To : Caroline Terquem

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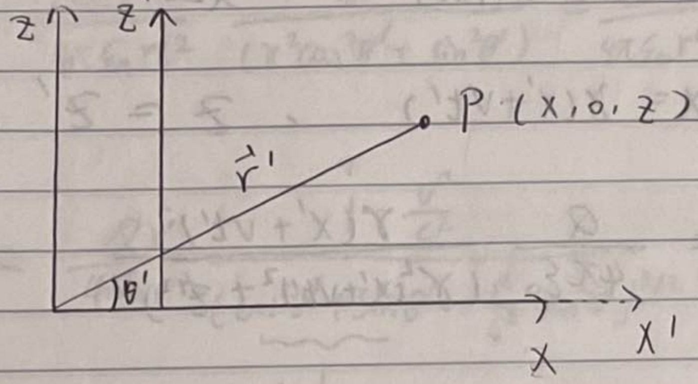
Electromagnetism 4

Excellent work!
In question 7, explain how you calculate the inductance.

1.



a)

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^3}\vec{r} \Rightarrow \quad E_x = \frac{Q}{4\pi\varepsilon_0}\frac{x}{(x^2+z^2)^{3/2}}$$

$$E_y = 0$$

$$E_z = \frac{Q}{4\pi\varepsilon_0}\frac{z}{(x^2+z^2)^{3/2}}$$

No magnetic field in frame F, and velocity $\vec{v}$ is along direction $\hat{x}$

$$\Rightarrow \quad E_x' = E_x, \quad E_y' = 0, \quad E_z' = \gamma E_z$$

$$\left(\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$

$$E_x' = \frac{Q}{4\pi\varepsilon_0}\frac{x}{(x^2+z^2)^{3/2}}$$

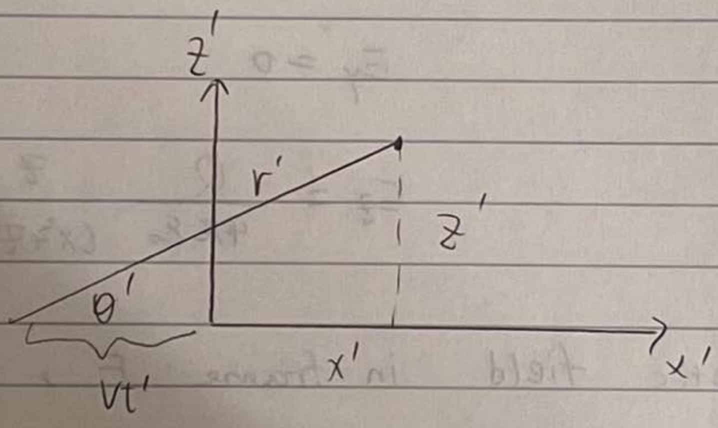
$$E_z' = \frac{Q}{4\pi\varepsilon_0}\frac{\gamma z}{(x^2+z^2)^{3/2}}$$

Lorentz Transformations $\Rightarrow$

$$x = \gamma(x' + vt') \quad , \quad z = z' \quad \checkmark$$

$$\therefore \quad E_x' = \frac{Q}{4\pi\varepsilon_0} \frac{\gamma(x'+vt')}{(\gamma^2(x'+vt')^2 + z'^2)^{3/2}}$$

$$E_z' = \frac{Q}{4\pi\varepsilon_0} \frac{\gamma z'}{(\gamma^2(x'+vt')^2 + z'^2)^{3/2}}$$



From Geometry we know

$$x' + vt' = r'\cos\theta' \quad , \quad z' = r'\sin\theta'$$ yes

$$\therefore \quad E_x' = \frac{Q}{4\pi\varepsilon_0} \frac{\gamma r'\cos\theta'}{(\gamma^2 r'^2\cos^2\theta' + r'^2\sin^2\theta')^{3/2}}$$

$$E_z' = \frac{Q}{4\pi\varepsilon_0} \frac{\gamma r'\sin\theta'}{(\gamma^2 r'^2\cos^2\theta' + r'^2\sin^2\theta')^{3/2}}$$

$$\vec{E}' = (E_x', 0, E_z') = \frac{Q}{4\pi\varepsilon_0} \frac{\gamma \overbrace{(\cos\theta', 0, \sin\theta')}^{\hat{r}'}}{(\gamma^2 r'^2\cos^2\theta' + r'^2\sin^2\theta')^{3/2}} \quad \checkmark$$

$$= \frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2} \frac{\gamma}{(\gamma^2\cos^2\theta' + \sin^2\theta')} = \frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2} \frac{\frac{1}{(1-\frac{v^2}{c^2})}(1-\frac{v^3}{c^2})^{3/2}}{(\frac{1}{1-\frac{v^2}{c^2}}\cos^2\theta' + \sin^2\theta')^{3/2}(1-\frac{v^3}{c^2})^{3/2}}$$

$$= \frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2} \frac{1-\frac{v^2}{c^2}}{(\underbrace{\cos^2\theta' + \sin^2\theta'}_{1} - \frac{v^2}{c^2}\sin^2\theta')^{3/2}} \quad (\text{let } \beta = \frac{v}{c})$$

$$= \boxed{\frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}}$$ yes

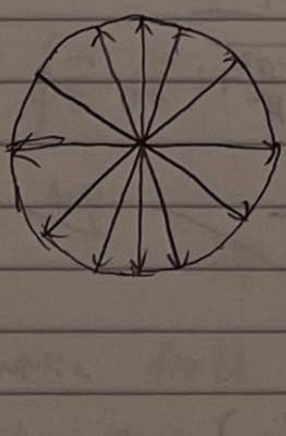
#

$\therefore \vec{E}'$ is parallel to $\hat{r}'$

When $v \ll c$, $\beta \to 0$

$\boxed{\vec{E}' \to \frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2}}$ becomes the Coulomb's Law.

b)



$$d\vec{S} = \hat{r}' r'^2 \sin\theta' d\theta' d\phi'$$

$$\vec{E}' = \frac{Q\hat{r}'}{4\pi\varepsilon_0 r'^2} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

$$\therefore \oiint \vec{E}\cdot d\vec{S} = \oiint \frac{Q}{4\pi\varepsilon_0 \cancel{r'^2}} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}} \cancel{r'^2} \sin\theta' d\theta' d\phi' \underbrace{\hat{r}'\cdot\hat{r}'}_{1}$$

$$= \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta' \frac{Q}{4\pi\varepsilon_0} \frac{(1-\beta^2)\sin\theta'}{(1-\beta^2\sin^2\theta')^{3/2}}$$

$$= 2\pi\left(\frac{Q}{4\pi\varepsilon_0}\right) \int_0^{\pi} d\theta' \frac{(1-\beta^2)\sin\theta'}{((1-\beta^2)+\beta^2\cos^2\theta')^{3/2}}$$

$$= \frac{Q}{2\varepsilon_0} \int_{-1}^{1} \frac{\left(\frac{1-\beta^2}{\beta^3}\right)}{\left(\left(\frac{1-\beta^2}{\beta^2}\right)+\cos^2\theta'\right)} d(\cos\theta')$$

$$= \frac{Q}{2\varepsilon_0}\frac{a^2}{\beta} \int_{-1}^{1} \frac{du}{(a^2+u^2)^{3/2}}$$

$$\boxed{\begin{aligned} u &= \cos\theta' \\ du &= -\sin\theta' d\theta' \\ a^2 &= \frac{1-\beta^2}{\beta^2} \end{aligned}}$$

