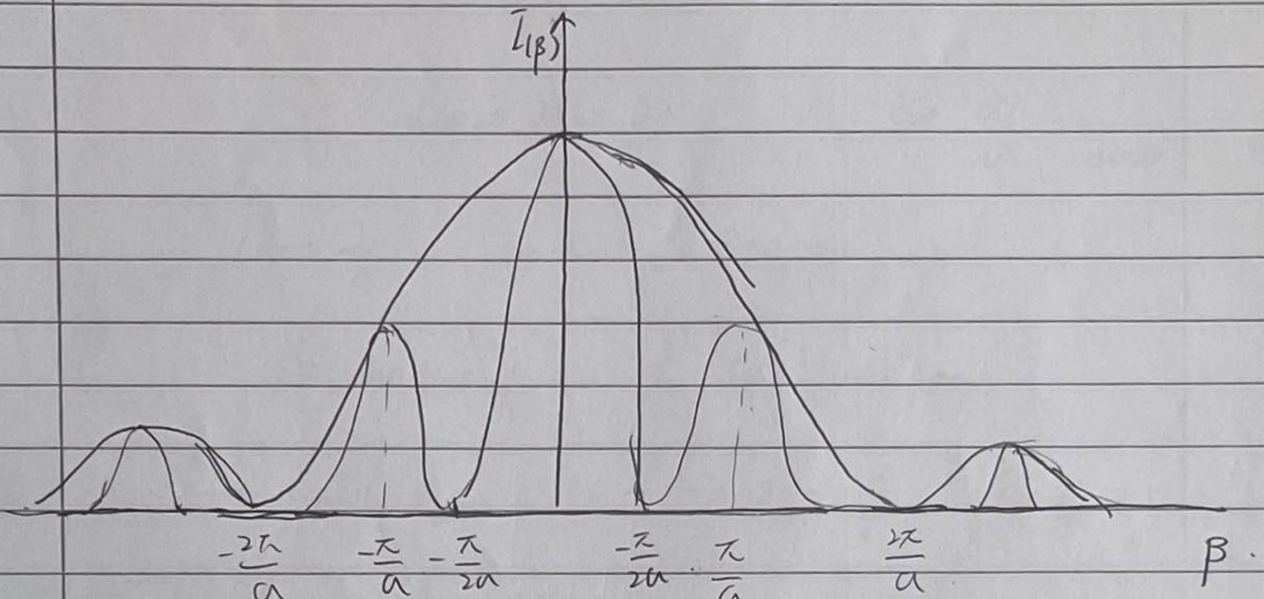


Third order is missing

not quite

(iii) for $d = 2a$.

$$I(\beta) = I(0)\cos^2(\beta a)\,\frac{\sin^2(\frac{1}{2}\beta a)}{(\frac{1}{2}\beta a)^2}$$



Second Order is missing

not quite

18. ~~We~~ Assume that we work on the first order

then $d \sin\theta = \lambda$

$\therefore \quad d\cos\theta \, \partial\theta = \partial\lambda \qquad \therefore \quad \cancel{\partial\lambda} \; \frac{\partial\theta}{\partial\lambda} = \frac{1}{d\cos\theta} = \frac{\nu}{\cos\theta}$

$\left(\nu = \frac{1}{d}\right.$ = spatial frequency).

ν = 500 lines/mm = 5×10^5 lines/m

for small θ, $\theta \approx \tan\theta = \frac{x}{f}$. $\cos\theta \approx 1$

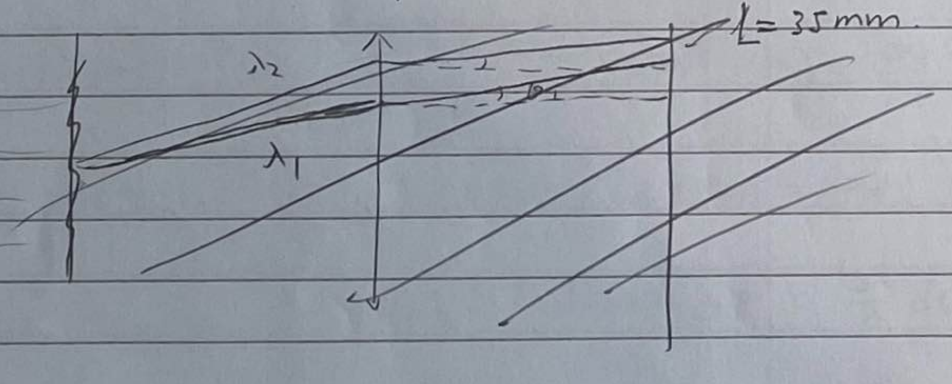
$\therefore \quad \cancel{\partial\lambda} \; \frac{\partial\theta}{\partial\lambda} \approx \nu \qquad \therefore \quad \Delta\theta \approx \nu\Delta\lambda$

