To: Michael Barnes

Quantum Mechanics I Ziyan Li

1.

$h = 6.626 \times 10^{-34}$ J·s $\lambda = 0.1 \times 10^{-9}$ m

$m_n = 1.67 \times 10^{-27}$ kg $m_e = 9.11 \times 10^{-31}$ kg

~~For neutron:~~

$\cancel{p_n = \frac{h}{\lambda}}$

All three matters / waves have the same momentum

$$p = \frac{h}{\lambda} = 6.626 \times 10^{-24} \text{ kg·m/s}$$

K.E of neutron:

$$T_n = \frac{p^2}{2m_n} = 1.31 \times 10^{-20} \text{ J} = \boxed{0.082 \text{ eV}}$$

K.E of electron:

$$T_e = \frac{p^2}{2m_e} = 2.41 \times 10^{-17} \text{ J} \cancel{\text{kg·m/s}} \cancel{= 150 \text{ eV}}$$

$$= \boxed{150 \text{ eV}}$$

K.E of EM wave:

$$T_M = h\nu = \frac{hc}{\lambda} = pc \cancel{= 1.99 \times 10}$$

$$= 2.0 \times 10^{-15} \cancel{\text{kg·m/s}} \text{ J}$$

$$= \boxed{1.2 \times 10^4 \text{ eV}}$$

It is reasonable to use non-relativistic formula to calculate the kinetic energy of neutron and electron because

For neutron:

$$p = \gamma_n m_n v_n = \frac{m_n v_n}{\sqrt{1-\frac{v_n^2}{c^2}}}$$

$$\Rightarrow \left(1-\frac{v_n^2}{c^2}\right)p^2 = m_n^2 v_n^2 \Rightarrow (c^2 - v_n^2)p^2 = m_n^2 v_n^2 c^2$$

$$c^2 p^2 = (m_n^2 c^2 + p^2) v_n^2$$

$$\therefore v_n = \frac{pc}{\sqrt{m_n^2 c^2 + p^2}} = 3.97 \times 10^3 \, m/s$$

For electron:

$$v_e = \frac{pc}{\sqrt{m_e^2 c^2 + p^2}} = 7.27 \times 10^6 \, m/s$$

$$\therefore \frac{v_e}{c} = 0.024, \quad \frac{v_n}{c} = 1.3 \times 10^{-4}$$

$\therefore$ In both cases $\frac{v}{c} \ll 1$

$\therefore$ we can treat both situations non-relativistically.

2 We make the assumption that if $\frac{v}{c} \geq 0.1$ then we cannot treat the problem non-relativistically any more.

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$, we take

$V = 0.1c$ so that

$$p = \frac{0.1mc}{\sqrt{1-0.01}} = \frac{0.1}{\sqrt{0.99}} mc$$

For electron :

$$P_e = \left(\frac{0.1}{\sqrt{0.99}}\right) m_e c = 2.75 \times 10^{-23} \text{ kg}\cdot\text{m/s}$$

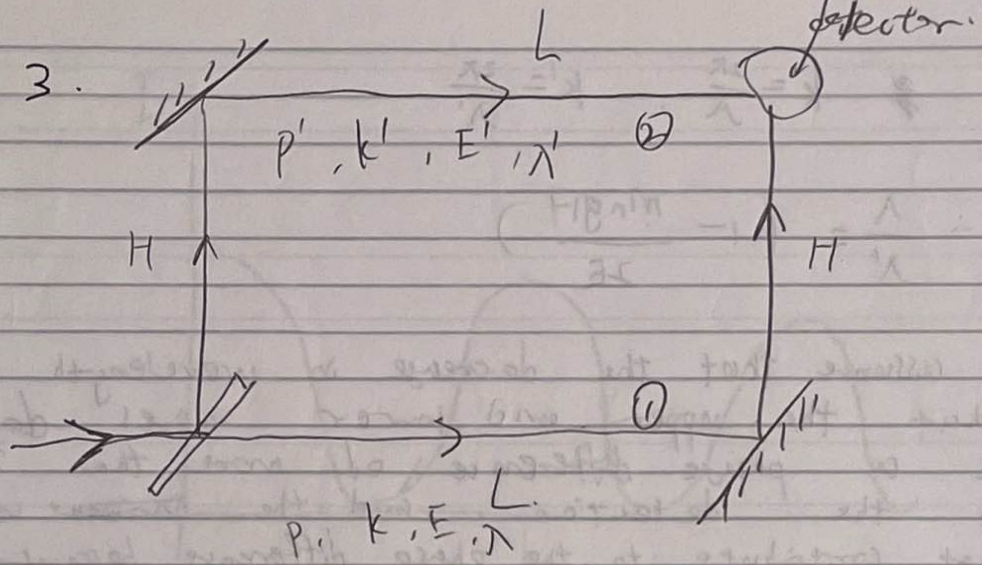
$$\lambda_e = \frac{h}{P_e} = 2.41 \times 10^{-11} \text{ m} = \boxed{0.0241 \text{ nm}}$$