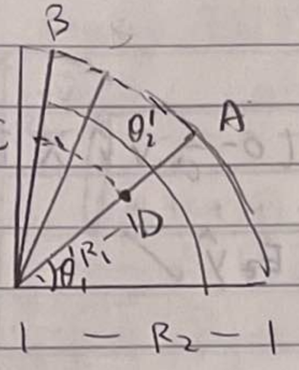
$$= \cancel{\frac{Q}{4}} \frac{Q}{2\varepsilon_0}\frac{a^2}{\beta}\frac{1}{a^2}\left[\frac{u}{\sqrt{a^2+u^2}}\right]_{-1}^{1}$$

$$= \frac{Q}{2\varepsilon_0\beta}\frac{2}{\sqrt{a^2+1}} = \frac{Q}{\varepsilon_0}\frac{1}{\beta\sqrt{\frac{1-\beta^2}{\beta^2}+1}}$$

$$= \frac{Q}{\varepsilon_0}\frac{1}{\sqrt{(\beta^2)\left(\frac{1}{\beta^2}-1+1\right)}} = \frac{Q}{\varepsilon_0}$$

yes

(Gauss's Law verified)

c) 

$\vec{dl} = \hat{r}' dr'$ along D → A and B → C

~~$\vec{dl} = -\hat{r}' dr'$ along B → C~~

$$\oint \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta_1')^{3/2}} \underbrace{(\hat{r}\cdot\hat{r})}_{1} dr' \frac{1}{r'^2}$$

$$+ \frac{Q}{4\pi\varepsilon_0}\int_{R_2}^{R_1} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta_2')} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta'^2)} (\hat{r}\cdot\hat{r})\, dr' \frac{1}{r'^2}$$

+ (line integral ~~on~~ along AB and ~~DE~~ CD, which are 0 because along those paths field lines are perpendicular) ✓

~~$\therefore \oint \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)(1-\beta^2)\left(\frac{1}{(1-\beta^2\sin^2\theta_1')^{3/2}} - \right.$~~

$$\oint \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)(1-\beta^3)\left(\frac{1}{(1-\beta^2\sin^2\theta_1')^{3/2}} - \frac{1}{(1-\beta^2\sin^2\theta_2')^{3/2}}\right)$$

$\neq 0$ ✓ A Moving charge produces magnetic field that changes with time at a fixed space coordinate in F', since $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$, $\oint \vec{E}\cdot d\vec{l} \neq 0$ by Stoke's Theorem ✓

d) $\vec{B}' = \vec{B}_\perp' + \vec{B}_\parallel'$

$$\vec{B}_\perp' = \gamma(\vec{B}_\perp - \tfrac{1}{c}\vec{v}\times\vec{E}_\perp) = \gamma(0 - \tfrac{1}{c^2} v\hat{x} \times E_z\hat{z})$$

$$= -\gamma \frac{v}{c^2} E_z(-\hat{y}) = \gamma \frac{v}{c^2} E_z \hat{y} \checkmark$$

$$\vec{B}_\parallel' = \vec{B}_\parallel = 0$$

$$\therefore \vec{B}' = \frac{\gamma v}{c^2} E_z \hat{y} = \frac{Q\hat{y}}{4\pi\varepsilon_0} \mu_0\varepsilon_0 \gamma v \frac{z}{(x^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 Q\hat{y}}{4\pi} \frac{\gamma v z}{(x^2+z^2)^{3/2}} = \frac{\mu_0 Q\hat{y}}{4\pi} \frac{\gamma v r' \sin\theta'}{(\gamma^2 r'^2 \cos^2\theta' + r'^2 \sin^2\theta')^{3/2}}$$

$$= \boxed{\frac{\mu_0 Q\hat{y}}{4\pi r'^2} \frac{v(1-\beta^2)\sin\theta'}{(1-\beta^2\sin^2\theta')^{3/2}}} \checkmark$$

$$\because v\hat{y}\sin\theta' = -v\hat{x} \times \underbrace{(\cos\theta'\hat{x} + \sin\theta'\hat{z})}_{\hat{r}'} = -\vec{v}\times\hat{r}'$$

$$\therefore \boxed{\vec{B}' = \frac{\mu_0 Q}{4\pi r'^2} \frac{(1-\beta^2)(-\vec{v})\times\hat{r}'}{(1-\beta^2\sin^2\theta')^{3/2}}}$$

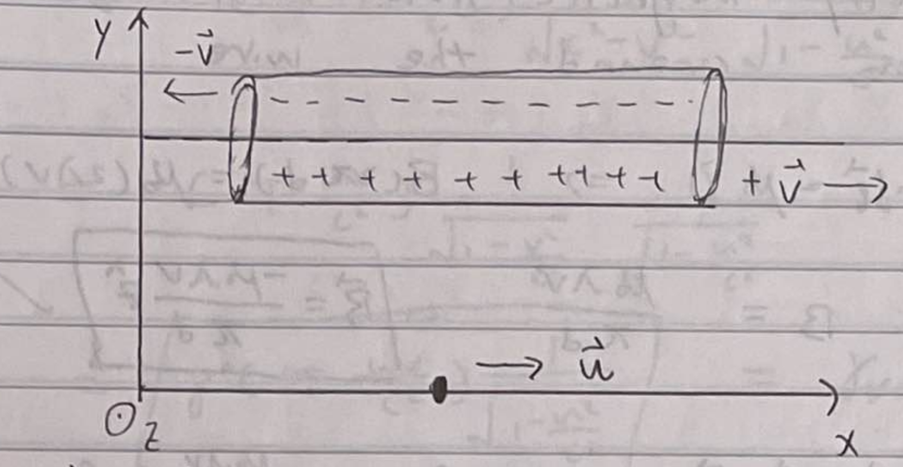
(In frame F' the velocity of ~~par~~ charge is $-\vec{v}$) ✓

When $v \ll c$, $\beta \to 0$, $\boxed{\vec{B}' \to \frac{\mu_0 Q(-\vec{v})\times\hat{r}'}{4\pi r'^2}}$

This becomes the Biot-Savart Law

yes

2.



a) In frame F

Equal amount of + an − charges

$\Rightarrow$ Net charge $\boxed{Q = 0}$ ✓

Current due to + charge :

$$I_+ = \frac{dq}{dt} = \left|\frac{\lambda d\vec{x}}{dt}\right| = \lambda v$$

Current due to − charge :

$$I_- = \frac{dq}{dt} = \left|-\lambda \frac{d\vec{x}}{dt}\right| = (-\lambda)(-v) = \lambda v$$

$\therefore$ Net current $I = I_+ + I_- = \boxed{2\lambda v}$ ✓

By symmetry Electric field must point radially and uniformly

$$\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} = 0 \Rightarrow E(2\pi d L) = 0$$

$$\Rightarrow \boxed{\vec{E} = 0}$$ ✓

By symmetry magnetic field must be circular around the wire

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi d) = \mu_0(2\lambda v)$$

$$\therefore B = \frac{\mu_0 \lambda v}{\pi d} \qquad \therefore \boxed{\vec{B} = \frac{-\mu_0 \lambda v}{\pi d}\hat{z}} \checkmark$$

$$\vec{f} = q(\vec{E} + \vec{u}\times\vec{B}) = q u \hat{x} \times \frac{\mu_0 \lambda v}{\pi d}(-\hat{z})$$

$$= \boxed{\frac{\mu_0 q \lambda u v \, \hat{y}}{\pi d}} \checkmark$$

b) $$V'_\pm = \frac{v \mp u}{1 \mp \frac{uv}{c^2}} \checkmark \qquad \gamma_\pm = \frac{1}{\sqrt{1 - \frac{v'^2_\pm}{c^2}}}$$

$$\therefore \gamma_\pm = \frac{c}{\sqrt{c^2 - \frac{(v\mp u)^2}{(1\mp\frac{uv}{c^2})^2}}}$$

$$= \frac{c}{\sqrt{\frac{c^2(1\mp\frac{uv}{c^2})^2 - (v\mp u)^2}{(1\mp\frac{uv}{c^2})^2}}}$$

$$= c\left(1\mp\frac{uv}{c^2}\right)\frac{1}{\sqrt{c^2 + \frac{u^2v^2}{c^2} \cancel{\mp 2uv} - v^2 - u^2 \cancel{\pm 2uv}}}$$

$$= c\left(1 \mp \frac{uv}{c^2}\right)\frac{1}{\sqrt{c^2-v^2}}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$= c\left(1 \mp \frac{uv}{c^2}\right)\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$= \boxed{\gamma\left(1 \mp \frac{uv}{c^2}\right)\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}} = \gamma_v \gamma_u \left(1 \mp \frac{uv}{c^2}\right)$$

$$\left(\gamma_v = \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

Define F_+ to be the rest frame of λ_+ and F_- to be the rest frame of λ_-

, and λ_0^+, λ_0^- be the rest charge densities of λ_+ and λ_-

Transfer from F_+ to F:

$$\frac{l_+}{\gamma_v} = l \quad \therefore l_+ = \gamma_v l \qquad \lambda_0^+ = \frac{Q}{l_+} = \frac{Q}{\gamma_v l} = \frac{\lambda}{\gamma_v}$$