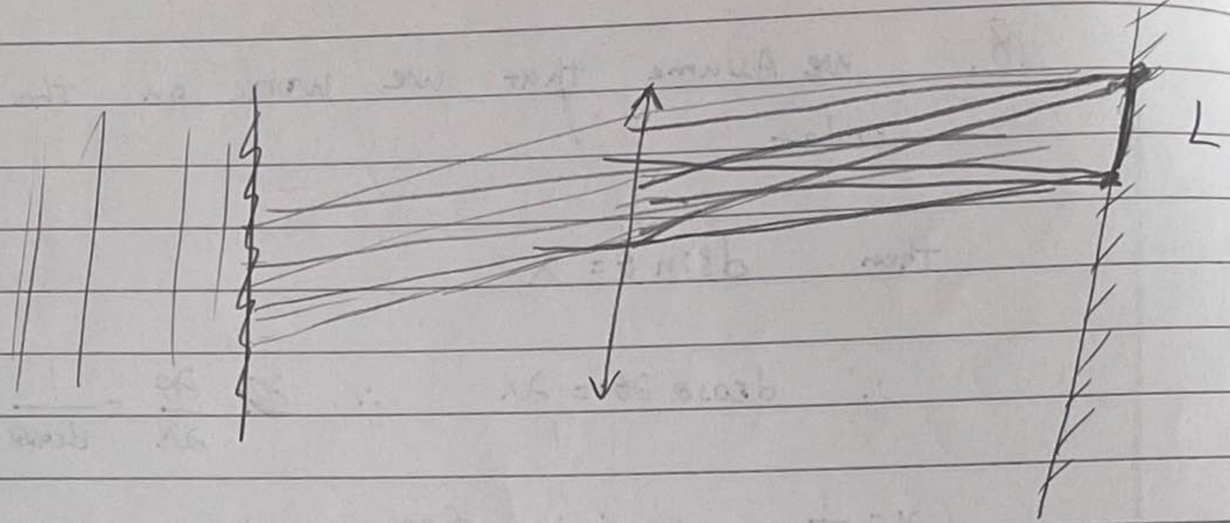
$\cancel{\therefore \Delta\theta \approx} \quad \because \quad \Delta\theta = \frac{\Delta x}{f}$

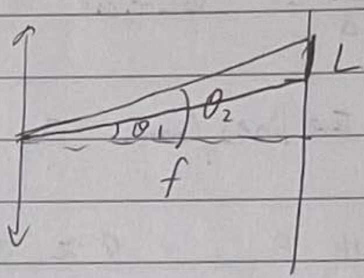
$\therefore \quad f = \frac{\Delta x}{\Delta\theta} = \frac{\Delta x}{\nu\Delta\lambda} = \frac{35\times10^{-3}\ m}{5\times10^5 \times (780-380)\times10^{-9}\ m^{-1}\times m}$

$= \boxed{0.175\ m}$ ✓

OR More accurately :