For ~~ne~~ nuetron :

$$\cancel{\lambda_n =}\ P_n = \left(\frac{0.1}{\sqrt{0.99}}\right) m_n c = 5.04 \times 10^{-20} \text{ kg}\cdot\text{m/s}$$

$$\lambda_n = \frac{h}{P_n} = 1.32 \times 10^{-14} \text{ m} = \boxed{1.32 \times 10^{-5} \text{ nm}}$$

3.



Energy of neutron in path ② and that in path ① has the relation

$$E' = E - m_n g H \quad \text{by Energy Conservation.}$$

~~$E = \sqrt{2m_n p}$, E'~~ ~~$p = \hbar k$ $p' = \hbar k'$~~

$\therefore$ ~~$E = \sqrt{2m_n p} = \sqrt{2m_n \hbar k}$~~

~~$E' = \sqrt{2m_n p'} = \sqrt{2m_n \hbar k'}$~~

$\therefore$ ~~E~~

$$p = \hbar k, \quad p' = \hbar k'$$

~~$E = \sqrt{\ }$~~

$$p = \sqrt{2m_n E}$$

$$p' = \sqrt{2m_n E'}$$

$$\therefore \frac{p'}{p} = \frac{k'}{k} = \sqrt{\frac{E'}{E}} = \sqrt{\frac{E - m_n g H}{E}} = \left(1 - \frac{m_n g H}{E}\right)^{1/2}.$$

For fast moving neutrons $m_n g H \ll E$ $\therefore$ we have

$$\frac{k'}{k} \simeq 1 - \frac{m_n g H}{2E} \quad \therefore \boxed{k' \simeq k\left(1 - \frac{m_n g H}{2E}\right)}$$

$\therefore$ $\cancel{\ }$ $k = \frac{2\pi}{\lambda}$, $k' = \frac{2\pi}{\lambda'}$

$$\therefore \frac{\lambda}{\lambda'} = \left(1 - \frac{m_n g H}{2E}\right)$$

We assume that the decrease in wavelength is small so that the upper and lower waves do not have a phase difference of more than 2π at the detector., and the ~~horizont~~ vertical paths H do not contribute to the phase difference because both waves undergo the same change in λ when climbing upwards.
$\therefore$ phase difference $\Delta\phi''$ is given by.

$$\cancel{\Delta\phi = \frac{L}{\lambda'} - \frac{L}{\lambda} = L\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) =}$$

$$\frac{\Delta\phi''}{2\pi} = \frac{L}{\lambda} - \frac{L}{\lambda'} = L\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{L}{\lambda}\left(1 - \frac{\lambda}{\lambda'}\right)$$

$$\therefore \Delta\phi'' = \frac{2\pi L}{\lambda}\left(1 - \frac{\lambda}{\lambda'}\right) = Lk\left(1 - \frac{\lambda}{\lambda'}\right)$$

$$p = \hbar k \quad \therefore k = \frac{p}{\hbar} = \frac{\sqrt{2m_n E}}{\hbar}$$

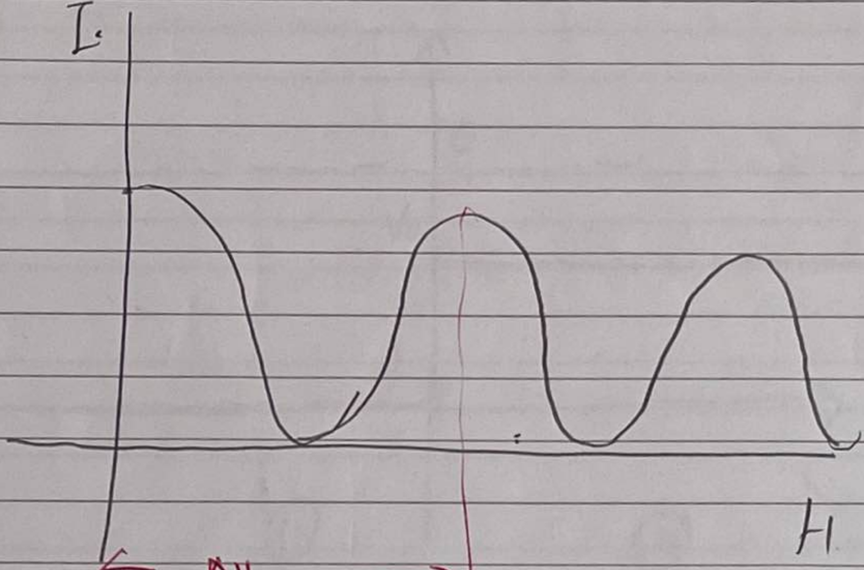
$$\therefore \Delta\phi'' = \left(\frac{\sqrt{2m_n E}}{\hbar}\right)\left(1 - \left(1 - \frac{m_n g H}{2E}\right)\right)(L)$$