Transfer from F_- to F:

$$\frac{l_-}{\gamma_v} = l \quad \therefore l_- = \gamma_v l \qquad \lambda_0^- = \frac{-Q}{l_-} = \frac{-Q}{\gamma_v l} = \frac{-\lambda}{\gamma_v}$$

$$\lambda_+' = \gamma_+ \lambda_0^+ \checkmark = \gamma_v \gamma_u \left(1 - \frac{uv}{c^2}\right)\left(\frac{\lambda}{\gamma_v}\right) = \gamma_u \left(1 - \frac{uv}{c^2}\right)\lambda$$

$$\lambda_-' = \gamma_- \lambda_0^- \checkmark = \gamma_v \gamma_u \left(1 + \frac{uv}{c^2}\right)\left(-\frac{\lambda}{\gamma_v}\right) = \gamma_u \left(-1 - \frac{uv}{c^2}\right)\lambda$$

$$\therefore \lambda' = \lambda_+' + \lambda_-' = -\frac{2\gamma_u u v \lambda}{c^2}$$ yes

Gauss's Law: $\oiint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

~~E (2πd~~ $E(2\pi d L) = \frac{\lambda' L}{\varepsilon_0}$

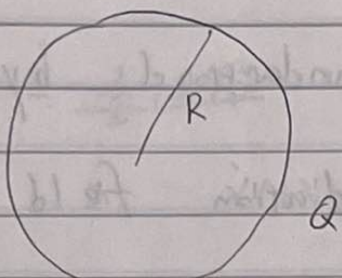
$$\vec{E} = \frac{-\lambda'}{2\pi\varepsilon_0 d}\hat{y} = (-\hat{y})\left(\frac{1}{2\pi\varepsilon_0 d}\right)\left(-\frac{2\gamma_u u v \lambda}{c^2}\right)$$

$$= \frac{\mu_0 \lambda \gamma_u u v \hat{y}}{\pi d} \checkmark$$

$$\therefore \vec{f}' = q\vec{E}' = \frac{\mu_0 q \lambda \gamma_u u v \hat{y}}{\pi d}$$

* Note that $\vec{f}' = \gamma_u \vec{f}$ ✓

3.



With radial oscillations the charge distribution is still spherically symmetric at all times

$\therefore \vec{E}$ points radially

$\Rightarrow$ Gauss's Law $\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0} \Rightarrow (4\pi r^2)E = \frac{Q}{\varepsilon_0}$

$$\Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}} \checkmark$$

$\therefore \vec{B}$ points radially

$\Rightarrow$ Gauss's Law for magnetism

$$\oint \vec{B} \cdot d\vec{S} = 0 \Rightarrow (4\pi r^2)B = 0 \Rightarrow \boxed{\vec{B} = 0} \checkmark$$

Larmor's formula : $P_{rad} = \frac{\omega^4 p^2}{6\pi\varepsilon_0 c^3}$

Dipole moment for the sphere :

$$\vec{p} = \int_{sphere} \vec{r}\, dq$$

This integral is 0 due to spherical symmetry $\therefore \vec{p} = 0 \Rightarrow P_{rad} = 0$

you can also use directly $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \vec{0}$

$\therefore$ No radiation

This can also be understood by noticing that the radiation field is E_θ and B_φ ~~and~~. Radiation field is 0 in radial direction for radial oscillation (for both electric and magnetic fields)

$\therefore$ ~~For electric field~~

All the radiation fields for each small oscillation charge cancel with each other due to spherical symmetry. So the only fields left is the radial field. yes

The field along direction of oscillation (in this case the radial field) is the same as if the charges are static

So for this problem we can treat ~~as~~ everything as being static.

4. a) $E(t) = \frac{1}{2} m\omega_0^2 x^2(t) + \frac{1}{2} m \dot{x}^2(t)$

$= \cancel{\frac{1}{2} m A^2}$ $\quad x(t) = A e^{-t/\tau} \cos(\omega_0 t)$

$\dot{x}(t) = A\left(-\frac{1}{\tau} e^{-t/\tau} \cos(\omega_0 t) - \omega_0 e^{-t/\tau} \sin(\omega_0 t)\right)$

$\therefore E(t) = \cancel{\frac{1}{2} m \omega_0^2} \; \frac{1}{2} m A^2 \Big(\omega_0^2 e^{-2t/\tau} \cos^2(\omega_0 t) + \omega_0^2 e^{-2t/\tau} \sin^2(\omega_0 t)$

$+ 2\omega_0 (\frac{1}{\tau}) e^{-2t/\tau} \sin(\omega_0 t) \cos(\omega_0 t)$

$+ (\frac{1}{\tau})^2 e^{-2t/\tau} \cos^2(\omega_0 t) \Big)$

$\because \omega_0 \tau \gg 1 \quad \therefore \omega_0 \gg \frac{1}{\tau}$, $\because \cos^2(\omega_0 t) + \sin^2(\omega_0 t) = 1$

and $1 > \cos^2(\omega_0 t)$

$1 > \sin(\omega_0 t) \cos(\omega_0 t)$

$\therefore \boxed{E(t) = \frac{1}{2} m A^2 \omega_0^2 e^{-2t/\tau}}$ ✓

b) Larmor's formula : $P_{rad} = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$

$\therefore \frac{dE}{dt} = -\frac{q^2 \ddot{x}^2(t)}{6\pi\varepsilon_0 c^3}$ ✓

$\ddot{x}(t) = A(-\frac{1}{\tau})(-\frac{1}{\tau} e^{-t/\tau} \cos(\omega_0 t) - \omega_0 e^{-t/\tau} \sin(\omega_0 t))$

$- A\omega_0 (-\frac{1}{\tau} e^{-t/\tau} \sin(\omega_0 t) + \omega_0^2 e^{-t/\tau} \cos(\omega_0 t))$

When $\omega_0 \gg \frac{1}{\tau}$, $\ddot{x}(t) \approx -\omega_0^2 A e^{-t/\tau} \cos(\omega_0 t)$

$$\therefore \frac{dE}{dt} = -\frac{q^2 \omega_0^4 A^2 e^{-2t/\tau}}{6\pi \varepsilon_0 c^3} \cos^2(\omega_0 t)$$ ✓

Over a period :

change in E is

$$\Delta E = \frac{1}{2} m A^2 \omega_0^2 \left[e^{-2(t+\frac{2\pi}{\omega_0})/\tau} - e^{-2t/\tau} \right]$$

$$= -\frac{1}{2} m A^2 \omega_0^2 \left(e^{-2t/\tau}\right)\left(1 - e^{-\frac{4\pi}{\omega_0 \tau}}\right)$$

$\because \omega_0 \tau \gg 1 \quad \therefore \omega_0 \tau \gg 4\pi \quad \therefore \frac{4\pi}{\omega_0 \tau} \ll 1$

~~$\therefore \Delta E = -\frac{1}{2} m A^2 \omega_0^2 e^{-2t/\tau} \left(\frac{4\pi}{\omega_0 \tau}\right)$~~

$\therefore \frac{4\pi}{\omega_0 \tau} \to 0 \qquad \therefore 1 - e^{-\frac{4\pi}{\omega_0 \tau}} \to 0$

$\therefore \quad \Delta E \to 0$ The variation of E is very small over a period of oscillation ✓

$\therefore$ Over a period of oscillation

$\because \quad E(t) = \frac{1}{2} m A^2 \omega_0^2 e^{-2t/\tau}$ varies very little

$\therefore \quad e^{-2t/\tau}$ varies very little.

