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Quantum Mechanics 2 Ziyan Li

1.

wavefunction $\psi(x)$ written in Dirac's ~~#~~ notation

is $|\psi\rangle$ $(\psi(x) = \langle x|\psi\rangle)$

Knowledge of a system is encoded in the quantum state (wave function)

$|\psi(x)|^2$ represents the probability density at position x.

2. $|\psi\rangle = e^{i\pi/5}|a\rangle + e^{i\pi/4}|b\rangle$

taking bra :

$\cancel{\langle\psi| = e^{-i\pi/5}\langle a|}$

$\langle\psi| = e^{-i\pi/5}\langle a| + e^{-i\pi/4}\langle b|$

3. $|\psi\rangle = a|A\rangle + b|B\rangle$

$\cancel{P_1 = |a|^2 \quad P_2 = |b|^2}$

$$1 = \langle\psi|\psi\rangle = \left(a^*\langle A| + b^*\langle B|\right)\left(a|A\rangle + b|B\rangle\right)$$

$$= |a|^2 \underbrace{\langle A|A\rangle}_{1} + |b^2| \underbrace{\langle B|B\rangle}_{1} + a^* b \underbrace{\langle A|B\rangle}_{0}$$

$$+ ab^* \underbrace{\langle B|A\rangle}_{0}$$

(~~assuming~~ Given that $|A\rangle$ and $|B\rangle$ are eigenstates and states are normalised)

$\therefore \; |a|^2 + |b|^2 = 1$

$$P_A = |\langle A|\psi\rangle|^2 = |a|^2$$

$$P_B = |\langle B|\psi\rangle|^2 = |b|^2$$

$$1 = P_A + P_B \quad \Rightarrow \quad P_A = |a|^2 = 1 - |b|^2$$

(a) $P_A = |a|^2 = \left|\frac{i}{2}\right|^2 = \left(\frac{i}{2}\right)\left(-\frac{i}{2}\right) = \boxed{\frac{1}{4}}$

(b) $P_A = 1 - |b|^2 = 1 - |e^{i\pi}|^2 = \boxed{0}$

(c) $P_A = 1 - |b|^2 = 1 - (\frac{1}{3} + i/\sqrt{2})(\frac{1}{3} - i/\sqrt{2})$

$= \cancel{1 - (\frac{1}{3})^2} \quad 1 - ((\frac{1}{3})^2 + (\frac{1}{2}))$

$= 1 - \frac{1}{9} - \frac{1}{2} = \frac{1}{2} - \frac{1}{9} = \frac{9-2}{18} = \boxed{\frac{7}{18}}$

4. (a) $\langle\psi|\hat{Q}|\psi\rangle = \langle\hat{Q}\rangle$ is the average value of operator $\hat{Q}$ in state $|\psi\rangle$

$|\langle q_n|\psi\rangle|^2$ is the probability ~~of~~ for the system to be in state $|q_n\rangle$

(The probability ~~of~~ for the system to collapse to state $|q_n\rangle$ when a measurement is made)

~~$|\psi\rangle = \sum_n q_n|q_n\rangle$~~ $\quad |\psi\rangle = \sum_n a_n|q_n\rangle \quad \hat{Q}|q_n\rangle = q_n|q_n\rangle$

(b) $\left(\sum_n |q_n\rangle\langle q_n|\right)|\psi\rangle = \sum_n \underbrace{\langle q_n|\psi\rangle}_{a_n}|q_n\rangle$

$= \sum_n a_n|q_n\rangle$ ~~$= \sum_n \ldots$~~ $= |\psi\rangle = I|\psi\rangle$

$\therefore \sum_n |q_n\rangle\langle q_n| = I$ is the identity operator.

$$\left(\sum_n q_n|q_n\rangle\langle q_n|\right)|\psi\rangle = \sum_n q_n|q_n\rangle\underbrace{\langle q_n|\psi\rangle}_{a_n}$$

$$= \sum_n a_n q_n|q_n\rangle = \sum_n a_n\hat{Q}|q_n\rangle$$

$$= \hat{Q}\left(\sum_n a_n|q_n\rangle\right) = \hat{Q}|\psi\rangle$$

$$\boxed{\sum_n q_n|q_n\rangle\langle q_n| = \hat{Q}}$$

(c)