$$= \left(\frac{\sqrt{2m_n E}}{\hbar}\right)\left(\frac{m_n g (HL)}{2E}\right). = 54.942$$

$$= 8(2\pi) + 4.68$$

$\therefore$ reduce to the interval $[0, 2\pi]$ gives

$$\boxed{\Delta\phi' = 4.68 \text{ rad}}$$



ΔH

$$= \frac{4\pi E}{L m_n g} \frac{\hbar}{(2m_n E)^{1/2}}$$

$$I \propto |1 + e^{i\Delta\phi}|^2$$
$$\propto \cos^2 \frac{\Delta\phi}{2}$$

$$I = |A|^2 |e^{i\phi_1} + e^{i\phi_2}|.$$

$$= |A|^2 |1 + e^{i\Delta\phi}|^2$$

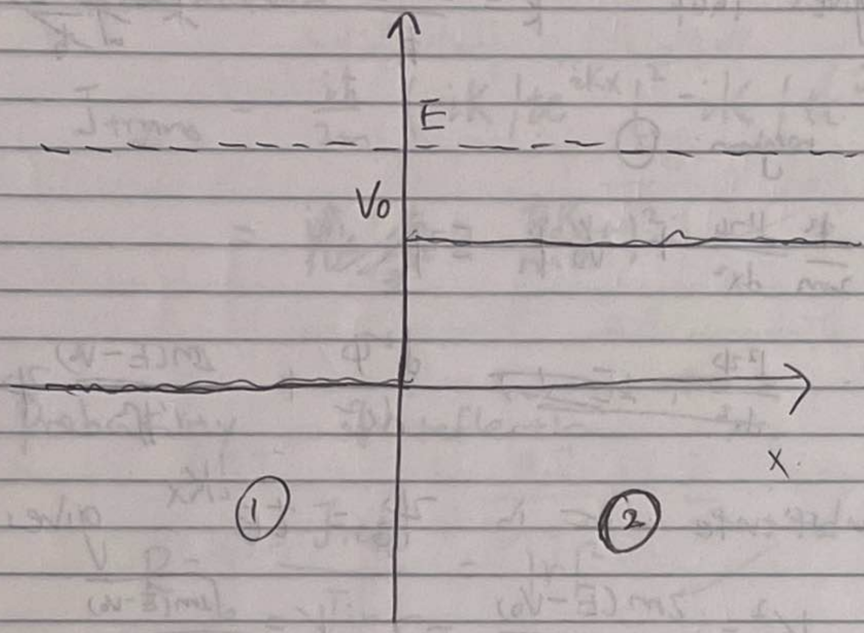
$$= |A|^2 (2 + 2\cos\Delta\phi)$$

$$= 4|A|^2 \cos^2 \frac{\Delta\phi}{2}$$

$$\frac{\Delta\phi}{2} = n\pi \Rightarrow \Delta\phi = 2n\pi$$

$$\Delta H = \frac{4\pi E \hbar}{m g L (2mE)^{1/2}}$$

4.



incident $\psi_1 = e^{ikx}$

transmitted $\psi_3 = t e^{iKx}$

reflected $\psi_2 = r e^{-ikx}$

The Time independent schrödinger equation in one-dimension:

$$\hat{H}\psi = E\psi$$

$$\Rightarrow \cancel{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

In region 1 $\quad V = 0$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

Substitute in $\psi_1 = e^{ikx}$ and $\psi_2 = re^{-ikx}$

gives that $k^2 = \frac{2mE}{\hbar^2} \Rightarrow \cancel{k = \sqrt{\frac{2mE}{\hbar}}}$ $k = \frac{\sqrt{2mE}}{\hbar}$

In region 2

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\therefore \cancel{\frac{d^2\psi}{dx^2} + (E-V_0)} \quad \frac{d^2\psi}{dx^2} + \frac{2m(E-V_0)}{\hbar^2}\psi = 0$$

substitute in $\psi_3 = te^{ikx}$ gives

$$k^2 = \frac{2m(E-V_0)}{\hbar^2} \Rightarrow k = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

The boundary conditions:

At $x = 0$.