$$L = f(\tan\theta_2 - \tan\theta_1)$$

$$\theta_1 = \cancel{\arcsin(\nu d)} \rightarrow \arcsin(\nu\lambda_1) = 0.191$$

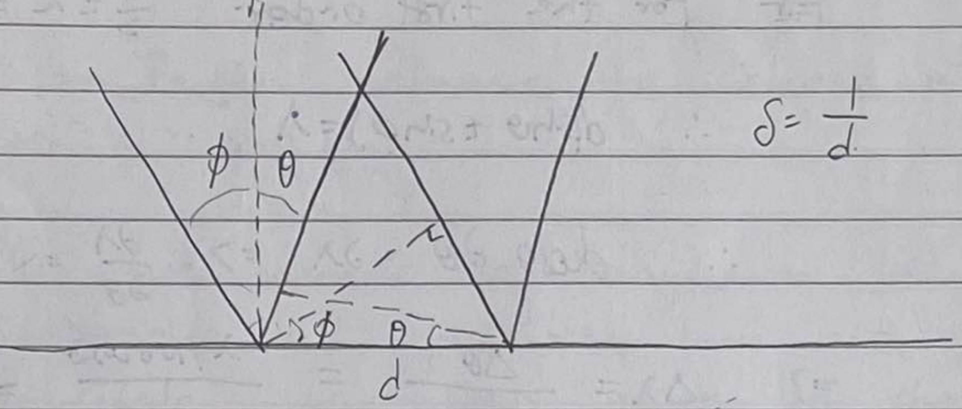
$$\theta_2 = \arcsin(\nu\lambda_2) = 0.400$$

$\lambda_1 = 380\,nm$ $\qquad \lambda_2 = 780\,nm$ $\qquad L = 35\times10^{-3}\,m$

$$\therefore f = \frac{L}{\tan\theta_2 - \tan\theta_1} = \frac{35\times10^{-3}\,m}{\tan(0.400) - \tan(0.191)}$$

$$= \boxed{0.153\,m}$$

19.



$\delta = \frac{1}{d}$

If we treat θ at ~~either~~ both sides of the normal being ~~both~~ positive, then for the first order maximum ($m=1$)

$$d(\sin\theta \pm \sin\phi) = \lambda$$

$$\therefore \quad \sin\theta \pm \sin\phi = \frac{\lambda}{d} = \delta\lambda$$

$$\therefore \quad \sin\theta = \delta\lambda \pm \sin\phi$$

$$\therefore \quad \theta = \sin^{-1}(\lambda\delta \pm \sin\phi)$$

We now have $\beta = (\sin\theta \pm \sin\phi)k$

Intensity distribution is $I(\beta) = I(0)\dfrac{\sin^2\left(\frac{N\beta d}{2}\right)}{\sin^2\left(\frac{\beta d}{2}\right)}$

$I(\delta) = I(0)\dfrac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$ $\Rightarrow$ $\Delta\delta = \frac{2\pi}{N}$ for rayleigh creterion.

$$\delta = \beta d = kd(\sin\theta \pm \sin\phi)$$

$$\Delta\delta = kd\cos\theta\,\Delta\theta = \frac{2\pi}{\lambda}d\cos\theta\,\Delta\theta$$

$$\therefore \quad \Delta\theta = \frac{\lambda}{Nd\cos\theta}$$

~~Ex~~ For the first order $\frac{\delta}{2} = \pi \Rightarrow \delta = 2\pi$.

$\therefore \quad d(\sin\theta \pm \sin\phi) = \lambda$

$\therefore \quad d\cos\theta \, \partial\theta = \partial\lambda \Rightarrow \frac{\partial\lambda}{\partial\theta} = \frac{1}{d\cos\theta}$

$\Rightarrow \quad \Delta\lambda = \frac{\Delta\theta}{\partial\theta/\partial\lambda} = \frac{\lambda/Nd\cos\theta}{1/d\cos\theta} = \frac{\lambda}{N}$

$\therefore \quad \Delta\lambda = \frac{\lambda}{N} = \boxed{\frac{\lambda}{\delta W}}$ ✓

(a) for the light to bounce straight back from the grating, we take the "+" sign and set $\theta = \phi$ ✓

$\therefore \quad 2\sin\phi = \lambda\delta$

$\therefore \quad \phi = \sin^{-1}\left(\frac{\lambda\delta}{2}\right)$

$= \sin^{-1}\left(\frac{(600\times10^{-9})(2400\times10^{3})}{2}\right)$

$= 0.8038$ rad

$= \boxed{46.1^\circ}$ ✓

(b)

