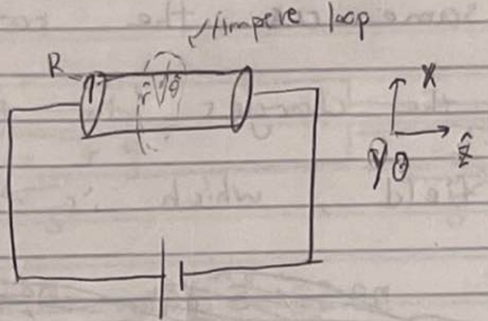
To: Caroline Terquem:

Electromagnetism 3 Ziyan Li

1)



What you have done is excellent! Congratulations.

Try to do Pb 6 and Pb 7.

a) ~~$\vec{E}$~~

$$\boxed{\vec{E} = \frac{V}{L}\hat{z}}$$

~~$\vec{B}$~~ Ampere's Law: $\oint_{\partial\Sigma} \vec{B}\cdot d\vec{l} = \mu_0 I_\Sigma$

$\therefore B(2\pi a) = \mu_0 I \quad \therefore \boxed{B = \frac{\mu_0 I}{2\pi a}\hat{\theta}}$

b) Poynting vector $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$

$$\vec{S} = \frac{V}{L}\hat{z}\times\frac{\mu_0 I}{2\pi r}\hat{\theta}\left(\frac{1}{\mu_0}\right)$$

$$= \frac{VI}{2\pi a L}(\hat{z}\times\hat{\theta}) = \boxed{\frac{-VI\hat{r}}{2\pi a L}}$$

Power flowing into the surface is

$$P_{in} = \cancel{\vec{S}\cdot\vec{A}} \left|\iint \vec{S}\cdot d\vec{\Sigma}\right| = \left|\cancel{\frac{-VI}{2\pi a L}\iint \hat{r}\cdot 2\pi a}\right.$$

$$= \left|\int_0^L\int_0^{2\pi} -\frac{VI}{2\pi a L}\hat{r}\cdot\hat{r}\, a\, d\theta\, dz\right|$$

$$= \left|\left(-\frac{VI}{2\pi a L}\right)(2\pi a L)\right| = |-VI| = \boxed{VI}$$

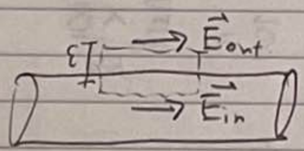
This is the same as the rate at which work is done on the charges in the resistor by the electric field, which is

$$P = \frac{dW}{dt} = \cancel{\frac{nqS\,\vec{v}\cdot\vec{E}\,(dt)}{dt}} \quad \cancel{nq\vec{v}\cdot\vec{E}}$$

$$= \frac{nq(\vec{v}\,dt)\cdot\vec{E}\,L}{dt} = nqvSL\left(\frac{V}{L}\right) = \underbrace{nqvS}_{I}\,V$$

$$= \boxed{VI} \checkmark$$

(c) Consider Boundary Conditions for $\vec{E}$



$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S} \to 0 \quad \text{as } \varepsilon \to 0$$

$$\therefore \vec{E}_{out}^{\,\parallel} L - \vec{E}_{in}^{\,\parallel} L = 0$$

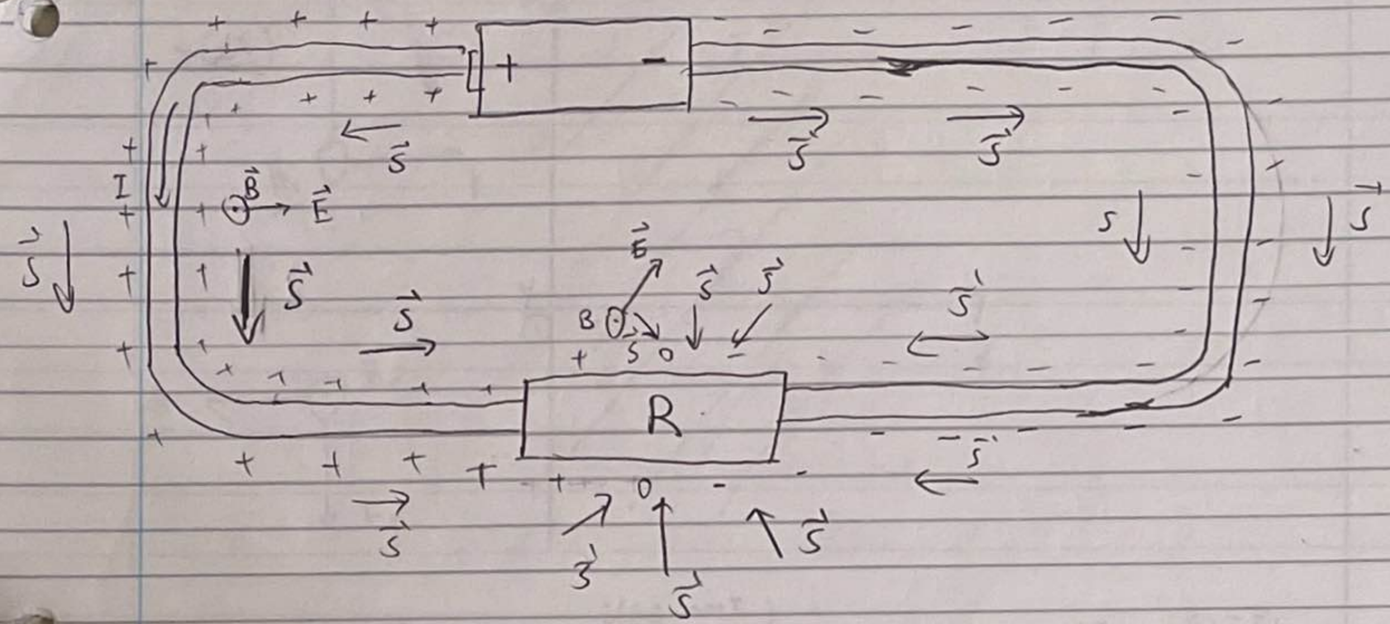
$$\therefore \vec{E}_{out}^{\,\parallel} = \vec{E}_{in}^{\,\parallel}$$

$\vec{E}_{in}^{\,\parallel}$ is ~~cons~~ a constant electrifield inside the resistor to keep the current flowing

$\Rightarrow$ $\vec{E}_{out}^{\,\parallel}$ is also a constant electric field just outside of the wire.