$\frac{d^2\psi}{dx^2}$ is finite $\Rightarrow \frac{d\psi}{dx}$ is continuous

$\Rightarrow \psi$ is continuous

The probability currents

incident: $J_{in} = \frac{i\hbar}{2m}\left(\psi\frac{d\psi^*}{dx} - \psi^*\frac{d\psi}{dx}\right)$

$$= \frac{i\hbar}{2m}\left(-ik|\psi|^2 - ik|\psi|^2\right).$$

$$= \frac{\hbar k}{m}|e^{ikx}| = \frac{\hbar k}{m}$$

reflected: $J_{refl} = \frac{i\hbar}{2m}\left(ik|re^{ikx}|^2 + ik|re^{-ikx}|^2\right).$

$$= -\frac{\hbar k}{m}|r|^2$$

Transmitted :

$$J_{trans} = \frac{i\hbar}{2m}\left(-iK\,|te^{iKx}|^2 - iK\,|te^{iKx}|\right)$$

$$= \cancel{\frac{i\hbar}{2}} \quad \frac{\hbar K}{m}|t|^2.$$

Probability of reflection

$$R = \frac{-J_{refl}}{J_{in}} = |r|^2$$

Probability of transmission

$$T = \frac{J_{trans}}{J_{in}} = \frac{K}{k}|t|^2$$

Apply the boundary conditions

$$\psi_1(0) + \psi_2(0) = \psi_3(0)$$

$$\cancel{e^{ikx} + re^{i}}$$

$$e^0 + re^0 = te^0 \Rightarrow 1 + r = t$$

$$\psi_1'(0) + \psi_2'(0) = \psi_3'(0)$$

$$ik(1-r) = iKt$$

$$\therefore k - rk = K + rK$$

$$\therefore r = \frac{k-K}{k+K}, \quad t = \frac{2k}{k+K}$$

$\therefore$ Reflection Probability

$$R = \left(\frac{k-K}{k+K}\right)^2$$

Transmission Probability

$$T = \frac{K}{k}\left(\frac{2k}{k+K}\right)^2 = \frac{4kK}{(k+K)^2}$$

$$R+T = \left(\frac{k-K}{k+K}\right)^2 + \frac{4kK}{(k+K)^2}$$

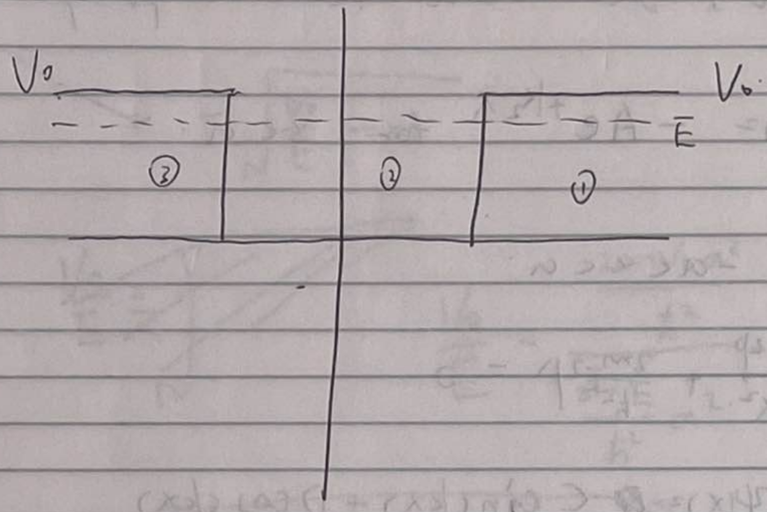
$$= \frac{k^2+K^2-2kK+4kK}{(k+K)^2}$$

$$= \frac{(k+K)^2}{(k+K)^2} = 1$$

$\therefore$ The flux of particles moving away from the origin is equal to the incident particle flux.

5.

$$V(x) = \begin{cases} 0 & , |x| < a \\ V_0 > 0 & , \text{otherwise} \end{cases}$$



Bound state $\Rightarrow$ $E < V_0$

Stationary state $\Rightarrow$ Solve the Time Independent Schrödinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\therefore \frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V(x) - E)\psi$$

For $x > a$ or $x < a$.

$$\cancel{\psi'' = \frac{2m}{\hbar}\psi}$$

$$\psi'' = \frac{2m}{\hbar^2}(V_0 - E)\psi \qquad \text{let } k_2 = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)} > 0.$$

$$\psi'' - k_2\psi = 0$$

$$\therefore \cancel{\psi(x)}\ \psi(x) = Ae^{k_2 x} + B^{-k_2 x}$$

Bound State $\Rightarrow$ $\psi(x)$ is finite at $\pm\infty$

$\therefore \psi_1(x) = Ae^{-k_2 x}$ for $x > a$

~~and $\psi(x) = Be$~~ and the odd-parity requires that

$\psi_3(x) = -Ae^{+k_2 x}$ for $x < a$.