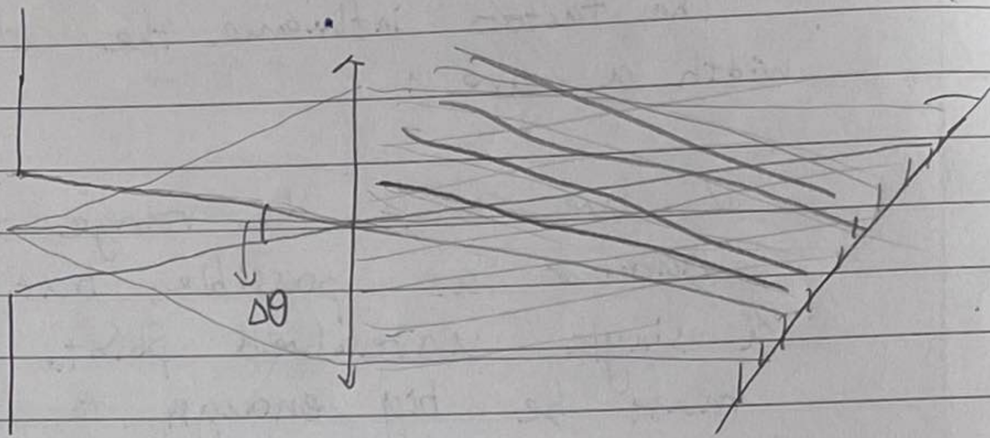
The factors influence the choice of slit width a are :

① we want the range to be as narrow as possible but not just a single unresolved point, so the width must be big enough to allow ~~the~~ two reflected beams with wavelengths differ by $\Delta\lambda$, the minimum resolved wavelength difference, to both pass through.

② The width must be as small as possible provided that ① is satisfied because we want the narrowest range of resolved wavelengths

We want the slit to be large enough to allow through most light energy but small enough to give maximum spectral resolution.

(c)



$$\Delta\theta = \frac{\lambda}{Nd\cos\theta} \qquad \because \theta = \phi$$

$$\therefore \quad \Delta\theta = \frac{\lambda}{Nd\cos\phi} = \frac{\lambda}{W\cos\phi}$$

$$a \simeq f\Delta\theta = \frac{f\lambda}{W\cos\phi}$$

$$= \frac{(200\times10^{-3})(600\times10^{-9})}{50\times10^{-3}\ \cos(46.1^\circ)}$$

$$= \boxed{3.5\times10^{-6}\ \text{m}}$$

$$\sum_{n_1} \sum_{n_2} \cdots \sum_{n_N} e^{-\beta n_i(\varepsilon_i - \mu)}$$

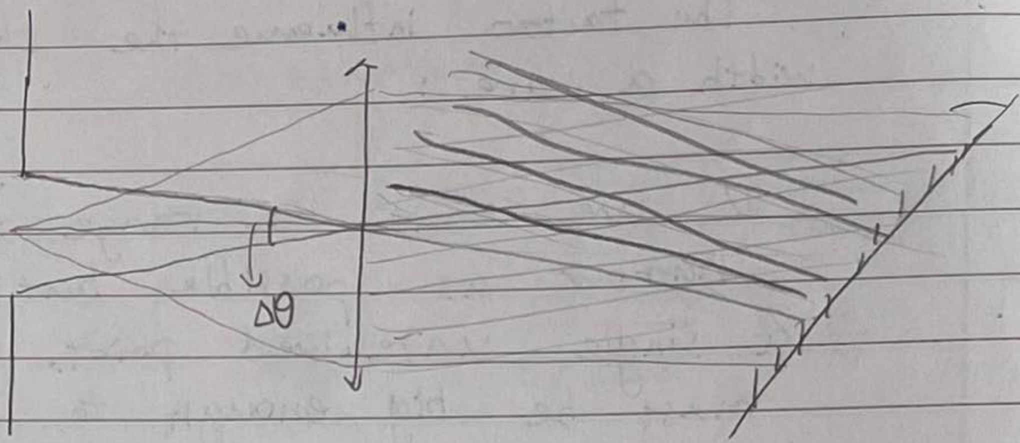
list of Questions 1

Huygen's principle?

integral . diff from the Fresnel-Kirchhoff?

. Microscope, incidence the slit on an angle?