$$\langle q_n | \psi \rangle = \langle q_n | I | \psi \rangle$$

$$\because \; I = \int_{-\infty}^{\infty} dx \, |x\rangle\langle x|$$

$$\therefore \; \langle q_n | \psi \rangle = \langle q_n | \int_{-\infty}^{\infty} dx \, |x\rangle\langle x| \, |\psi\rangle$$

$$= \int_{-\infty}^{\infty} dx \, \underbrace{\langle q_n | x \rangle}_{= \langle x | q_n \rangle^* = u_n^*(x)} \underbrace{\langle x | \psi \rangle}_{\psi(x)}$$

$$= \boxed{\int_{-\infty}^{\infty} dx \, u_n^*(x) \psi(x)}$$

5. TISE:

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

In 1-D : $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}|\psi\rangle + V|\psi\rangle = E|\psi\rangle$

TDSE :

$$\hat{H}|\psi\rangle = i\hbar\frac{\partial|\psi\rangle}{\partial t}$$

In 1-D

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}|\psi\rangle + V|\psi\rangle = i\hbar\frac{\partial|\psi\rangle}{\partial t}.$$

It is necessary for the wavefunction to satisfy TDSE because TDSE is the most general form of ~~st~~ schrödinger equation ~~(postulat~~

$|\psi\rangle$ must satisfy the schrödinger equation by ~~post~~ postulate of Quantum Mechanics

For $|\psi\rangle$ to satisfy the TISE, we have

$$E|\psi\rangle = i\hbar\frac{\partial|\psi\rangle}{\partial t}$$

$$\therefore \frac{d|\psi\rangle}{|\psi\rangle} = \cancel{\frac{i\hbar}{E}dt}\ -\frac{iE}{\hbar}dt$$

$$\Rightarrow \boxed{|\psi(x,t)\rangle = |\psi(x,0)\rangle e^{-\frac{iE}{\hbar}t}}$$

$\therefore$ As long as the state $|\Psi\rangle$ evolves as equation ①, i.e. all time dependence is contained in the exponential $e^{\frac{iE}{\hbar}t}$, then $|\Psi(x)\rangle$ satisfies the TISE

6. Because we need a wave function which has complete knowledge of the quantum systems and is able to predict the future evolution of the system.

Also due to uncertainty principle we cannot measure momentum and position accurately at the same time. Thus the wave function has to be expressed with only the initial condition of itself instead of its first derivative. Therefore the equation has to be first order in time so the wave function can be calculated with one boundary condition of time.

7.

$|\psi(0)\rangle = 0.2|1\rangle + 0.3|2\rangle + 0.4|3\rangle + 0.843|4\rangle$

(a) $E = n^2 \varepsilon$, for $E < 6\varepsilon$.

$n = 1$ or 2

$\therefore P = P_1 + P_2 = (0.2)^2 + (0.3)^2 = \boxed{0.13}$

(b)

$\langle E \rangle = \sum_i P_i E_i = (0.2)^2 (1^2)\varepsilon + (0.3)^2 (2^2)\varepsilon + (0.4)^2 (3^2)\varepsilon + (0.843)^2 (4^2)\varepsilon = \boxed{13.21\varepsilon}$

$\Delta E_{rms} = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} = \boxed{4.67\varepsilon}$

(c) $\because$ $|1\rangle, |2\rangle, |3\rangle, |4\rangle$ are ~~eigenvect~~ eigen-states $\therefore |\psi(t)\rangle = |\psi(0)\rangle e^{-\frac{i\varepsilon}{\hbar}t}$

$\therefore |\psi(t)\rangle = 0.2|1\rangle e^{-\frac{i\varepsilon}{\hbar}t} + 0.3|2\rangle e^{-\frac{4i\varepsilon}{\hbar}t} + 0.4|3\rangle e^{-\frac{9\varepsilon}{\hbar}t} + 0.843|4\rangle e^{-\frac{16\varepsilon}{\hbar}t}$

because all probability amplitudes are independent of time.