For $-a < x < a$.

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\therefore \psi(x) = C\sin(kx) + D\cos(kx)$$

For odd-parity we require that

$$\psi_2(x) = C\sin(kx) \quad \text{where } k = \frac{\sqrt{2mE}}{\hbar}$$

Boundary Conditions:

$$\psi_1(a) = \psi_2(a)$$

$$\Rightarrow C\sin(ka) = Ae^{-k_2 a} \quad (1)$$

$$\psi_1'(a) = \psi_2'(a)$$

$$kC\cos(ka) = -k_2 Ae^{-k_2 a} \quad (2)$$

$$\psi_2(a) = \psi_3(a) \Rightarrow -C\sin(ka) = -Ae^{-k_2 a} \quad (3)$$

$$\psi_2'(-a) = \psi_3'(-a) \Rightarrow -kC\cos(ka) = kAe^{-k_2 a} \quad (4)$$

$$\frac{(4)}{(3)} = \frac{(2)}{(1)} \Rightarrow \cot(ka) = -\frac{k_2}{k}$$

$$\therefore \cot(ka) = -\frac{k_2}{k} = -\sqrt{\frac{\frac{2m}{\hbar^2}(V_0 - E)}{\frac{2m}{\hbar^2}E}}$$

$$= -\sqrt{\frac{V_0}{E} - 1}$$

$$\cancel{\frac{V_0}{E} = \sqrt{}}$$

$$\frac{V_0}{E} = \frac{\frac{2mV_0 a^2}{\hbar^2}}{\frac{2mE}{\hbar^2}a^2} = \frac{W^2}{(ka)^2}$$

$$\therefore \boxed{\cot(ka) = -\sqrt{\frac{W^2}{(ka)^2} - 1}}$$

If $W \leq \frac{\pi}{2}$ (we know that $W \geq 0$ by definition)

then for $\cot(ka)$ to be real, $W^2 \geq (ka)^2$.

$\therefore |ka| \leq \frac{\pi}{2}$

$\because \cot(ka) \leq 0 \quad \therefore -\frac{\pi}{2} \leq ka \leq 0$

$\because k = \frac{\sqrt{2mE}}{\hbar^2}$ and $E \neq 0 \quad \therefore k \neq 0 \quad \therefore ka \neq 0$

$\because E > 0 \quad \therefore k > 0$ and $\because a > 0 \quad \therefore ka > 0$

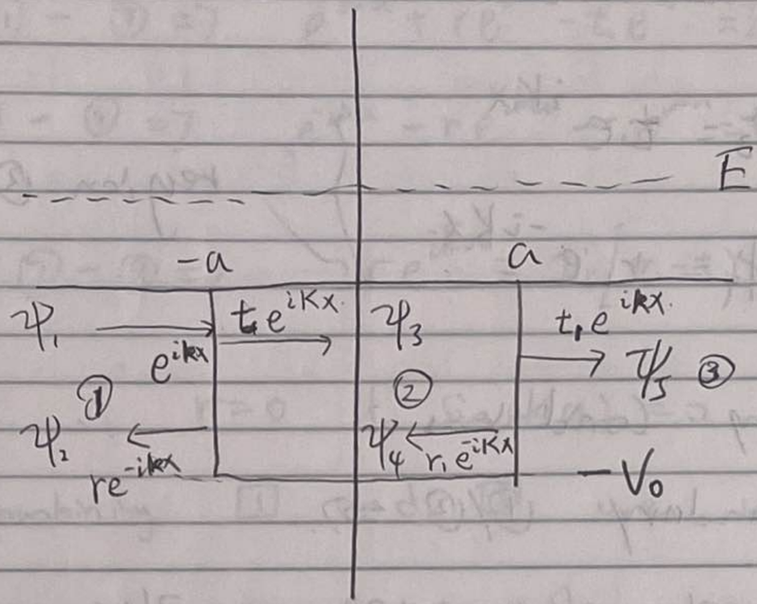
There is a contradiction between

$-\frac{\pi}{2} \leq ka \leq 0$ and $ka > 0$

$\therefore W \leq \frac{\pi}{2}$ is wrong

$$\therefore \boxed{W > \pi/2}$$

6.