(c)



$$\Delta\theta = \frac{\lambda}{Nd\cos\theta} \quad \because \theta = \phi$$

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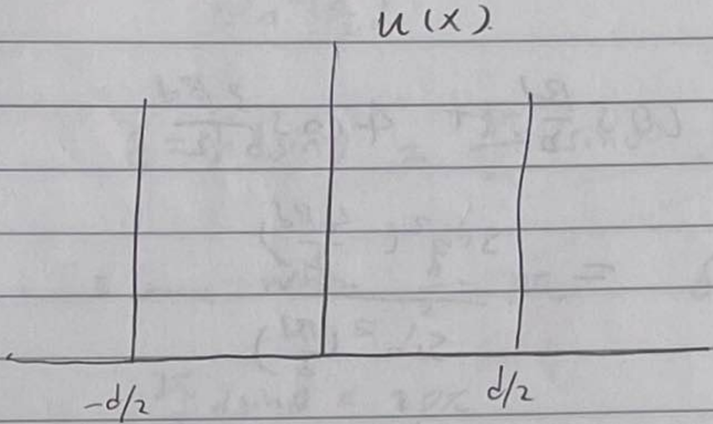
$$= \frac{(200\times10^{-3})(600\times10^{-9})}{50\times10^{-3}\ \cos(46.1^\circ)}$$

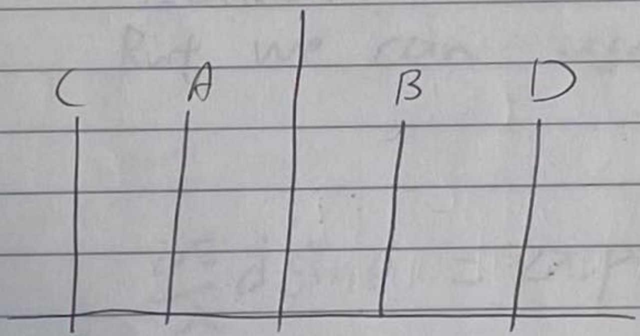
$$= \boxed{3.5\times10^{-6}\ m}$$

14 $u(x) = \delta(x - \frac{d}{2}) + \delta(x + \frac{d}{2})$



$u(\beta) = e^{-i\beta\frac{d}{2}} + e^{i\beta\frac{d}{2}} = 2\cos\frac{\beta d}{2}$

$I = I_0 \cos^2(\frac{\beta d}{2})$



25% of intensity

AB : $\sqrt{\frac{I_0}{4}} \cos(\frac{\beta d}{2})$

CD : $\sqrt{I_0} \cos(\frac{\beta 3d}{2})$

$I \propto \left(\sqrt{\frac{I_0}{4}} \cos(\frac{\beta d}{2}) + \sqrt{I_0} \cos(\frac{\beta 3d}{2}) \right)$

$\boxed{I_{max} = N^2 I_0}$

$$I \propto I_0 \left(\frac{1}{2}\cos\frac{\beta d}{2} + \cos\frac{3\beta d}{2}\right)^2$$

$$4\cos\frac{\beta d}{2} + 4\cos\frac{3\beta d}{2}$$

$$= \frac{\sin^2\left(\frac{4\beta d}{2}\right)}{\sin^2\left(\frac{\beta d}{2}\right)}$$

$$I = 4I_0\left(\frac{1}{2}\cos\frac{\beta d}{2} + \cos\frac{3\beta d}{2}\right)^2$$

$4I_0(\frac{1}{2}+1)^2 = 9I_0$ central maxima.

15. $$I = I_0 \frac{\sin^2(\frac{N\delta}{2})}{\sin^2(\frac{\delta}{2})}$$

$$\delta = kd\sin\theta = \frac{2\pi}{\lambda} d\sin\theta$$

$I = I_{max}$ when $\frac{\delta}{2} = p\pi$ $\delta = 2p\pi$.

$$\frac{2\pi}{\lambda} d\sin\theta = 2p\pi$$

$\lambda = \frac{\lambda_0}{n}$. but $\frac{\lambda}{\Delta\lambda} = Np$ indep. of n.