$\therefore P_1(t) = (0.2)^2 e^{\frac{i\varepsilon}{\hbar}t} \cdot e^{-\frac{i\varepsilon}{\hbar}t} = (0.2)^2 = P_1(0)$

similarly $P_i(t) = P_i(0)$ for $i = 1, 2, 3, 4$

$\therefore$ Results found in (a) and (b) are

still valid.

d) After ~~measure the~~ measurement the state

of the system ~~becomes~~ collapses to $\boxed{|4\rangle}$

~~wave f~~

$$|\psi'(t)\rangle = |4\rangle e^{\frac{-16i\varepsilon}{\hbar}t}$$

The probability of obtaining energy 16ε

is $\underline{1}$

8 ~~(a)~~ Uncertainty Principle states that ~~to~~ 2 quantities can be ~~known~~ well-defined simultaneously if and only if the commutator of their corresponding ~~of~~ operators is zero.

$$\Delta A \Delta B \geq \frac{1}{2}\left| i\langle [A,B]\rangle \right|.$$ (energy is well ~~known~~ known. in this question).

(a) $[\hat{H},\hat{p}] = [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), -i\hbar\frac{\partial}{\partial x}] = [\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}, -i\hbar\frac{\partial}{\partial x}}_{0}] + [\hat{V},\hat{p}] = [\hat{V},\hat{p}]$

~~(a)~~ for $\hat{p}$ and $\hat{V}$'s commutator:

$$[\hat{p},\hat{V}]\psi = \hat{p}\hat{V}\psi - \hat{V}\hat{p}\psi$$

$$= -i\hbar\frac{\partial}{\partial x}(V(x)\psi(x)) - V(x)(-i\hbar\frac{\partial}{\partial x}\psi(x)).$$

$$= -i\hbar \cancel{V(x)\frac{\partial\psi}{\partial x}} - i\hbar\psi\frac{\partial V}{\partial x} + i\hbar \cancel{V\frac{\partial\psi}{\partial x}}.$$

$$= -i\hbar\psi\frac{\partial V}{\partial x} \neq 0 \quad \text{unless} \quad \frac{\partial V}{\partial x} = 0$$

$\therefore [\hat{H},\hat{p}] \neq 0$ unless $\frac{\partial V}{\partial x} = 0$

$\therefore$ If $V(x)$ is a constant function then the particle can have well-defined momentum

If $V(x)$ is not a constant function ($\frac{\partial V}{\partial x} \neq 0$) then the particle cannot have well defined momentum

(b) $[\hat{H}, \hat{x}] = [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}, \hat{x}] + \underbrace{[\hat{V}, \hat{x}]}_{0}$.

$$[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}, \hat{x}]\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(x\psi) + \frac{\hbar^2}{2m}x\frac{\partial^2\psi}{\partial x^2}$$

$$= -\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left(x\frac{\partial\psi}{\partial x} + \psi\right) + \frac{\hbar^2}{2m}x\frac{\partial^2\psi}{\partial x^2}.$$

$$= -\frac{\hbar^2}{2m}\left(\cancel{x\frac{\partial^2\psi}{\partial x^2}} + 2\frac{\partial\psi}{\partial x}\right) + \cancel{\frac{\hbar^2}{2m}x\frac{\partial^2\psi}{\partial x^2}}.$$

$$= -\frac{\hbar^2}{m}\frac{\partial\psi}{\partial x}$$

ψ is a general wave function so

$\frac{\partial\psi}{\partial x}$ is not 0 in general

$[\hat{H}, \hat{x}] = -\frac{\hbar^2}{m}\frac{\partial}{\partial x} \neq 0$

$\therefore$ Particle cannot have simultaneously well-defined energy and position.

9. $\cancel{A}$ $\hat{A}^\dagger = \hat{A}$ $\hat{B}^\dagger = \hat{B}$

(1) $(\hat{A}+\hat{B})^\dagger = \cancel{\hat{A}+\hat{B}}\ \hat{A}^\dagger + \hat{B}^\dagger = \hat{A}+\hat{B}$ $\therefore$ is hermitian.

(2) $(c\hat{A})^\dagger = c^*\hat{A}^\dagger = c^*\hat{A}$ $\therefore$ This is not hermitian unless c is real.