$\therefore$ We can treat $e^{-2t/\tau}$ as constant when $\cos^2(\omega_0 t)$ varies over a period.

$\therefore$ we can take the time-average of $\cos^2(\omega_0 t)$, which is $\frac{1}{2}$

$$\therefore \quad \frac{dE}{dt} = -\frac{q^2 \omega_0^4 A^2 e^{-2t/\tau}}{12\pi\varepsilon_0 c^3}$$

c) $E(t) = \frac{1}{2} m A^2 \omega_0^2 e^{-2t/\tau}$

$$\therefore \quad \frac{dE}{dt} = -\frac{mA^2\omega_0^2}{\tau} e^{-2t/\tau}$$

$$\therefore \quad -\frac{mA^2\omega_0^{\cancel{2}}}{\tau} \cancel{e^{-2t/\tau}} = -\frac{q^2 \omega_0^{\cancel{4}\,2} A^2}{12\pi\varepsilon_0 c^3} \cancel{e^{-2t/\tau}}$$

$$\therefore \quad \boxed{\tau = \frac{12\pi\varepsilon_0 c^3 m}{q^2 \omega_0^2}}$$

$\lambda_0 = 5000\,\text{Å} = 5 \times 10^{-7}\,\text{m} \Rightarrow \omega_0 = \frac{2\pi c}{\lambda_0} \Rightarrow \tau = \frac{3\varepsilon_0 c m \lambda_0^2}{\pi q^2}$

$$\therefore \quad \tau = \frac{3(8.854\times10^{-12})(3.0\times10^8)(9.11\times10^{-31})(5\times10^{-7})^2}{\pi(1.6\times10^{-19})^2}$$

$= \boxed{2.26 \times 10^{-8}\,s} \sim 10^{-8}\,s$ = lifetime of a typical excited state in a free-decaying atom

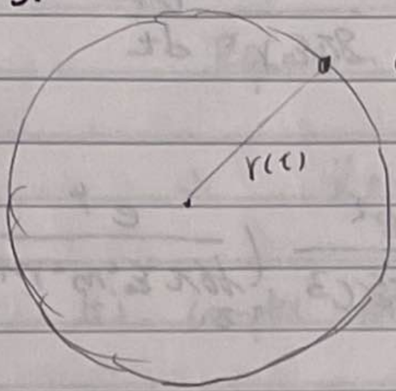
5.

$$\omega_0 = \frac{2\pi c}{\lambda_0} = 3.77 \times 10^{15} s^{-1}$$

$$\therefore \quad \omega_0 \tau = (3.77 \times 10^{15} s^{-1})(2.26 \times 10^{-8} s)$$

$$= \underline{8.52 \times 10^{7} >> 1} \checkmark$$

5.



a) Coulomb Force:

$$ma = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$\therefore \boxed{a = \frac{e^2}{4\pi\varepsilon_0 m r^2}}$$

b) $E = T + V$

near circular motion $\Rightarrow a = \frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2 m}$

$$\Rightarrow v^2 = \frac{e^2}{4\pi\varepsilon_0 r m}$$

$$T = \frac{1}{2} m v^2 = \frac{e^2}{8\pi\varepsilon_0 r}$$

$$\because V = -\frac{e^2}{4\pi\varepsilon_0 r} \qquad \therefore \boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

c) Larmor's Formula:

$$P_{rad} = \frac{e^2 a^2}{6\pi\varepsilon_0 c^3}$$

$$\therefore \frac{dE}{dt} = -P_{rad} = -\left(\frac{e^2}{6\pi\varepsilon_0 c^3}\right)\left(\frac{e^4}{16\pi^2\varepsilon_0^2 m^2 r^4}\right)$$

6.

$$\frac{dE}{dt} = \frac{d}{dt}\left(-\frac{e^2}{8\pi\varepsilon_0 r^2}\right) = \frac{e^2}{8\pi\varepsilon_0 r^2}\frac{dr}{dt}$$

$$\therefore \quad \frac{e^2}{8\pi\varepsilon_0 r^2}\frac{dr}{dt} = -\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{e^4}{\cancel{16}\,2\pi^2\varepsilon_0^2 m^2 r^{\cancel{4}\,2}}\right)$$

$$\therefore \quad \frac{dr}{dt} = -\left(\frac{e^4}{12\pi^2\varepsilon_0^2 m^2 c^3}\right)\frac{1}{r^2}$$

$$\therefore \quad \int_a^0 r^2\,dr = -\frac{e^4}{12\pi^2\varepsilon_0^2 m^2 c^3}\int_0^\tau dt$$

$$\therefore \quad -\frac{a_0^3}{3} = -\frac{e^4}{12\pi^2\varepsilon_0^2 m^2 c^3}\tau$$

$$\therefore \quad \boxed{\tau = \frac{4\pi^2\varepsilon_0^2 c^3 m^2 a_0^3}{e^4}} \checkmark$$

$$\tau = \frac{(4\pi^2)(8.854\times10^{-12})^2(3.0\times10^{8})^3(9.11\times10^{-31})^2(0.53\times10^{-10})^3}{(1.6\times10^{-19})^4}$$

$$= \boxed{1.57\times10^{-11}\,s} \checkmark$$

Orbital period $T = \frac{2\pi a_0}{v} = \frac{2\pi a_0}{\sqrt{\frac{e^2}{4\pi\varepsilon_0 a_0 m}}} = 2\pi a_0 e^{-1}\sqrt{4\pi\varepsilon_0 a_0 m}$

$$= \frac{4\pi^{3/2}}{e}\sqrt{\varepsilon_0 m}\,a_0^{3/2} = \boxed{1.53\times10^{-16}s} \checkmark$$

$$\tau \approx 10^5 T \implies \boxed{\tau \gg T}$$

τ is small, so the electron will spiral into the nucleus very quickly. But in reality this does not happen. So the classical model must be wrong and we require the quantum model to explain photon emission

yes

6. a)

Equation of motion: $m\ddot{x} = -m\omega_0^2 x + qE_0\cos(\omega t)$

$\therefore \quad \ddot{x} + \omega_0^2 x = \frac{qE_0}{m}\cos(\omega t)$

It's complex form $\Rightarrow \quad \ddot{\tilde{x}} + \omega_0 \tilde{x} = \frac{qE_0}{m} e^{i\omega t}$

$(x = Re(\tilde{x}))$

Try $\tilde{x} = Ae^{i\omega t}$, $\cancel{-\omega^2 A} - \omega^2 A + \omega_0^2 A = \frac{qE_0}{m}$

$\Rightarrow \quad A = \frac{qE_0}{m(\omega_0^2 - \omega^2)}$

$\therefore \quad \tilde{x} = \frac{qE_0}{m(\omega_0^2 - \omega^2)} e^{i\omega t}$

$\therefore \quad \boxed{x(t) = \frac{qE_0 \cos(\omega t)}{m(\omega_0^2 - \omega^2)}}$ in steady ~~Stea~~ state

yes

b) $\omega_0 \gg \omega \quad \Rightarrow \quad x(t) = \frac{qE_0}{m\omega_0^2}\cos(\omega t)$

$\ddot{x} = -\frac{qE_0\omega^2}{m\omega_0^2}\cos(\omega t)$

Larmor's formula: $P_{rad} = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$

$\therefore \quad P_{rad} = \frac{e^2 \ddot{x}^2}{6\pi\varepsilon_0 c^3} = \left(\frac{e^2}{6\pi\varepsilon_0 c^3}\right)\left(\frac{e^2 E_0^2 \omega^4}{m^2\omega_0^4}\right)\cos^2\omega t$