$\Rightarrow$ There is an electric field $\vec{E}_{out}^{\,\parallel}$ outside the resistor giving an inward radial Poynting vector. yes

(d)

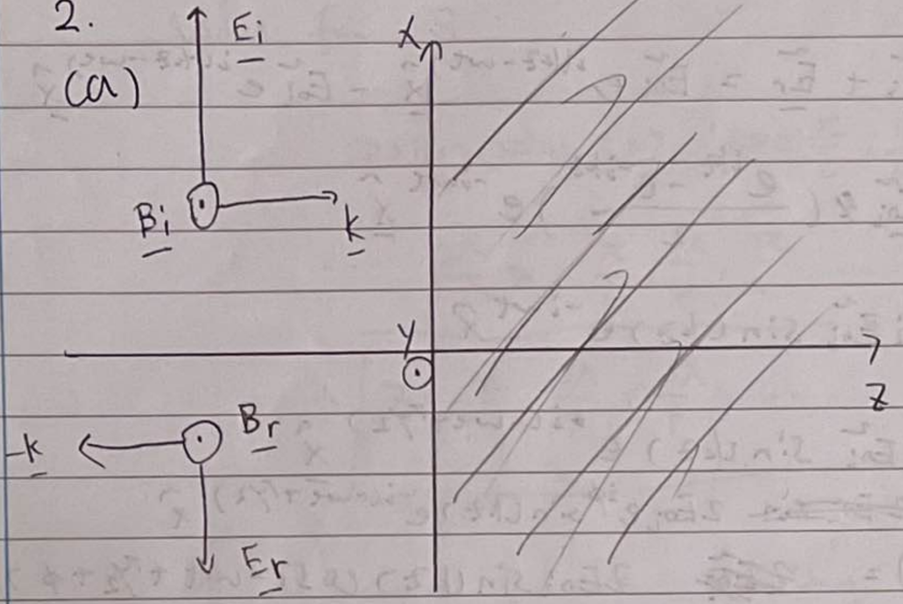


The energy flows out of the battery, ~~from~~ out of both the positive and negative terminals, along the wires and ~~then~~ finally goes into the resistor.

No energy is returning to the battery. ✓

2.

(a)



$$\tilde{\underline{E_i}} = \tilde{E_{0i}}\, e^{i(kz-\omega t)}\,\hat{\underline{x}} \qquad \tilde{E_{0r}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}\tilde{E_{0i}}$$

$$\tilde{\underline{E_r}} = \tilde{E_{0r}}\, e^{i(-kz-\omega t)}\,\hat{\underline{x}} \qquad \tilde{E_{0t}} = \frac{2Z_1}{Z_1 + Z_2}\tilde{E_{0i}}$$

$$\tilde{\underline{E_t}} = \tilde{E_{0t}}\, e^{i(k_2 z-\omega t)}\,\hat{\underline{x}}$$

$z > 0$ there is a perfect conductor ($\sigma \to \infty$)

$\therefore Z_2 \to 0$

$\therefore \tilde{E_{0r}} = -\tilde{E_{0i}} \qquad \tilde{E_{0t}} = 0$

$$\therefore \boxed{\tilde{\underline{E_t}} = \underline{0}, \quad \tilde{\underline{E_r}} = -\tilde{E_{0i}}\, e^{i(-kz-\omega t)}\,\hat{\underline{x}}}$$

we get total reflection

$$\tilde{\underline{B_t}} = \frac{\underline{k} \times \tilde{\underline{E_t}}}{\omega} = \boxed{\underline{0}}$$

$$\tilde{\underline{B_r}} = \frac{-\underline{k} \times \underline{E_r}}{\omega} = \frac{-k\hat{z}}{\omega} \times -\tilde{E_{0i}}\, e^{i(-kz-\omega t)}\,\hat{\underline{x}}$$

$$= \frac{k\tilde{E_{0i}}}{\omega}\, e^{i(-kz-\omega t)}\,\hat{\underline{y}}$$

$$= \boxed{\tilde{B_{0i}}\, e^{i(-kz-\omega t)}\,\hat{\underline{y}}}$$

b)

$$\tilde{\underline{E}} = \tilde{\underline{E}}_i + \tilde{\underline{E}}_r = \tilde{E}_{0i} e^{i(kz-\omega t)} \hat{\underline{x}} - \tilde{E}_{0i} e^{i(-kz-\omega t)} \hat{\underline{x}}$$

$$= 2\tilde{E}_{0i} \cancel{i} \left(\frac{e^{ikz} - e^{-ikz}}{2} \right) e^{-i\omega t} \hat{\underline{x}}$$

$$= 2i\tilde{E}_{0i} \sin(kz) e^{-i\omega t} \hat{\underline{x}}$$

$$= 2\tilde{E}_{0i} \sin(kz) e^{i(-\omega t + \pi/2)} \hat{x}$$

$$= \cancel{2E_{0i}\sin} \; 2E_{0i} e^{i\phi} \sin(kz) e^{i(-\omega t + \pi/2)} \hat{x}$$

$$\therefore \; \underline{E} = Re(\tilde{\underline{E}}) = \cancel{2\tilde{E}_{0i}} \; 2E_{0i} \sin(kz) \cos(-\omega t + \pi/2 + \phi) \hat{\underline{x}}$$

$$= \cancel{2E_{0i}\sin(kz)\cos} = \boxed{2E_{0i} \sin(kz) \sin(\omega t - \phi) \hat{\underline{x}}} \checkmark$$

$$\tilde{\underline{B}} = \tilde{\underline{B}}_i + \tilde{\underline{B}}_r = \tilde{B}_{0i} e^{i(kz-\omega t)} \hat{\underline{y}} + \tilde{B}_{0i} e^{i(-kz-\omega t)} \hat{\underline{y}}$$

$$= 2\tilde{B}_{0i} \left(\frac{e^{ikz} + e^{-ikz}}{2} \right) e^{-i\omega t} \hat{\underline{y}}$$

$$= \cancel{2\tilde{B}_{0i}} \; 2B_{0i} e^{i\phi} \cos(kz) e^{-i\omega t} \hat{\underline{y}}$$

$$= 2B_{0i} \cos(kz) e^{i(-\omega t + \phi)} \hat{\underline{y}}$$

$$\underline{B} = Re(\tilde{\underline{B}}) = \boxed{2B_{0i} \cos(kz) \cos(\omega t - \phi) \hat{\underline{y}}} \checkmark$$

Nodes for $\underline{E} \Rightarrow \sin(kz) = 0 \Rightarrow kz = 0, \pi, 2\pi, \ldots$

$$\therefore \cancel{z = 0} \; \therefore \frac{2\pi}{\lambda} z = 0, \pi, 2\pi, 3\pi \ldots$$

$$\therefore \boxed{z = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots}$$

yes but $z < 0$

Nodes for $\underline{B}$:

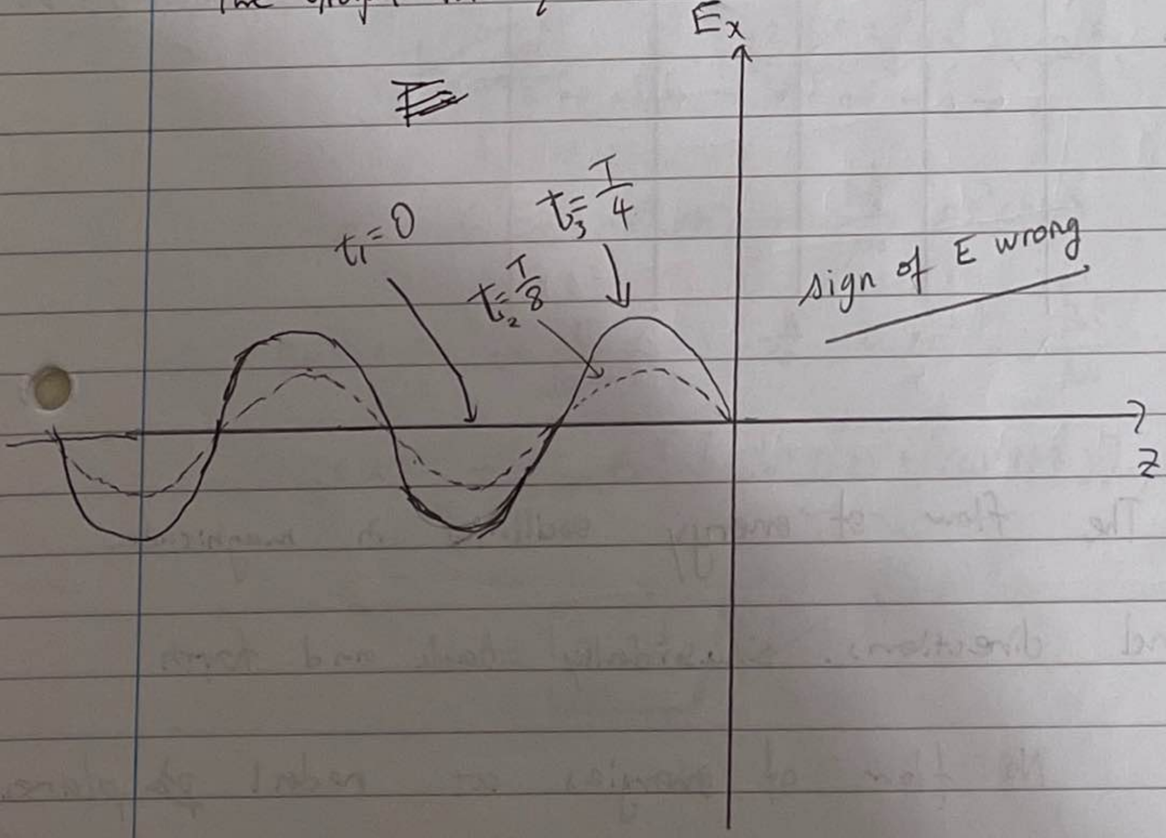
$$\cos(kz) = 0 \implies kz = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\therefore \quad \frac{2\pi}{\lambda} z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\therefore \boxed{z = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots}$$ yes but $z < 0$

$\underline{E}$ and $\underline{B}$ are standing waves because the time and space dependence are ~~sep~~ ~~separ~~ separated.

The Graph for $\phi = 0$: ~~to~~ $T = \frac{2\pi}{\omega}$

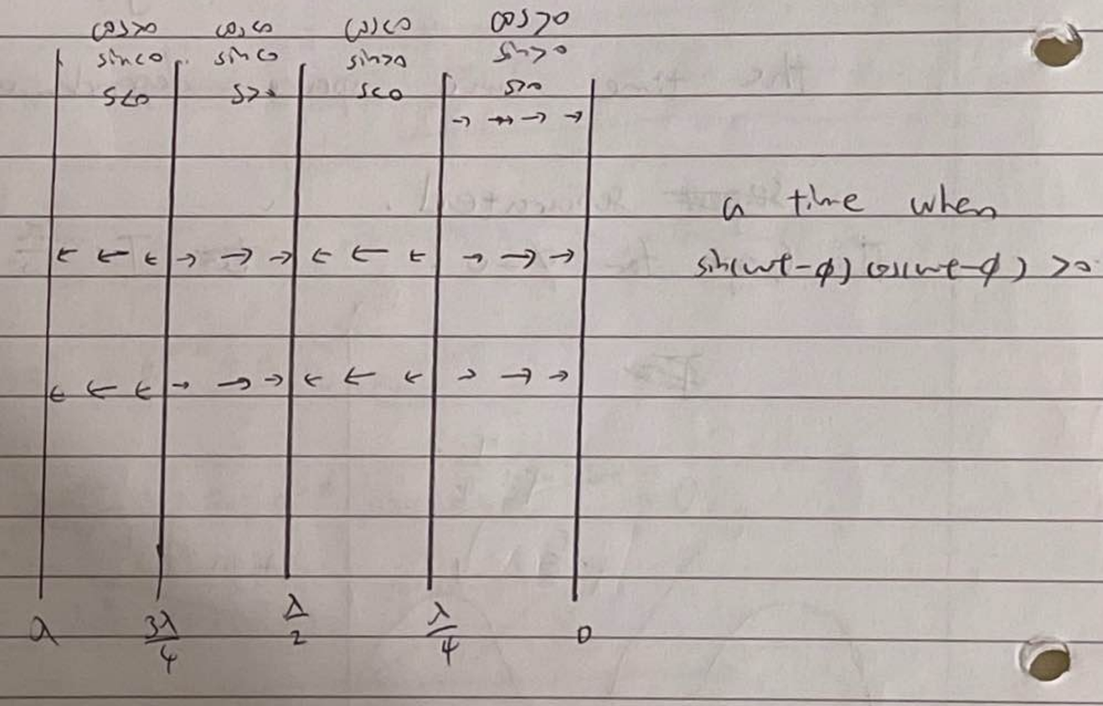


c) $E_{0i} = cB_{0i}$

$$\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B} = \boxed{4 E_{0i} B_{0i} \sin(kz)\cos(kz)\sin(\omega t - \phi)\cos(\omega t - \phi)\, \hat{\underline{z}}}$$