$\cancel{AB}$

We prove that $(AB)^\dagger = B^\dagger A^\dagger$. (Here $\cancel{A^\dagger, B}$ $\hat{A}, \hat{B}$ doesn't have to be hermitian)

$\langle \psi | (\hat{A}\hat{B})^\dagger | \phi \rangle^* = \langle \phi | AB | \psi \rangle$ $= \cancel{\int \phi^*(x) \hat{A} (\hat{B}}$

$\cancel{= \int \phi^*(x) \hat{A} (\hat{B}\phi(x))\, dx = \int}$

$= \int \phi^*(x) \hat{A} (\hat{B}\psi(x))\, dx = \int (\hat{A}^\dagger \phi(x))^* \hat{B} \psi(x)\, dx$

$= \int (\hat{B}^\dagger \hat{A}^\dagger \phi(x))^* \psi(x)\, dx = \left(\int \psi^*(x) \hat{B}^\dagger \hat{A}^\dagger \phi(x)\, dx \right)^*$

$= \langle \psi | \hat{B}^\dagger \hat{A}^\dagger | \phi \rangle^*$

This is true for any ψ and ϕ

$\therefore$ $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$

$\therefore$ (3) $(\hat{A}\hat{B})^\dagger = \cancel{\hat{B}\hat{A}}\ \hat{B}^\dagger \hat{A}^\dagger = \hat{B}\hat{A}$ This is not hermitian unless $[\hat{A}, \hat{B}] = 0$

(4) $(\hat{A}\hat{B}+\hat{B}\hat{A})^\dagger = (\hat{A}\hat{B})^\dagger + (\hat{B}\hat{A})^\dagger$

$= \hat{B}^\dagger\hat{A}^\dagger + \hat{A}^\dagger\hat{B}^\dagger = \hat{B}\hat{A} + \hat{A}\hat{B} = \hat{A}\hat{B} + \hat{B}\hat{A}$

This is hermitian.

(5) $\int \phi^*(x)\frac{\partial}{\partial x}\psi(x)\,dx = [\phi^*(x)\psi(x)]_{-\infty}^{\infty} - \int \psi(x)\frac{\partial}{\partial x}\phi^*(x)\,dx$

$= \left[\int \cancel{\phi}\,\psi^*(x)\left(-\frac{\partial}{\partial x}\right)\phi(x)\,dx\right]^*$

$\therefore \left(\frac{\partial}{\partial x}\right)^\dagger = -\frac{\partial}{\partial x}$

$\therefore$ Not hermitian

(6) $\int \phi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x)\,dx = \cancel{[\phi}$

$[-i\hbar\phi^*(x)\psi(x)]_{-\infty}^{\infty} - \int -i\hbar\,\psi(x)\frac{\partial\phi^*(x)}{\partial x}\,dx.$

$= i\hbar\int \psi(x)\frac{\partial}{\partial x}\phi^*(x)\,dx = \left[\int \psi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\right)\phi(x)\,dx\right]^*$

$\therefore \left(-i\hbar\frac{\partial}{\partial x}\right)^\dagger = -i\hbar\frac{\partial}{\partial x}$ $\therefore$ Is hermitian.

$$\int \phi^*(x) \frac{\partial^2}{\partial x^2} \psi(x)\, dx = \left[\phi^*(x) \frac{\partial \psi}{\partial x}\right]_{-\infty}^{\infty} - \int \frac{\partial \psi}{\partial x} \cdot \frac{\partial \phi^*}{\partial x}\, dx$$

$$= -\left[\frac{\partial \phi^*}{\partial x} \psi\right]_{-\infty}^{\infty} + \int \frac{\partial^2 \phi^*}{\partial x^2} \psi(x)\, dx$$

$$= \left[\int \psi^*(x) \frac{\partial^2}{\partial x^2} \phi(x)\, dx\right]^*$$

$$\therefore \left(\frac{\partial^2}{\partial x^2}\right)^\dagger = \frac{\partial^2}{\partial x^2}$$

$\therefore$ Is hermitian.

10.

$$\left[i[\hat{A},\hat{B}] \right]^{\dagger} = [i\hat{A}\hat{B} - i\hat{B}\hat{A}]^{\dagger}$$

$$= -i\hat{B}^{\dagger}\hat{A}^{\dagger} + i\hat{A}^{\dagger}\hat{B}^{\dagger} = i\hat{A}\hat{B} - i\hat{B}\hat{A} = i[\hat{A},\hat{B}].$$

$\therefore$ $i[\hat{A},\hat{B}]$ is hermitian.