For $x < -a$ and $x > a$.

Solve the TISE

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \qquad \text{let } k = \frac{\sqrt{2mE}}{\hbar}$$

Incident $\psi_1 = e^{ikx}$
reflected $\psi_2 = re^{-ikx}$ } region 1

$\cancel{\psi_3 = te^{iKx}}$
$\cancel{\psi_4 = r_1 e^{-iKx}}$ } ~~region 2~~

$\psi_5 = t_1 e^{ikx}$ => region 3

Solve the TISE

$$\cancel{\frac{d^2\psi}{dx^2}} \left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} - V_0\right)\psi = E\psi$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} + \frac{2m(V_0+E)}{\hbar^2}\psi = 0$$

let $K = \frac{\sqrt{2m(V_0+E)}}{\hbar}$ then we have waves

$$\psi_3 = t e^{iKx}$$
$$\psi_4 = r_1 e^{-iKx}$$
} region ②

Boundary Conditions :

At boundary ①/② =>

$\cancel{\psi_1 + \psi_2}$ $\psi_1(-a) + \psi_2(-a) = \psi_3(-a) + \psi_4(-a)$

$$\therefore \; e^{-ika} + r e^{ika} = t e^{-iKa} + r_1 e^{iKa} \quad ①$$

$$\psi_1'(-a) + \psi_2'(-a) = \psi_3'(-a) + \psi_4'(-a)$$

$$\therefore \; ik e^{-ika} - ikr e^{ika} = iKt e^{-iKa} - \cancel{iKe^{iKa}} \; iKr_1 e^{iKa} \quad ②$$

At boundary ②/③ =>

$$t e^{iKa} + r_1 e^{-iKa} = t_1 e^{ika} \quad ③$$

$$iKt e^{iKa} - iKr_1 e^{-iKa} = ikt_1 e^{ika} \quad ④$$

① + ③ => $e^{-ika} + re^{ika} + t_1 e^{ika} = 2(t + r_1) \cdot \cos(Ka)$ ⑤

② + ④ =>

$e^{-ika} - re^{ika} + t_1 e^{ika} = \frac{2K}{k}(t - r_1)\cos(Ka)$ ⑥

⑤ − ⑥ => $2re^{ika} = \left[2(t + r_1) - \frac{2K}{k}(t - r_1)\right]\cos(Ka)$

$\therefore \; r = 0$ if $\cos(Ka) = 0$ $\boxed{1}$

① − ③ ⟹ $e^{-ika} + re^{ika} - t_1 e^{ika} = 2i(r_1 - t)\sin(Ka)$ ⑦

② − ④ ⟹ $e^{-ika} - re^{ika} - t_1 e^{ika} = -2i\frac{K}{k}(t + r_1)\sin(Ka)$ ⑧

⑦ − ⑧ ⟹ $2re^{ika} = 2i\left[(r_1 - t) + \frac{K}{k}(r_1 + t)\right]\sin(Ka)$

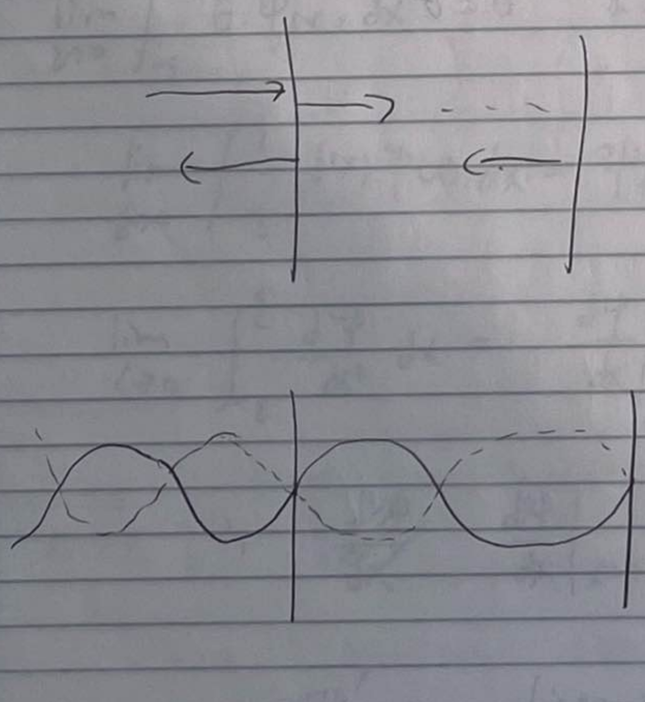
$\therefore$ $r = 0$ if $\sin(Ka) = 0$ $\boxed{2}$