At nodal planes $\sin(kz) = 0$ or $\cos(kz) = 0$

$\therefore$ $\boxed{\underline{S} = 0 \text{ at nodal planes}}$



The flow of energy oscillates in magnitude and directions. sinusoidally back and forth

No flow of energies at nodal ~~ph~~ planes.

$$\langle \underline{S} \rangle = 4 E_{0i} B_{0i} \sin(kz)\cos(kz) \underbrace{\langle \sin(\omega t-\phi)\cos(\omega t-\phi) \rangle}_{0} \hat{z}$$

$$= \boxed{\underline{0}}$$ ✓

time average of poynting vector is $\underline{0}$

d) energy density

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 \langle \underline{E} \cdot \underline{E} \rangle + \frac{1}{2\mu_0} \langle \underline{B} \cdot \underline{B} \rangle$$

$$= \frac{1}{2}\varepsilon_0 \cdot 4 E_{0i}^2 \sin^2(kz) \overbrace{\langle \sin^2(\omega t-\phi) \rangle}^{1/2}$$

$$+ \cancel{\frac{1}{2\mu_0} 4B_{0i}^2} \frac{1}{2\mu_0} \cdot 4B_{0i}^2 \cos^2(kz) \overbrace{\langle \cos^2(\omega t-\phi) \rangle}^{1/2.}$$

$$= \varepsilon_0 E_{0i}^2 \sin^2(kz) + \frac{B_{0i}^2}{\mu_0} \cos^2(kz)$$

$$E_{0i} = c B_{0i} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} B_{0i} \Rightarrow \varepsilon_0 E_{0i}^2 = \frac{B_{0i}^2}{\mu_0}$$

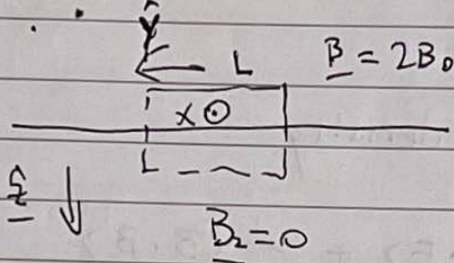
$$\therefore \langle u \rangle = \varepsilon_0 E_{0i}^2 [\underbrace{\sin^2(kz) + \cos^2(kz)}_{1}]$$

$$= \boxed{\varepsilon_0 E_{0i}^2}$$ ✓

e) at the surface $\underline{E} \cdot \hat{\underline{z}} = 0$

$\therefore$ $E_n = 0$ $\qquad \sigma = \varepsilon_0 E_n = \boxed{0}$

$\therefore$ No surface charge density.



$\underline{B} = 2B_{0i} \cos(kz)\cos(\omega t - \phi)\,\hat{\underline{y}}$

$\underline{B_2} = 0$

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_s \quad \text{(Ampere's Law)}.$$

$BL = \mu_0 K L \implies B = \mu_0 K.$

$\therefore$ ~~$\underline{K} = \frac{2B_{0i}}{\mu_0} \cos(kz)\cos(\omega t - \phi)\,\hat{\underline{x}}$~~

At $z=0$, $\boxed{\underline{K} = \frac{2B_{0i}}{\mu_0} \cos(\omega t - \phi)\,\hat{\underline{x}}}$

The ~~magnitude~~ magnetic field due to this $\underline{K}$ is $\underline{B}' = \frac{\mu_0 \underline{K}}{2}\hat{\underline{y}}$ ~~$\underline{B_2}$~~ for $z = 0^-$

and $\underline{B}_2' = -\frac{\mu_0 \underline{K}}{2}\hat{\underline{y}}$ for $z = 0^+$

We know: $\underline{B} = 2B_{0i}\cos(kz)\cos(\omega t - \phi)\,\hat{\underline{y}}$

~~$\underline{B}' = B_{0i}\cos(kz)\cos(\omega t - \phi)\,\hat{\underline{y}}$~~ ~~$\underline{B}' = B_{0i}\cos(\omega t - \phi)\,\hat{\underline{y}}$~~

~~$\underline{B}'$ due to surface current is one half of the total field $\underline{B}$~~

we know $\boxed{\underline{B}' = B_{0i} \cos(\omega t - \phi)\, \hat{\underline{y}}}$ due to surface current. at $z = 0^-$

~~The~~ The reflected magnetic field. at $z=0^-$ is

~~$\underline{B_r} = Re(\hat{\underline{B_r}}) =$~~

$$\underline{Br}_{0^-} = Re(\underline{\tilde{Br}}_{0^-}) = Re(B_{0i}\, e^{i\phi} e^{-i\omega t}.)\, \hat{y}$$

$$\cancel{= B_{0i}(\omega t - \phi)} = \boxed{B_{0i} \cos(\omega t - \phi)\, \hat{\underline{y}}}$$

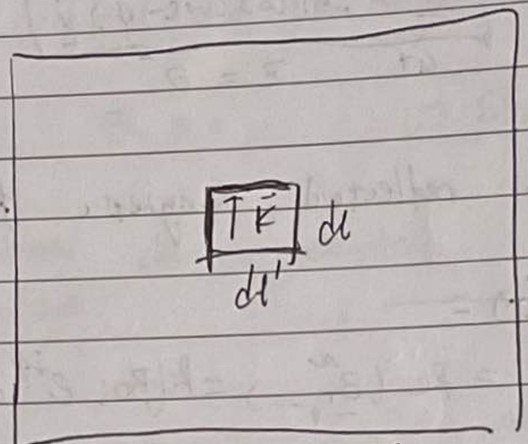
$\therefore$ two fields are equal

$$\underline{B}' = \underline{Br}_{0^-} \checkmark$$

f) The reflected field is complete due to the surface current so the force experienced by the ~~surface~~ boundary is entirely ~~due~~ due to the incident $\underline{B}$ field.

$$\cancel{\underline{B_i} =} \quad \underline{B_i}_{0^-} = Re(\underline{\hat{B_i}}_{0^-}) = Re\left[B_{i0} e^{i\phi} e^{-i\omega t} \hat{\underline{y}} \right]$$

$$= B_{0i} \cos(\omega t - \phi)\, \hat{\underline{y}}$$



$$d^2\vec{F} = I\,d\vec{l} \times \vec{B} = \cancel{K}\,(\vec{K}\cdot d\vec{l}')\,d\vec{l} \times \vec{B}$$

$$= \cancel{(K\,dl')\,dl\ B_{0i}}$$

$$= K(dl'\,dl)\,B_{0i}\cos(\omega t-\phi)\,\underbrace{\hat{x}\times\hat{y}}_{\hat{z}}$$

$$= \frac{2B_{0i}^2}{\mu_0}\cos^2(\omega t-\phi)\,\hat{z}\,(dl'\,dl).$$

$$P = \frac{dF}{dA} = \frac{d^2F}{dl'\,dl} = \frac{2B_{0i}^2}{\mu_0}\cos^2(\omega t-\phi)\,\hat{z}$$