But we can use higher order

$$\frac{2\pi}{\lambda} d\sin\theta = 2\pi p$$

$$\sin\theta = \boxed{\frac{p\lambda}{d} \leq 1.}$$

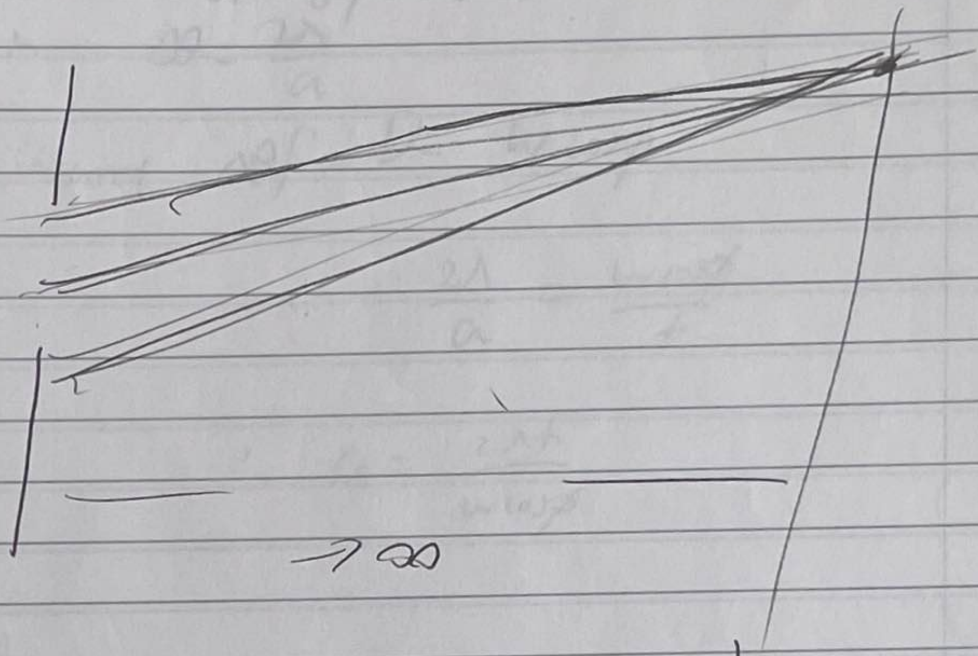
$n > 1$ λ becomes smaller

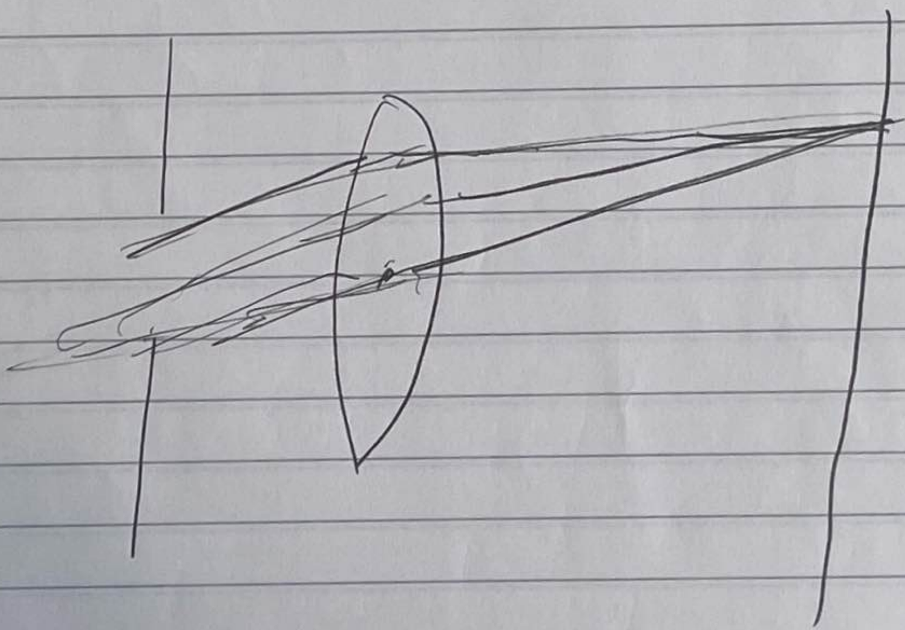
$\therefore$ p becomes larger.

pth order with $n = 1$ $\quad \sin\theta = \frac{p\lambda_0}{d}$

pth order with $n \neq 1$ $\quad \sin\theta = \frac{p\lambda_0}{nd}$

lens ~~ab~~ and Fraunhofer diffraction





$\tan\theta = \frac{y}{f}$

small θ $\quad \theta \approx \frac{y}{f}$

most of energy is emitted within $\Delta\theta = \frac{2\lambda}{a}$

want $\Delta\theta f = D = w\cos\phi$

$$\therefore \frac{2\lambda}{a} = \frac{w\cos\phi}{f}$$

$$\therefore a = \frac{2\lambda f}{w\cos\phi}$$