Combining $\boxed{1}$ and $\boxed{2}$ we see that

$r = 0$ if $\cos(Ka) = 0$ or $\sin(Ka) = 0$

⟹ $r = 0$ if $Ka = \frac{n\pi}{2}$

$\therefore$ $\boxed{\text{Reflection probability } R = |r|^2 = 0 \text{ if } Ka = \frac{n\pi}{2}}$



$Ka = \frac{n\pi}{2}$

⟹ $2Ka = n\pi$

$\because K = \frac{2\pi}{\lambda}$

$\therefore$ we have $n\lambda = 4a$

$\therefore$ To have total transmission, we must have the wavelength λ times a +ve integer n equal to twice the width of the well.

The explanation to this phenomenon is that when the ~~we~~ incident wave hits the boundary at $x = -a$ the reflected ~~l~~ wave has a phase shift of π ~~because it is going~~ whereas at the ~~to~~ other boundary there is no phase shift.

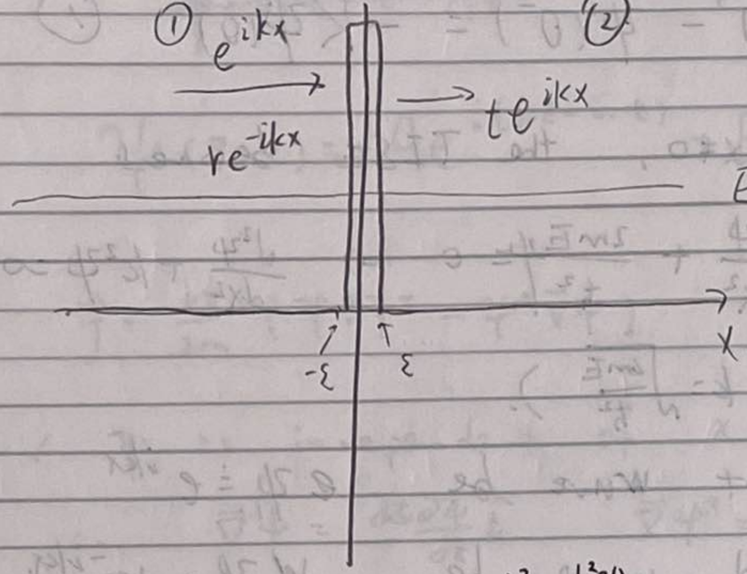
The path difference between wave reflected at $x = -a$ and that at $x = a$ is $4a$. So as long as $4a$ = integer multiple of π then the interference between these two ~~to~~ reflected waves is completely ~~de~~ destructive. So the total reflected ~~wa~~ wave ~~goes~~ vanishes.

$Ka = \frac{n\pi}{2}$

$\Delta\phi = 2(2a)K$

$= 2\pi n.$

7. $V(x) = V_\delta \delta(x)$



TISE $\Rightarrow$ $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_\delta \delta(x)\psi = E\psi$

Integrate this equation from $-\varepsilon$ to $+\varepsilon$ and take the limit $\varepsilon \to 0$ yields.

$$\lim_{\varepsilon\to 0}\left\{-\frac{\hbar^2}{2m}\int_{-\varepsilon}^{\varepsilon}\frac{d^2\psi}{dx^2}dx + V_\delta\int_{-\varepsilon}^{\varepsilon}\delta(x)\psi(x)dx = \int_{-\varepsilon}^{\varepsilon}E\psi(x)dx\right\}$$

$\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}E\psi(x)dx = 0$ as ψ is a finite function.

$$\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}\delta(x)\psi(x)dx = \psi(0)$$

$$\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}\frac{d^2\psi}{dx^2}dx = \left.\frac{d\psi}{dx}\right|_{x=+\varepsilon} - \left.\frac{d\psi}{dx}\right|_{x=-\varepsilon}$$

$$\therefore \cancel{\frac{d^2\psi}{dx}} \quad \left.\frac{d\psi}{dx}\right|_{x=+\varepsilon} - \left.\frac{d\psi}{dx}\right|_{x=-\varepsilon} = +\frac{2mV_\delta}{\hbar^2}\psi(0)$$

ψ' is discontinuous at $x=0$.

$$\psi'(0^+) - \psi'(0^-) = -K\psi(0) \quad (1)$$

at $x \neq 0$, the TISE becomes

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0.$$

$$\left(k = \sqrt{\frac{2mE}{\hbar^2}}\right).$$

let incident wave be $\psi_1 = e^{ikx}$

reflected wave be $\psi_2 = re^{-ikx}$.