Radiation pressure :

$$\langle \vec{P} \rangle = \frac{2B_{0i}^2}{\mu_0}\,\underbrace{\langle \cos^2(\omega t-\phi)\rangle}_{1/2}\,\hat{z}$$

$$= \frac{B_{0i}^2}{\mu_0}\,\hat{z}$$

$$\langle P \rangle = \frac{B_{0i}^2}{\mu_0} = \boxed{\varepsilon_0 E_{0i}^2}$$

9) After contration of dz
The increase of electromagnetic energy per unit area is

$$\Delta U = u\,\Delta V / A = u A\, dz / A = u\, dz$$

energy density   volume

The work done by electromagnetic field is.
per unit area
$$\Delta W = P\,dz$$

∵ ~~AE~~ Total reflection

∴ All energies bounds back at the boundary.

∴ ~~AE~~ The energy ~~is conserved~~ stored in the electromagnetic fields ~~is~~ to the left of the conductor is conserved

∴ After contraction of conductor change in energy of EM field is

$$\Delta E = \Delta U - \Delta W$$

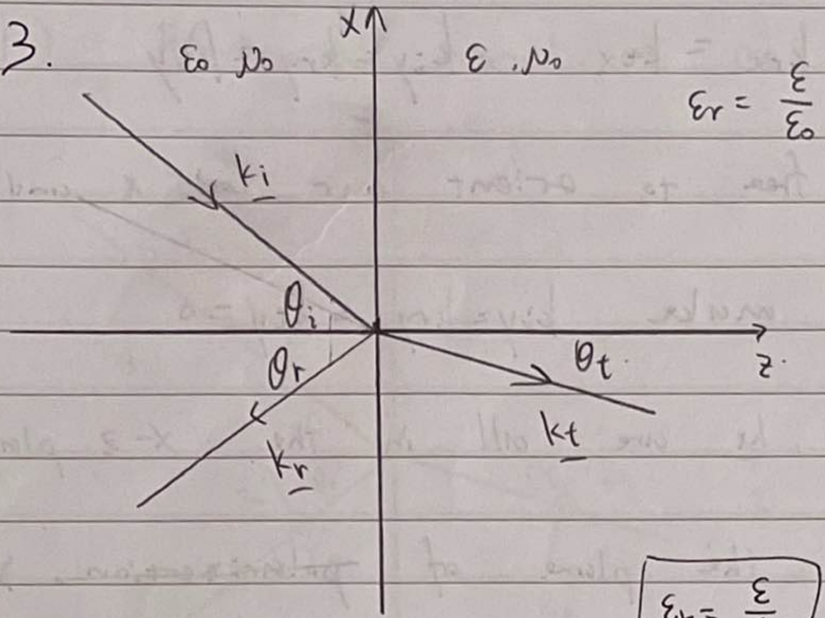
Energy Conservation :

$$\Delta E = 0 \Rightarrow \Delta U = \Delta W$$

$$\Rightarrow P\,dz = u\,dz \Rightarrow \boxed{P = u}$$

We can conclude that for collimated ~~plan~~ linearly polarized plane waves the radiation pressure is equal to the energy density.

3.



$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

$$\tilde{\underline{E}}_i = \underline{E}_{0i}\, e^{i(\underline{k}_i\cdot\underline{r}-\omega t)} \qquad \tilde{\underline{B}}_i = \frac{1}{c}\hat{\underline{k}}_i \times \tilde{\underline{E}}_i$$

$$\tilde{\underline{E}}_r = \tilde{\underline{E}}_{0r}\, e^{i(\underline{k}_r\cdot\underline{r}-\omega t)} \qquad \tilde{\underline{B}}_r = \frac{1}{c}\hat{\underline{k}}_r \times \tilde{\underline{E}}_r$$

$$\tilde{\underline{E}}_t = \tilde{\underline{E}}_{0t}\, e^{i(\underline{k}_t\cdot\underline{r}-\omega t)} \qquad \tilde{\underline{B}}_t = \frac{1}{v}\hat{\underline{k}}_t \times \tilde{\underline{E}}_t$$

$$\boxed{n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu_0}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r}}$$

Boundary Condition:

$$\cancel{E_{0iz}\, e^{i(k_{ix}x+k}} \quad E_{0iz}\, e^{i(\underline{k}_i\cdot\underline{r}-\omega t)} + E_{0rz}\, e^{i(\underline{k}_r\cdot\underline{r}-\omega t)}$$

$$= \varepsilon_r E_{0tz}\, e^{i(\underline{k}_t\cdot\underline{r}-\omega t)} \quad \text{at } z=0$$

$$\Rightarrow \underline{k}_i\cdot\underline{r} = \underline{k}_r\cdot\underline{r} = \underline{k}_t\cdot\underline{r} \quad \text{at } z=0.$$

$$\therefore\ k_{ix}x + k_{iy}y = k_{rx}x + k_{ry}y = k_{tx}x + k_{ty}y$$

is true for all x and y

$\therefore k_{ix} = k_{rx} = k_{tx}$, $k_{iy} = k_{ry} = k_{ty}$

$\therefore$ we are free to orient our ~~co~~ x and y

axis to make $k_{iy} = k_{ry} = k_{ty} = 0$

$\therefore$ $\underline{k_i}$, $\underline{k_r}$, $\underline{k_t}$ are all in the x-z plane

This is the plane of ~~polarization~~ incidence.