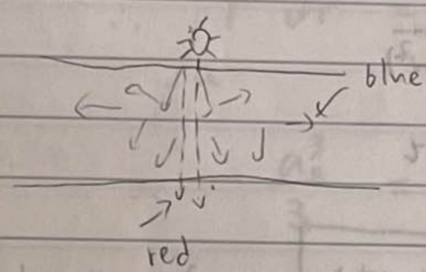
Time average ~~gives~~ $\langle\cos^2\omega t\rangle$ gives $\frac{1}{2}$ $\Rightarrow$

$$\boxed{P_{rad} = \frac{e^4 E_0^2 \omega^4}{12\pi\varepsilon_0 c^3 \omega_0^4 m^2}}$$

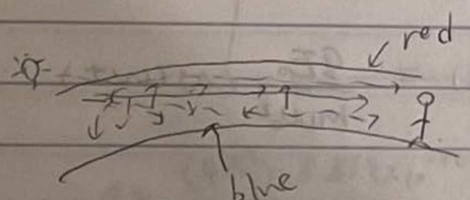
Noticing that $P_{rad} \propto \omega^4 \propto \frac{1}{\lambda^4}$ ✓

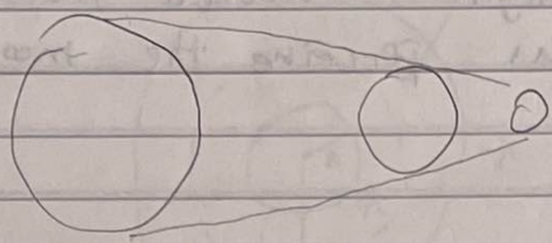
Hence the ~~cool~~ ~~color~~ colour with shorter wavelengths radiate more energy to all the directions ~~other that the direction of~~ than longer wavelengths. The light with short wave-length gets scattered more by air molecules. ~~So~~ ✓ ~~the~~

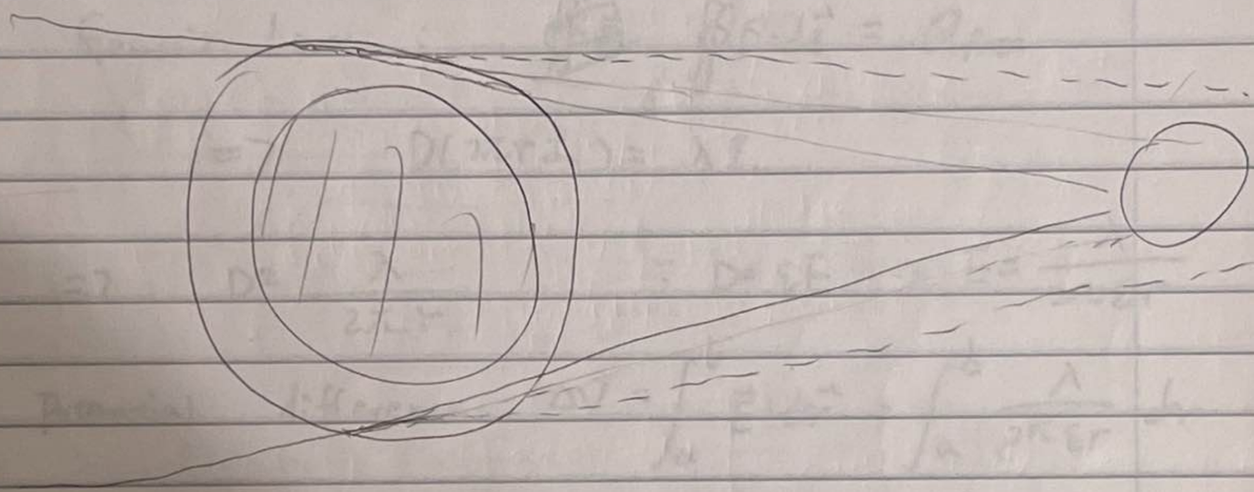
At day time, when we look at the part of sky that is ~~fa~~ not too close to the sun, ~~the~~ ~~sky~~ we ~~will~~ see the scattered light, which is mostly blue. So the sky is blue ✓



At ~~night~~ sunset, The blue colour ~~is~~ is still scattered more than red light, But in this case, the observer sees the light ~~that~~ from the Sun that travels tangentially and thus a much longer distance. The blue light will be scattered ~~out~~ and effectively filtered out by the atmosphere and leave the red light to enter the eye of an observer ~~.~~ So the sunset looks red. yes







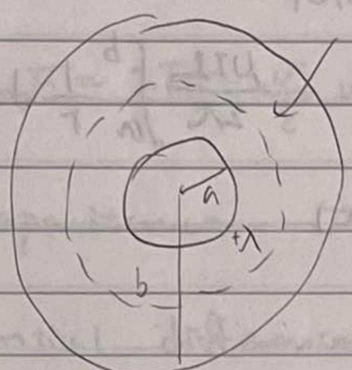
Same as the case for sunset, in the total lunar eclipse the light from the sun travels tangential to the Earth and thus travels the whole atmosphere. ~~Bute~~ Blue light is scattered out due to its high scattering efficiency so only red light is able to pass the atmosphere. ~~and~~ And ~~due~~ since the sunlight is refracted at the atmosphere, it is bent towards the moon and able to reach the moon (without the refraction ~~the~~ no light will be able to reach ~~mo~~ the moon)
The moon then reflect the light that reaches it, which is red, back to

Earth's surface. So observers on Earth

see ~~sees~~ the red light reflected from the

yes Moon and thus perceive the moon as red.

7 a)



Gauss's Law : $\cancel{\oiint \vec{B}}$ $\oiint \vec{D}\cdot d\vec{s} = Q_{free}$

$\Rightarrow \quad D(2\pi r L) = \lambda L$

$\Rightarrow \quad D = \dfrac{\lambda}{2\pi r} \qquad \because D = \varepsilon E \quad \therefore E = \dfrac{\lambda}{2\pi\varepsilon r}$

Potential difference $\Delta V = \int_a^b \vec{E}\cdot d\vec{r} = \int_a^b \dfrac{\lambda}{2\pi\varepsilon r}\,dr$

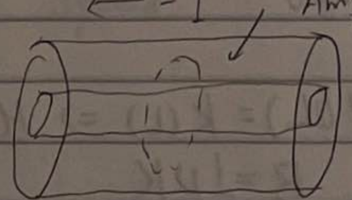
$= \dfrac{\lambda}{2\pi\varepsilon}\ln(b/a)$

Capacitance per unit length

$C = \dfrac{\tilde{C}}{L} = \dfrac{Q}{\Delta V L} = \dfrac{\lambda L}{\frac{\lambda L \ln(b/a)}{2\pi\varepsilon}} = \boxed{2\pi\varepsilon/\ln(b/a)}$

yes

denominator



Ampere's Law : $\oint \vec{H}\cdot d\vec{l} = I_{free}$

$\Rightarrow \quad H(2\pi r) = I \quad \Rightarrow \quad H = \dfrac{I}{2\pi r}$ ✓

~~Ans~~ $B = \mu H = \frac{\mu I}{2\pi r}$, Flux $\Phi_B = \int \vec{B}\cdot d\vec{s}$

$\therefore \Phi_B = \int B(\ell\, dr) = \frac{\mu I \ell}{2\pi}\int_a^b \frac{1}{r}\, dr$ explain

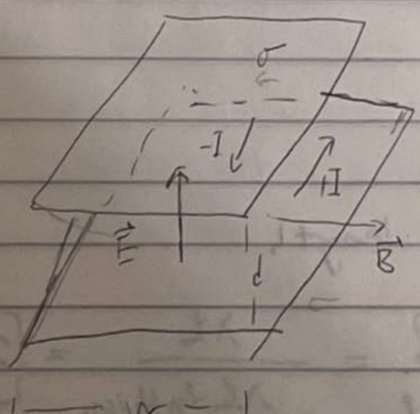
$= \frac{\mu I \ell}{2\pi} \ln(b/a)$

Inductance per unit length is

$$L = \frac{\tilde{L}}{\ell} = \frac{\Phi_B}{I\ell} = \boxed{\frac{\mu}{2\pi}\ln(b/a)}$$