transmitted wave be $\psi_3 = te^{ikx}$

then ψ is continuous at $x=0$ gives

$$1 + r = t \quad (2) \Rightarrow r = t - 1$$

Boundary condition (1) gives $\Rightarrow$

$$ikt - (ik - ikr) = +Kt \quad (3)$$

(2) $\rightarrow$ (3) $\Rightarrow$ $\therefore ikt - ik + ik(t-1) = +Kt$.

$$\therefore 2ik(t-1) = +Kt$$

$$\therefore t(K + 2ik) = 2ik$$

$$\therefore t = \frac{2ik}{-K + 2ik}$$

$$P_{tun} = |t|^2 = \frac{4k^2}{K^2 + 4k^2} = \frac{1}{1 + K^2/4k^2} = \boxed{\frac{1}{1 + (K/2k)^2}}$$

8. $\cancel{\psi(x,t) = Ae^{i(kz-\omega t)}}$

$$\psi(z,t) = Ae^{i(kz-\omega t)} + Be^{-i(kz+\omega t)}$$

$$\vec{J} = \frac{i\hbar}{2m}(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi)$$

$\because$ ψ is independent of x and y

$\therefore$ $\vec{\nabla}\psi = \frac{\partial\psi}{\partial z}\hat{z}$ $\quad \vec{\nabla}\psi^* = \frac{\partial\psi^*}{\partial z}\hat{z}$

$$\therefore \vec{J} = \frac{i\hbar}{2m}\Big[(Ae^{i(kz-\omega t)} + Be^{-i(kz+\omega t)})((-ik)A^*e^{-i(kz-\omega t)} + ikB^*e^{+i(kz+\omega t)})$$

$$- (A^*e^{-i(kz-\omega t)} + B^*e^{i(kz+\omega t)})(ikAe^{i(kz-\omega t)} - ikBe^{-i(kz+\omega t)})\Big]\hat{z}$$

$$= \frac{i\hbar}{2m}\Big[-2ik|A|^2 + 2ik|B|^2 - ik(\cancel{BA^*e^{-2ikz}}) + \cancel{AB^*e^{+2ikz}}(ik)$$

$$-ik\cancel{AB^*e^{+2ikz}} + ik\,\cancel{BA^*e^{-2ikz}}\Big]\hat{z}$$

$$= \frac{\hbar k}{m}[|A|^2 - |B|^2]\hat{z} = v(|A|^2 - |B|^2)\hat{z}$$

Physically the probability current density of the superposition of a wave travelling in positive direction and a wave travelling in negative direction is the difference between the probability density current of each individual ~~wave~~

wave.

superposition ~~of~~ of

$J \to$ the signed sum

of the currents

of each wave.

~~If~~ $|A| = |B| \to$ standing wave

probability goes to

0

9.

(a) $\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \psi^* \hat{x} \psi \, dx = \int_{-\infty}^{\infty} x |\psi|^2 dx$

(b) $\langle \hat{x}^2 \rangle = \int_{-\infty}^{\infty} \psi^* \hat{x}^2 \psi \, dx = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx$

(c) $\langle \hat{P}_x \rangle = \int_{-\infty}^{\infty} \psi^* \hat{P}_x \psi \, dx = -i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} dx$

(d) $\langle \hat{P}_x^2 \rangle = \int_{-\infty}^{\infty} \psi^* \hat{P}_x^2 \psi \, dx = -\hbar^2 \int_{-\infty}^{\infty} \psi^* \frac{\partial^2 \psi}{\partial x^2} dx$

(e) $\langle E \rangle = \int_{-\infty}^{\infty} \psi^* \hat{H} \psi \, dx$

$$= \int_{-\infty}^{\infty} \psi^* \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi \right) dx$$

The Probability that the particle will be found in the interval (x_1, x_2) is

$$P_{x_1, x_2} = \frac{\int_{x_1}^{x_2} |\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

7

$|\psi\rangle$

$\langle x|\psi\rangle = \psi(x)$

$$|\psi\rangle = \int |x\rangle\langle x|\psi\rangle\, dx$$

$$= \int dx\, \langle x|\psi\rangle |x\rangle$$

$$= \int dx\, \psi(x) |x\rangle$$

$|x\rangle = \hat{e}_x$

basis vector in position space

$|\alpha\rangle$ represents the state in which ~~that~~ you are definitely ~~at~~ ~~state~~ at position x.

$$\langle x|\psi\rangle = \psi(x)$$

$$|\psi\rangle = |x\rangle$$

$$\psi(x) = \langle x|\psi\rangle = \langle x|x\rangle = \delta(x-x) = \delta(0) = 1$$