$$k_{ix} = k_{rx} = k_{tx} \Rightarrow \underline{k_i} \cdot \hat{\underline{x}} = \underline{k_r} \cdot \hat{x} = \underline{k_t} \cdot \hat{\underline{x}}$$

$$\Rightarrow |k_i| \cos(90^\circ - \theta_i) = |k_r| \cos(90^\circ - \theta_r) = |k_t| \cos(90^\circ - \theta_t)$$

we know $c = \frac{\omega}{|k_i|} = \frac{\omega}{|k_r|}$ $\quad v = \frac{\omega}{|k_t|}$ (dispersion relation)

$$\Rightarrow |k_i| = |k_r| \Rightarrow \sin\theta_i = \sin\theta_r \Rightarrow \boxed{\theta_i = \theta_r}$$ ✓

$$|k_i| c = |k_t| v \quad \therefore |k_t| = \frac{c}{v} |k_i| = n |k_i|.$$

$$\therefore |k_i| \sin\theta_i = n |k_i| \sin\theta_t.$$

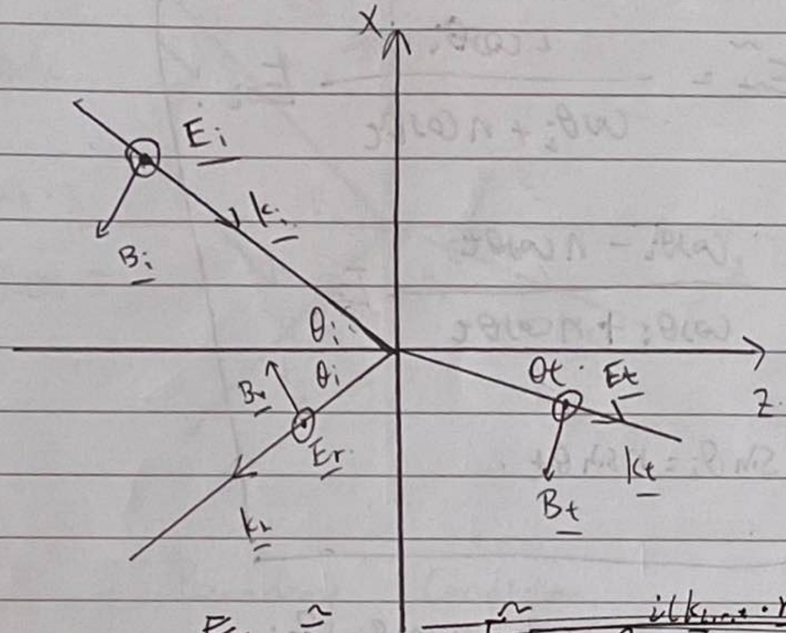
$$\therefore \boxed{\sin\theta_i = n \sin\theta_t}$$ ✓

For $n < 1$.

At total reflection, $\theta_t = 90^\circ$

$$\therefore \boxed{\sin\theta_{ic} = n} \Rightarrow$$ ✓

b) Polarization normal to the plane of incidence



~~$\tilde{E}_{i,r,t} = \tilde{E}_{0i,r,t}\, e^{i(k_{i,r,t}\cdot r - \omega t)} = \hat{E}_{0i,r,t}\, e^{i(k_{i,r,t}\cdot r - \omega t)}\hat{x}$~~

~~$E_{0i} + E_{0r} = E_{0t}$~~

∴ E field is along the $\hat{x}$- direction.

∴ $$E_{0i} + \tilde{E}_{0r} = \tilde{E}_{0t}. \quad ✓ \quad ①$$

$$\tilde{B}_{0i}\sin\theta_i + \tilde{B}_{0r}\sin\theta_i = \tilde{B}_{0t}\sin\theta_t \quad ✓ \quad ②$$

$$\tilde{B}_{0i}\cos\theta_i - \tilde{B}_{0r}\cos\theta_i = \tilde{B}_{0t}\cos\theta_t \quad ✓ \quad ③$$

$$\tilde{E}_{0r} = c\tilde{B}_{0r} \qquad \cancel{E_{0t} = vB_{0t}} \qquad E_{0i} = cB_{0i}.$$

$$\tilde{E}_{0t} = v\tilde{B}_{0t} \Rightarrow c\tilde{B}_{0t} = \frac{c}{v}\tilde{E}_{0t} = n\tilde{E}_{0t}.$$

From these we see ① and ② are the same thing,

$$c\,③ \Rightarrow \tilde{E}_{0i}\cos\theta_i - \tilde{E}_{0r}\cos\theta_i = n\tilde{E}_{0t}\cos\theta_t \quad ④$$

$$\cos\theta_i\,① \Rightarrow \tilde{E}_{0i}\cos\theta_i + \tilde{E}_{0r}\cos\theta_i = \tilde{E}_{0t}\cos\theta_i. \quad ⑤$$

④ + ⑤ => $2E_{0i}\cos\theta_i = \tilde{E}_{0t}(\cos\theta_i + n\cos\theta_t)$

$$\boxed{\begin{aligned} \tilde{E}_{0t} &= \frac{2\cos\theta_i}{\cos\theta_i + n\cos\theta_t} E_{0i} \\ \tilde{E}_{0r} &= \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t} E_{0i} \\ \text{where} \quad & \sin\theta_i = n\sin\theta_t. \end{aligned}}$$

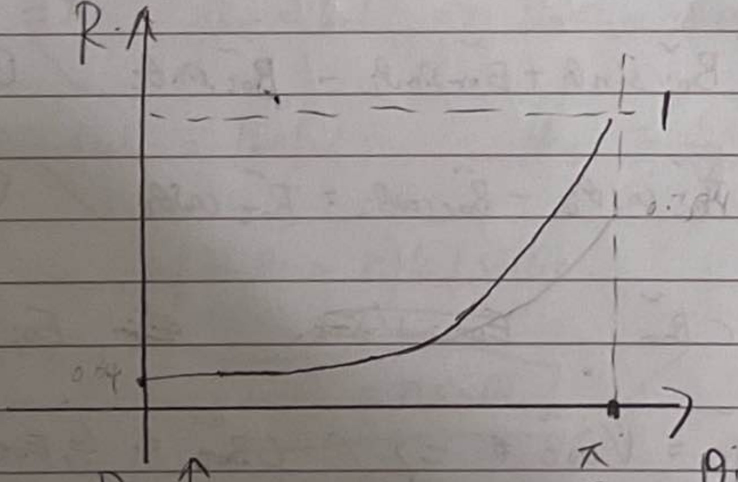
$\sin\theta_t = \frac{1}{n}\sin\theta_i$

$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$

$= \cos\theta_t = \sqrt{1-\frac{1}{n^2}\sin^2\theta_i}$

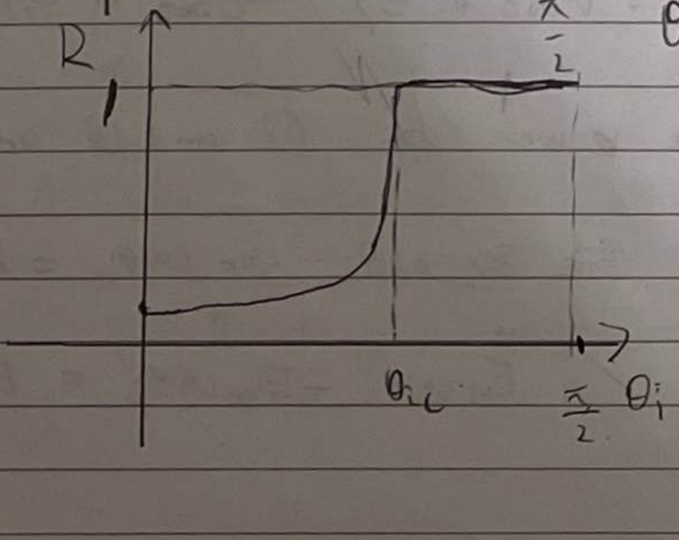
$$R = \left|\frac{\tilde{E}_{0r}}{\tilde{E}_{0i}}\right|^2 = \left(\frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_t}\right)^2.$$

$$= \left(\frac{\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n^2-\sin^2\theta_i}}\right)^2$$