11. $(\hat{A}\hat{B}\hat{C}\hat{D})^{\dagger} = (\hat{B}\hat{C}\hat{D})^{\dagger}\hat{A}^{\dagger} = \cancel{\hat{C}\hat{D}}\ (\hat{C}\hat{D})^{\dagger}\hat{B}^{\dagger}\hat{A}^{\dagger}$

$= \hat{D}^{\dagger}\hat{C}^{\dagger}\hat{B}^{\dagger}\hat{A}^{\dagger}.$

12. $\hat{A}|\varphi_n\rangle = a_n|\varphi_n\rangle$

$\hat{B}|\varphi_n\rangle = b_n|\varphi_n\rangle$ ($|\varphi_n\rangle$ is the complete set of eigenkets)-

then ~~$\hat{A}\hat{B}$~~ any state $|\psi\rangle$ ~~can~~ can be expressed as

$$|\psi\rangle = \sum_n c_n|\varphi_n\rangle$$

then ~~$\hat{A}\hat{B}$~~ $[\hat{A},\hat{B}]|\psi\rangle = \hat{A}\hat{B}|\psi\rangle - \hat{B}\hat{A}|\psi\rangle$

$= \cancel{c_n} \sum_n c_n(\hat{A}\hat{B}|\varphi_n\rangle - \hat{B}\hat{A}|\varphi_n\rangle)$

$= \sum_n c_n(\underbrace{a_n b_n - b_n a_n}_{0})|\varphi_n\rangle = 0$

$\because$ $|\psi\rangle$ is a general state

$\therefore$ $[\hat{A},\hat{B}] = 0$

~~13. If $\hat{A}$ has degenerate eigenstates of the eigenvalue~~ ~~corresponding~~ ~~for to the~~ ~~eigenvector~~ ~~eigen~~

13 If $\hat{A}$ has degenerate eigenstates for the eigenvalue that corresponds to ~~the~~ an eigenstate $|\phi\rangle$, then.

even though $[\hat{A}, \hat{B}] = 0$, $|\phi\rangle$ may still not be an eigenstate of $\hat{B}$.

Counterexample.

$$\hat{T}_x = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad \hat{P}_x = -i\hbar\frac{\partial}{\partial x}.$$

$[\hat{T}_x, \hat{P}_x] = 0$ clearly, let $\langle x|\phi\rangle = \cos(kx)$

$$\cancel{\hat{T}_A} \quad \hat{T}_x|\phi\rangle = \frac{\hbar^2 k^2}{2m}\cos(kx) = \frac{\hbar^2 k^2}{2m}|\phi\rangle$$

$\therefore$ $|\phi\rangle$ is an eigenstate of $\hat{T}_x$, but

$$\hat{P}_x|\phi\rangle = -i\hbar k\sin(kx)$$

$\therefore$ $|\phi\rangle$ is not an eigenstate of $\hat{P}_x$.

14. (a)

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B} = \hat{A}\hat{B}\hat{C} - \hat{A}\hat{C}\hat{B} + \hat{A}\hat{C}\hat{B} - \hat{C}\hat{A}\hat{B}$$

$$= \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$$

(b)

$$[\hat{A}\hat{B}\hat{C}, \hat{D}] = \hat{A}\hat{B}\hat{C}\hat{D} - \hat{D}\hat{A}\hat{B}\hat{C}$$

$$= \hat{A}\hat{B}\hat{C}\hat{D} - \hat{A}\hat{B}\hat{D}\hat{C} + \hat{A}\hat{B}\hat{D}\hat{C} - \hat{A}\hat{D}\hat{B}\hat{C} + \hat{A}\hat{D}\hat{B}\hat{C} - \hat{D}\hat{A}\hat{B}\hat{C}$$

$$= \hat{A}\hat{B}[\hat{C}, \hat{D}] + \hat{A}[\hat{B}, \hat{D}]\hat{C} + [\hat{A}, \hat{D}]\hat{B}\hat{C}$$

~~(c)~~ This is ~~similarly~~ similar to differentiation in that $\hat{D}$ is like a differential operator — ~~when~~ $\hat{D}$ acting on the product of ~~[illegible]~~ $\hat{A}, \hat{B}, \hat{C}$ is like the sum of $\hat{D}$ acting on them individually.

(c) $[\hat{x}, \hat{p}]\psi = \cancel{[\hat{x}\hat{p}]}\ -i\hbar x \frac{\partial \psi}{\partial x} + i\hbar x \frac{\partial \psi}{\partial x} + i\hbar \frac{\partial x}{\partial x}\psi$

$= i\hbar \psi$

$\therefore [\hat{x}, \hat{p}] = i\hbar$

$\therefore [\hat{x}^1, \hat{p}] = i\hbar\,(1)\,\hat{x}^{1-1}$

$\therefore\ [\hat{x}^n, \hat{p}] = i\hbar n \hat{x}^{n-1}$ is true for $n=1$

Use induction, assume proposition true for $n=k$

$[\hat{x}^k, \hat{p}] = i\hbar k \hat{x}^{k-1}$. then for $n=k+1$.