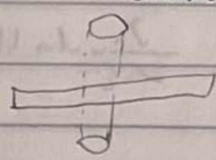
Speed $v = \frac{1}{\sqrt{CL}} = \boxed{\frac{1}{\sqrt{\mu\varepsilon}}}$

b)



Surface charge density σ

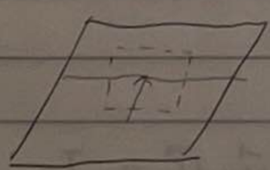
~~Linear~~ Linear current density K



Gauss's Law: $D(S) = \sigma S$

$\therefore D = \sigma \quad \therefore E_1 = \frac{\sigma}{2\varepsilon}$

Ampere's Law:



~~K~~ $H(2L) = K(L) \Rightarrow H = K$

$\therefore B_1 = \frac{1}{2}\mu K$

Superposition of 2 slabs gives

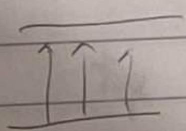
$$E = 2E_1 = \frac{\sigma}{\varepsilon} \checkmark , \quad B = 2B_1 = \mu K \checkmark$$

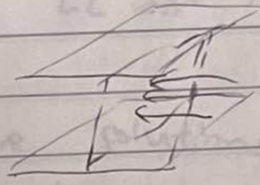
~~Capacitance~~ $\cancel{\tilde{C}}$

potential difference : $\Delta V = Ed = \frac{\sigma d}{\varepsilon}$ ✓

magnetic flux : $\Phi_B = B(d\ell) = \mu K d\ell$

explain





Capacitance / length θ :

$$C = \frac{\tilde{C}}{\ell} = \frac{Q}{\Delta V \ell} = \frac{\cancel{\sigma} w \cancel{\ell}}{\frac{\cancel{\sigma} d}{\varepsilon}\cancel{\ell}} = \boxed{\frac{\varepsilon w}{d}} \checkmark$$

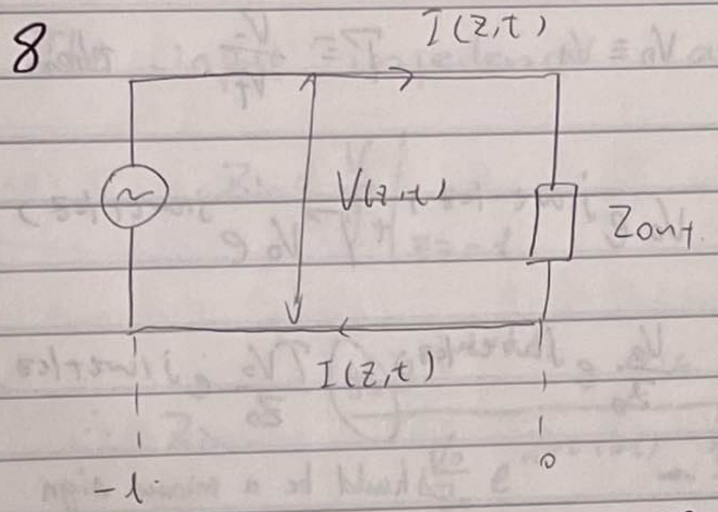
Inductance ~~p~~ / length :

$$L = \frac{\tilde{L}}{\ell} = \frac{\Phi_B}{I\ell} = \frac{\mu K d \ell}{K w \ell} = \boxed{\frac{\mu d}{w}} \checkmark$$

Speed

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\varepsilon w}{d}\frac{\mu d}{w}}} = \boxed{\frac{1}{\sqrt{\mu\varepsilon}}} \checkmark$$

8



$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial z^2}, \quad \frac{\partial^2 I}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 I}{\partial z^2}$$

wave solutions:

$$V(z,t) = V_+ e^{j(\omega t - kz)} + V_- e^{j(\omega t + kz)}$$

$$I(z,t) = I_+ e^{j(\omega t - kz)} + I_- e^{j(\omega t + kz)}$$

We know $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$

$$\therefore \quad -jkV_+ e^{j(\omega t - kz)} + jkV_- e^{j(\omega t + kz)}$$

$$= -L I_+ e^{j(\omega t - kz)}(j\omega) - L I_- e^{j(\omega t + kz)}(j\omega)$$

$$\Rightarrow \quad \cancel{jk}\ kV_+ = L\omega I_+ , \quad kV_- = -L\omega I_-$$

$$\therefore \frac{L\omega}{k} = Lv = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} = Z_0$$

$$\therefore \quad \cancel{\frac{V_+}{I_+}} \quad I_+ = \frac{V_+}{Z_0}, \quad I_- = -\frac{V_-}{Z_0}$$

~~Let~~ Define $V_0 \equiv V_+$, $\Gamma \equiv \frac{V_-}{V_+}$. then we have

$$V(z,t) = V_0 e^{j(\omega t - kz)} + \Gamma V_0 e^{j(\omega t + kz)}$$

$$I(z,t) = \frac{V_0}{Z_0} e^{j(\omega t - kz)} + \frac{\Gamma V_0}{Z_0} e^{j(\omega t + kz)}$$

should be a minus sign

At $z=0$, we must have $\frac{V}{I} = Z_{out}$

to satisfy Ohm's Law ~~(the capacitance~~

(General Ohm's Law, Z_{out} can have ~~ka~~ reactive coponents)

$$\therefore \frac{V_0 e^{j\omega t} + \Gamma V_0 e^{j\omega t}}{\frac{V_0}{Z_0} e^{j\omega t} + \frac{\Gamma V_0}{Z_0} e^{j\omega t}} = Z_{out}$$

$$\therefore \frac{Z_{out}}{Z_0} = \frac{1+\Gamma}{1-\Gamma} \Rightarrow (1-\Gamma) Z_{out} = (1+\Gamma) Z_0$$

yes...

$$\therefore \Gamma (Z_0 + Z_{out}) = Z_{out} - Z_0$$

$$\therefore \Gamma = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$$

The input impedence at $z = -l$ is

$$Z_{in} = \frac{V}{I}\bigg|_{z=-l}$$

$$\therefore Z_{in} = \frac{V_0 e^{j(\omega t + kl)} + \Gamma V_0 e^{j(\omega t - kl)}}{\frac{V_0}{Z_0} e^{j(\omega t + kl)} - \frac{\Gamma V_0}{Z_0} e^{j(\omega t - kl)}}$$

$$= Z_0 \frac{e^{jkl} + \Gamma e^{-jkl}}{e^{jkl} - \Gamma e^{-jkl}}$$

yes, but you didn't need to do all that...

$$= Z_0 \frac{e^{jkl} + \frac{Z_{out} - Z_0}{Z_{out} + Z_0} e^{-jkl}}{e^{jkl} - \frac{Z_{out} - Z_0}{Z_{out} + Z_0} e^{-jkl}}$$

$$= Z_0 \frac{(Z_{out} + Z_0) e^{jkl} + (Z_{out} - Z_0) e^{-jkl}}{(Z_{out} + Z_0) e^{jkl} - (Z_{out} - Z_0) e^{-jkl}}$$

$$= Z_0 \frac{Z_{out}(e^{jkl} + e^{-jkl}) + Z_0(e^{jkl} - e^{-jkl})}{Z_{out}(e^{jkl} - e^{jkl}) + Z_0(e^{jkl} + e^{-jkl})}$$

$$= Z_0 \frac{Z_{out}\cos(kl) + j Z_0 \sin(kl)}{Z_0 \cos(kl) + j Z_{out} \sin(kl)}$$

Note that $\boxed{\frac{\omega}{k} = v = \frac{1}{\sqrt{LC}}, \quad k = \frac{\omega}{v}}$