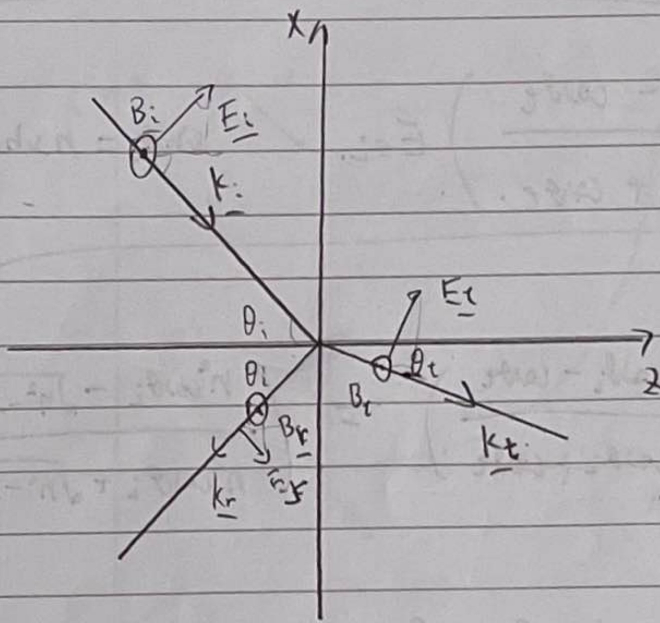
$n > 1$

yes



$n < 1$.

c)



Boundary Conditions:

$$B_{0i} + B_{0r} = B_{0t}. \quad ①$$

$$E_{0i}\sin\theta_i + E_{0r}\sin\theta_i = \cancel{\varepsilon_r E_{0t}} \quad ② \quad \varepsilon_r E_{0t}\sin\theta_t = n^2 E_{0t}\sin\theta_t$$

$$E_{0i}\cos\theta_i - E_{0r}\cos\theta_i = E_{0t}\cos\theta_t. \quad ③ \checkmark$$

① and ② both gives

$$E_{0i} + E_{0r} = nE_{0t}. \quad ④$$

$\cos\theta_i$ ④ $\Rightarrow$ $E_{0i}\cos\theta_i + E_{0r}\cos\theta_i = nE_{0t}\cos\theta_i$ ⑤

③ + ⑤ $\Rightarrow$ $2E_{0i}\cos\theta_i = (n\cos\theta_i + \cos\theta_t)E_{0t}$.

$$\therefore \boxed{E_{0t} = \frac{2\cos\theta_i}{n\cos\theta_i + \cos\theta_t} E_{0i}} \checkmark \quad \boxed{\sin\theta_i = n\sin\theta_t}$$

~~E_{0r}~~ =

⑤ − ③ $\Rightarrow$ $2E_{0r}\cos\theta_i = (n\cos\theta_i - \cos\theta_t)E_{0t}$.

$$\therefore E_{0r} = \cancel{\frac{2\cos\theta_i}{n\cos\theta_i - \cos\theta_t}E_{0t}}\ \frac{n\cos\theta_i - \cos\theta_t}{2\cos\theta_i}E_{0t}$$

$$= \cancel{\frac{2\cos\theta_i}{n\cos\theta_i - \cos\theta_t}}\ \cancel{2\cos\theta_i}$$

$$\boxed{E_{or} = \left(\frac{n\cos\theta_i - \cos\theta_t}{n\cos\theta_i + \cos\theta_t}\right) E_{oi}.\ \checkmark \quad \sin\theta_i = n\sin\theta_t.}$$

$$R = \left(\frac{n\cos\theta_i - \cos\theta_t}{n\cos\theta_i + \cos\theta_t}\right)^2 = \left(\frac{n^2\cos\theta_i - \sqrt{n^2-\sin^2\theta_i}}{n^2\cos\theta_i + \sqrt{n^2-\sin^2\theta_i}}\right)^2.$$

For $R=0$, let $\theta_i = \theta_b$, then.

$$\cancel{n\cos\theta_b = n\cos\theta_i = \cos\theta_t.}$$

$$\cancel{\sin\theta_i = n\sin\theta_t.}$$

$$\cancel{\therefore n^2\cos\theta_i = n\cos\theta_t.}$$

$$\cancel{\sin\theta_i = n\sin\theta_t.}$$

$$\cancel{n^2 = (n\cos\theta_t)^2 + (n\sin\theta_t)^2 = n^4\cos^2\theta_i + \sin^2\theta_i}$$

$$n^2\cos\theta_b = \sqrt{n^2-\sin^2\theta_b} \Rightarrow n^4\cos^2\theta_b = n^2 - \sin^2\theta_b$$

$$\Rightarrow n^4\cos^2\theta_b = n^2 - 1 + (1-\sin^2\theta_b) = n^2 - 1 - \cos^2\theta_b.$$

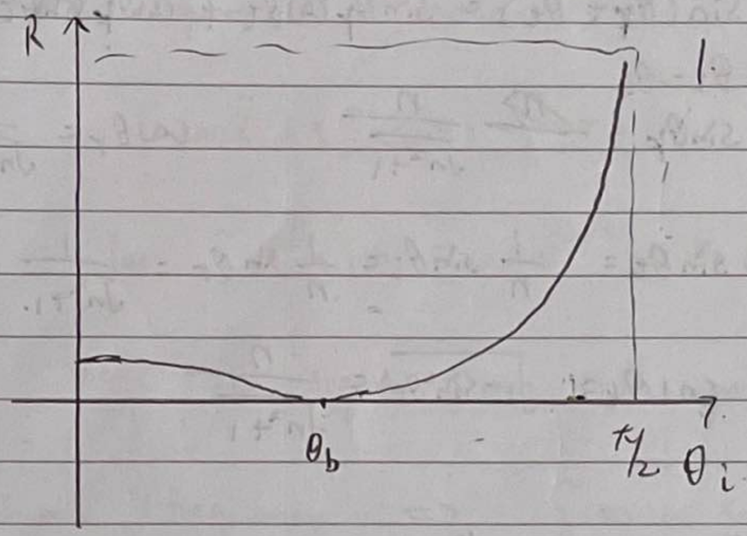
$$\therefore (n^4-1)\cos^2\theta_b = n^2-1$$