$[\hat{x}^{k+1}, \hat{p}] = \cancel{[\hat{x}\hat{x}^k, \hat{p}] = \hat{x}}$

$= [\hat{x}^k \hat{x}, \hat{p}] = \hat{x}^k [\hat{x}, \hat{p}] + [\hat{x}^k, \hat{p}] \hat{x}$

$= i\hbar \hat{x}^k + i\hbar k \hat{x}^{k-1} \hat{x}$

$= i\hbar (k+1) \hat{x}^k$

$\therefore$ Proposition is true for $n=k+1$, thus true for all n.

$\therefore [\hat{x}^n, \hat{p}] = i\hbar n \hat{x}^{n-1}$

(d)

$[f(\hat{x}), \hat{p}]\psi$

$= -i\hbar f(x) \frac{\partial \psi}{\partial x} - (-i\hbar) \frac{\partial}{\partial x}(\psi f(x))$.

$= \cancel{-i\hbar f(x) \frac{\partial \psi}{\partial x}} + \cancel{i\hbar f(x) \frac{\partial \psi}{\partial x}} + i\hbar \frac{\partial f}{\partial x} \psi$.

$= i\hbar \frac{\partial f}{\partial x} \psi$

$\therefore [f(\hat{x}), \hat{p}] = i\hbar \frac{df}{dx}$.

$$\hat{H}|\psi\rangle = E_n|\psi\rangle$$

$$\left(\frac{\hat{p}^2}{2m} + V(x)\right)|\psi\rangle = E_n|\psi\rangle$$

$$\Rightarrow \hat{p}\hat{p}|\psi\rangle = 2m(E_n - V(x))|\psi\rangle$$

if momentum well defined.

$$\hat{p}|\psi\rangle = p_n|\psi\rangle$$

$$\therefore p_n^2|\psi\rangle = 2m(E_n - V(x))|\psi\rangle$$

$\therefore$ $V(x)$ has to be a constant

for $\hat{p}$ to be well-defined.

$$[\hat{H}, \hat{x}] \propto \Delta E \, \Delta x = 0.$$

$$\left[\frac{\partial^2}{\partial x^2}, \hat{x}\right]\psi = \frac{\partial^2}{\partial x^2} x\psi - x\frac{\partial^2 \psi}{\partial x^2} \neq 0$$

13.

$$\hat{A}|\psi\rangle = \lambda|\psi\rangle$$

$$[\hat{A},\hat{B}]=0$$

$$[\hat{A},\hat{B}]|\psi\rangle = (AB-BA)|\psi\rangle$$

$$= AB|\psi\rangle - \lambda B|\psi\rangle = 0$$

$$\therefore A\cdot(B|\psi\rangle) = \lambda(B|\psi\rangle)$$

$$\Updownarrow$$

$$A|\psi\rangle = \lambda|\psi\rangle$$

$$|\psi\rangle = \sum_n a_n e^{-bt}|E_n\rangle.$$

$$\psi(x) = \langle x|\psi\rangle = \sum_n a_n e^{-bt}\langle x|E_n\rangle$$

$$|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$$

$$\psi(x) = \langle x|\psi\rangle = c_1\psi_1(x) + c_2\psi_2(x)$$

$\cancel{\int dx |\psi|^2}$

$$|\phi(x)|^2 = \langle \phi | x \rangle \langle x | \phi \rangle$$

$$\Delta E \Delta X = \frac{1}{2} | \langle i\hbar$$

$$[\frac{\partial}{\partial t}, x] \psi = \frac{\partial x \psi}{\partial t} - x \frac{\partial \psi}{\partial t}$$

$$= \cancel{\frac{\partial}{\partial t} [x \psi]}$$

$$\boxed{\hat{H} = i\hbar \frac{\partial}{\partial t}}$$

$$i\hbar \langle x | \frac{\partial |\psi\rangle}{\partial t} = \hat{H} \langle x | \psi \rangle = \cancel{E_n \psi(x)}$$

$$= E_n u_n(x).$$