Open circuit : $Z_{out} \to \infty$

$$\therefore Z_1 = Z_0 \frac{\cos kl}{j\sin kl} = \boxed{-jZ_0 \cot(kl)}$$ ✓

when $\cot kl > 0$, ~~phase~~ current leads voltage by 90° $\Rightarrow$ stub equivalent to a capacitor

yes ✓

when ~~tan kl~~ $\cot kl < 0$, current lags voltage by 90° $\Rightarrow$ stub equivalent to an inductor

Short circuit : $Z_{out} = 0$

$$\therefore Z_2 = Z_0 \frac{j\sin kl}{\cos kl} = \boxed{jZ_0 \tan(kl)}$$ ✓

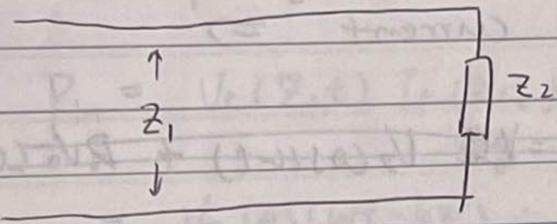
When $\tan kl > 0$, ~~the~~ current lags voltage by 90° $\Rightarrow$ inductor

yes ✓

When $\tan kl < 0$, current leads voltage by 90° $\Rightarrow$ capacitor

* $\boxed{Z_1 Z_2} = (-jZ_0 \cot(kl))(jZ_0 \tan(kl)) = (-j^2) Z_0^2 \tan(kl) \frac{1}{\tan kl} = \boxed{Z_0^2}$ ✓
* Stubs can be used to replace discrete capacitors and inductors ~~at~~ when the ~~fr~~ frequency is very high or low. Because at those frequencies there will be parasitic reactance ($j\omega L$ or $\frac{1}{j\omega C}$ gets large when ω is too large or too small) that affects the performance of discrete capacitors or inductors ✓

9.



$$T = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\tilde{V}(z,t) = V_0 e^{j(\omega t - kz)} + TV_0 e^{j(\omega t + kz)}$$

$$\tilde{I}(z,t) = \cancel{Z} \frac{V_0}{Z_1} e^{j(\omega t - kz)} - \frac{TV_0}{Z_1} e^{j(\omega t + kz)}$$

At the load, $z = 0$

$$\tilde{V}(0,t) = V_0 e^{j\omega t} + TV_0 e^{j\omega t}$$

$$\tilde{I}(0,t) = \frac{V_0}{Z_1} e^{j\omega t} - \frac{TV_0}{Z_1} e^{j\omega t}$$

let $T = Re^{j\theta}$ where θ, R ~~is~~ are real and $R = |T|$

$\theta = \arg(T)$ then

$$\tilde{V}(0,t) = V_0 e^{j\omega t} + RV_0 e^{j(\omega t + \phi)}$$
$$\tilde{I}(0,t) = \frac{V_0}{Z_1} e^{j\omega t} - \frac{RV_0}{Z_1} e^{j(\omega t + \phi)}$$

~~Power transmitted into the load is~~

~~$P_{load} = V$~~

Take the real part to get the actuall voltage and current $\Rightarrow$

~~$V(z, \quad V(0,t) = V_0 \cos(\omega t) + RV_0 (\cos$~~

$$V(0,t) = V_0 \cos(\omega t) + RV_0 \cos(\omega t + \phi)$$

$$I(0,t) = \frac{V_0}{Z_1} \cos(\omega t) - \frac{RV_0}{Z_1} \cos(\omega t + \phi)$$

yes

Power transmitted into the load is

$$P_{load} = V(0,t)\, I(0,t)$$

$$= \frac{V_0^2}{Z_1} \cos^2(\omega t) + \cancel{\frac{RV_0^2}{Z_1} \cos(\omega t)\cos(\omega t + \phi)}$$

$$- \cancel{\frac{RV_0^2}{Z_1} \cos(\omega t) \cos(\omega t + \phi)}$$

$$- \frac{R^2 V_0^2}{Z_1} \cos^2(\omega t + \phi)$$

$$= \frac{V_0^2}{Z_1} \cos^2(\omega t) - \frac{R^2 V_0^2}{Z_1} \cos^2(\omega t + \phi)$$

Time average yields

$$\langle P_{load} \rangle = \frac{V_0^2}{Z_1} \underbrace{\langle \cos^2(\omega t) \rangle}_{\frac{1}{2}} - \frac{R^2 V_0^2}{Z_1} \underbrace{\langle \cos^2(\omega t + \phi) \rangle}_{\frac{1}{2}}$$

$$= \frac{(1 - R^2) V_0^2}{2 Z_1}$$ ✓

Power input :

$$P_{in} = V_+(z,t)\, I_+(z,t)$$

$$= V_0 \cos(\omega t - kz) \cdot \frac{V_0}{Z_1} \cos(\omega t - kz)$$

$$= \frac{V_0^2}{Z_1} \cos^2(\omega t - kz)$$

Time average gives :

$$\langle P_{in} \rangle = \frac{V_0^2}{Z_1} \underbrace{\langle \cos^2(\omega t - kz) \rangle}_{\frac{1}{2}} = \frac{V_0^2}{2Z_1}$$

Hence, $$t = \frac{\langle P_{load} \rangle}{\langle P_{in} \rangle} = \frac{\frac{(1-R^2) V_0^2}{2Z_1}}{\frac{V_0^2}{2Z_1}}$$

$$= 1 - R^2 = 1 - \left| \frac{Z_2 - Z_1}{Z_2 + Z_1} \right|^2$$

$$= \frac{|Z_2 + Z_1|^2 - |Z_2 - Z_1|^2}{|Z_2 + Z_1|^2}$$

$$= \frac{|(\mathrm{Re}(Z_2) + Z_1) + i\,\mathrm{Im}(Z_2)|^2 - |(\mathrm{Re}(Z_2) - Z_1) + i\,\mathrm{Im}(Z_2)|^2}{|Z_2 + Z_1|^2}$$

$\left(Z_1 = \frac{1}{\sqrt{LC}} \text{ is Real} \right)$

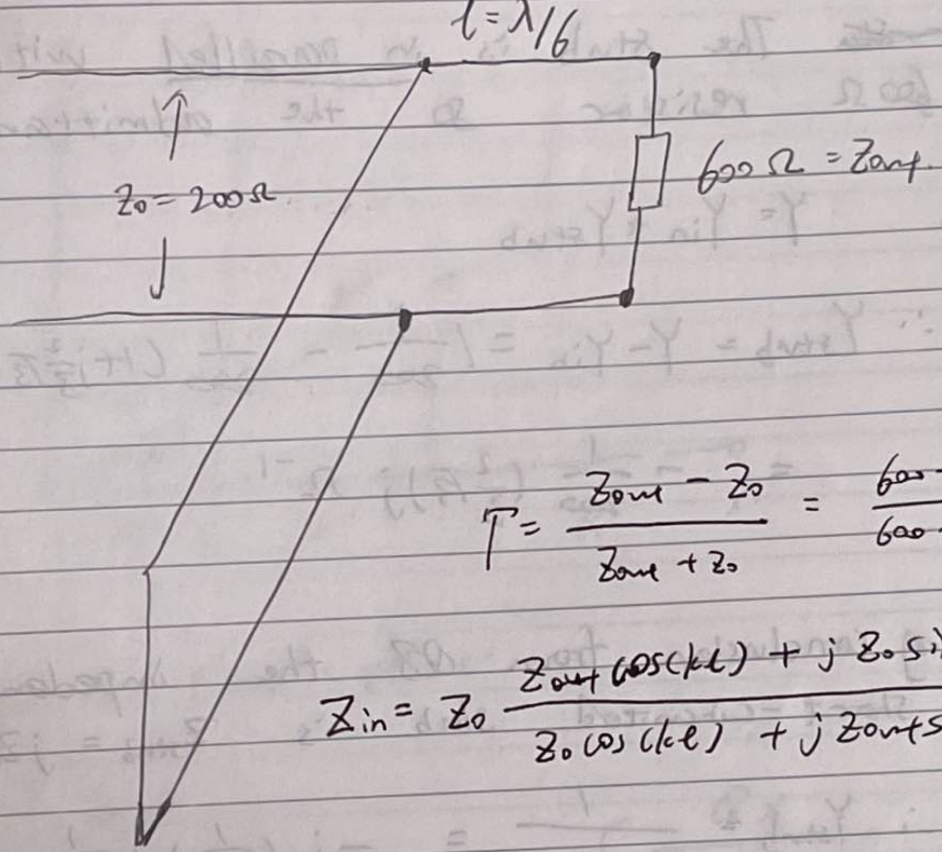
$$= \frac{(\mathrm{Re}(Z_2) + Z_1)^2 + \cancel{\mathrm{Im}^2(Z_2)} - [(\mathrm{Re}(Z_2) - Z_1)^2 + \cancel{\mathrm{Im}^2(Z_2)}]}{|Z_2 + Z_1|^2}$$

$$= \frac{\cancel{Re^2(z_2)} + 2Re(z_2)z_1 + \cancel{z_1^2} - (\cancel{Re^2(z_2)} - 2Re(z_2)z_1 + \cancel{z_1^2})}{|z_1 + z_2|^2}$$

$$= \boxed{\frac{4z_1 Re(z_2)}{|z_1 + z_2|^2}}$$

yes

10.



$$\Gamma = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} = \frac{600-200}{600+200} = \frac{1}{2}$$

$$Z_{in} = Z_0 \frac{Z_{out}\cos(kl) + jZ_0\sin(kl)}{Z_0\cos(kl) + jZ_{out}\sin(kl)}$$

$k = \frac{2\pi}{\lambda}$, $l = \frac{\lambda}{6}$ $\therefore kl = \frac{\pi}{3}$ $\therefore \cos(kl) = \frac{1}{2}$, $\sin(kl) = \frac{\sqrt{3}}{2}$

$$\therefore Z_{in} = 200\frac{600(\frac{1}{2}) + j(200)(\frac{\sqrt{3}}{2})}{200(\frac{1}{2}) + j600(\frac{\sqrt{3}}{2})}\Omega = 200\frac{3+j\sqrt{3}}{1+j3\sqrt{3}}\Omega$$

$$Y_{in} = \frac{1}{Z_{in}} = \cancel{1+j\ldots}\ \frac{1+j3\sqrt{3}}{200(3+j\sqrt{3})} = \frac{3+j2\sqrt{3}}{600}$$

$$= \boxed{\frac{1}{200}(1+j\tfrac{2}{3}\sqrt{3})\ \Omega^{-1}}$$

For no power reflection the net impedance must equal to the characteristic impedence 200Ω, so the net reactance is ~~Y = Z_0 = 1/200~~

$\cancel{Y = \frac{1}{200}}$ $Y = \frac{1}{200}\ \Omega^{-1}$

~~2~~ The stub is in parallel with the $600\,\Omega$ resistor so the admittance adds up

$\therefore \quad Y = Y_{in} + Y_{stub}$

$\therefore Y_{stub} = Y - Y_{in} = \left[\frac{1}{200} - \frac{1}{200}\left(1 + j\frac{2}{3}\sqrt{3}\right)\right] \Omega^{-1}$

$= -\frac{1}{200}\left(\frac{2}{3}\sqrt{3}\right)j \ \Omega^{-1}$

Using conclusion from Q8 the impedance of a short-circuited stub is $Z_{stub} = jZ_0 \tan(kL)$

$\therefore Y_{stub} = \frac{1}{jZ_0 \tan(kL)} = -j\left(\frac{1}{200}\right)\left(\frac{1}{\tan(kL)}\right) \ \Omega^{-1}$

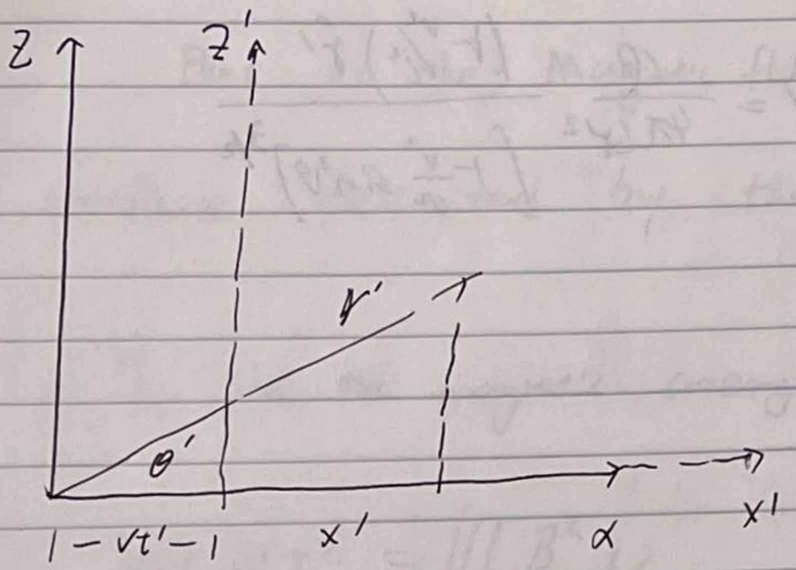
$\therefore \frac{2}{3}\sqrt{3} = \frac{1}{\tan(kL)}$ ✓ $\therefore \tan(kL) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore \tan(kL - n\pi) = \frac{\sqrt{3}}{2}$

$\therefore \quad kL = n\pi + \arctan\left(\frac{\sqrt{3}}{2}\right)$

$k = \frac{2\pi}{\lambda}$

$$\therefore \boxed{L = \frac{n\lambda}{2} + \frac{\lambda}{2\pi}\arctan\left(\frac{\sqrt{3}}{2}\right) \quad (n = 0, 1, 2, \ldots)}$$



$$r'\cos\theta' = vt' + x'$$

in F

$$\vec{E} = \frac{Q\vec{r}}{4\pi\varepsilon_0 r^3} = \begin{cases} \dfrac{Qx}{4\pi\varepsilon_0 (x^2+z^2)^{3/2}} \\[2ex] \dfrac{Qz}{4\pi\varepsilon_0 (x^2+z^2)^{3/2}} \end{cases}$$

$$E_x' = E_x \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E_z' = \gamma E_z$$

(F) is moving with velocity $-v\hat{x}$ with respect to F'

$$x = \gamma(x' + vt') = \gamma r'\cos\theta'$$
$$z = z' = \cancel{\gamma}\, r'\sin\theta'$$

$$\vec{E}' = \frac{q}{4\pi\epsilon_0 r'^2}\frac{\left(1-v^2/c^2\right)\hat{r}'}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}$$

Flux is the Magnetic flux of the surface delimited by the current.

7. Use the magnetic energy.

$$W = \frac{1}{2} L I^2 = \iiint \frac{B^2}{2\mu} d\tau.$$

$$= \oint \ell \int_{a}^{b} \frac{B^2}{2\mu} 2\pi r dr.$$

$$V = V_+ e^{j(\omega t - kx)} + r V_+ e^{j(\omega t - kx)}$$

$$I = \frac{V_+}{Z_1} e^{j(\omega t + kx)} + \frac{r V_+}{Z_1} e^{j(\omega t + kx)}$$

$$P_{reff} = \langle Re(V_{reff}) Re(I_{reff}) \rangle$$

$$r = |r| e^{j\phi}$$

$$V_{reff} = |r| V_+ e^{j(\omega t + kx + \phi)}$$

$$I_{reff} = -\frac{|r| V_+}{Z_1} e^{j(\omega t + kx + \phi)}$$

$$P_{reff} = \frac{|r|^2 V_+^2}{2 Z_1}$$

$$P = \langle Re(V_{inc}) Re(I_{inc}) \rangle$$

$$= \langle V_+ \cos^2(\omega t - kx) \frac{V_+}{Z_1} \rangle$$

$$= \frac{V_+^2}{2 Z_1}$$