For $n \neq 1$ $\therefore \cos^2\theta_b = \frac{n^2-1}{(n^2-1)(n^2+1)} = \frac{1}{n^2+1}.$

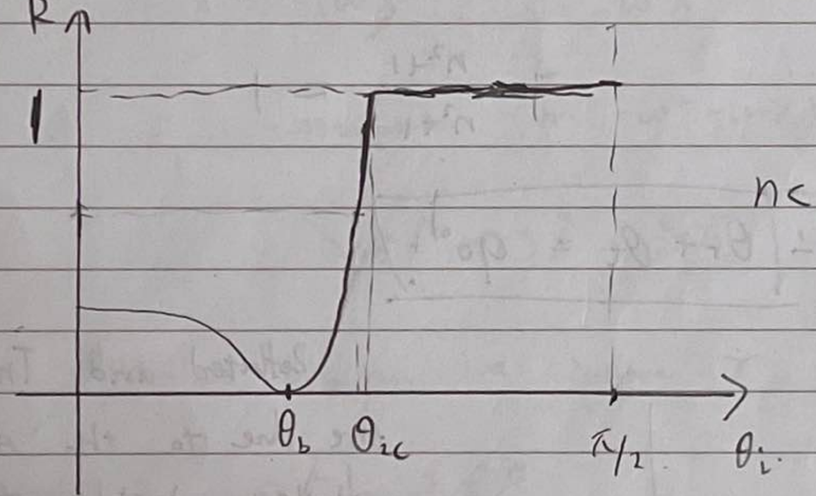
$$\therefore \cancel{\cos^2\theta_b}\ \cos^2\theta_b = \frac{1}{n^2+1}, \quad \sin^2\theta_b = \frac{n^2}{n^2+1}.$$

$$\therefore \tan^2\theta_b = n^2 \Rightarrow \cancel{\tan\theta_b}\ \boxed{\tan\theta_b = n.}\ \checkmark$$

θ_b is the brewster angle.



yes



When unpolarized light beam incident at Brewster's angle the component ~~parallel to~~ of $\vec{E}$ parallel to the plane of incidence is blocked.

So the reflected beam of light becomes polarized (with a polarization perpendicular to the plane of incidence). yes

(d) $\sin(\theta_r + \theta_t) = \sin\theta_r \cos\theta_t + \cos\theta_r \sin\theta_t.$

if $\theta_i = \theta_i = \theta_b$

$$\sin\theta_r = \frac{n}{\sqrt{n^2+1}} \qquad \cos\theta_r = \frac{1}{\sqrt{n^2+1}}$$

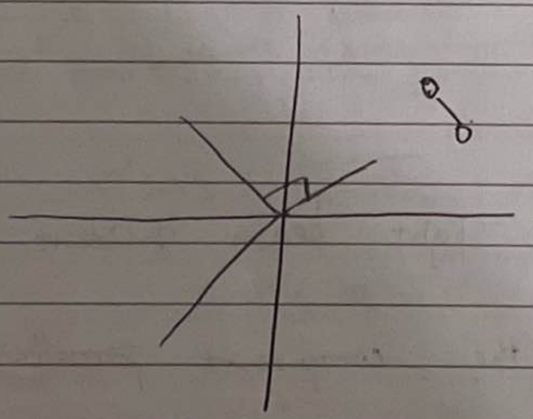
$$\sin\theta_t = \frac{1}{n}\sin\theta_i = \frac{1}{n}\sin\theta_r = \frac{1}{\sqrt{n^2+1}}.$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \frac{n}{\sqrt{n^2+1}}$$

$$\therefore \sin(\theta_r + \theta_t) = \left(\frac{n}{\sqrt{n^2+1}}\right)\left(\frac{n}{\sqrt{n^2+1}}\right) + \left(\frac{1}{\sqrt{n^2+1}}\right)\left(\frac{1}{\sqrt{n^2+1}}\right)$$

$$= \frac{n^2+1}{n^2+1} = 1$$

$$\therefore \boxed{\theta_r + \theta_t = 90^\circ} \checkmark$$



Reflected and Transmitted waves are due to the ~~dip~~ oscillating dipoles inside the dielectric

$\Rightarrow$ dipole oscillating along electric field

$\Rightarrow$ dipole moment is along the electric field inside dielectric, hence dipole moment is perpendicular to the transmitted wave. yes

$\Rightarrow$ At Brewster angle, reflected wave is perpendicular to transmitted wave ($\theta_r + \theta_t = 90^\circ$), so reflected wave is parallel to the dipole moments.

$\Rightarrow$ Since dipoles do not radiate energy in the direction of their moments, there is no energy in the reflected wave, Thus there is no reflected wave. ✓

4. (a) Equation of motion:

$$m\ddot{\tilde{\underline{x}}} = q\tilde{\underline{E}}(\underline{x},t) - m\gamma\dot{\tilde{\underline{x}}} \qquad (\gamma \text{ is damping term}) \quad (\underline{E} \text{ is external field})$$

let $\tilde{\underline{E}}(\underline{x},t) = \underline{E_0}(\underline{x})e^{-i\omega t} = \underline{E_0}e^{-i\omega t} = \tilde{\underline{E}}$

then at steady state $\underline{x}$ should vary with the same frequency $\Rightarrow \tilde{\underline{x}}(t) = \underline{x_0}e^{-i\omega t}$

$\therefore \dot{\tilde{\underline{x}}} = -i\omega\tilde{\underline{x}}, \quad \ddot{\tilde{\underline{x}}} = -\omega^2\tilde{\underline{x}}$

$\therefore$ $\therefore$ $\cancel{-m\omega^2 - mi}$ $m(-\omega^2 - i\omega\gamma)\tilde{\underline{x}} = q\tilde{\underline{E}}$

$\therefore (-\omega^2 - i\omega\gamma)\tilde{\underline{x}} = \frac{q}{m}\tilde{\underline{E}}$

For collisionless plasma we ignore γ so

$$-\omega^2\tilde{\underline{x}} = \frac{q}{m}\tilde{\underline{E}}$$

$$\therefore \tilde{\underline{x}} = -\frac{q}{\omega^2 m}\tilde{\underline{E}}$$

$q = -e \Rightarrow$ $\cancel{\tilde{\underline{x}} = \frac{e}{\omega^2 E}}$ $\tilde{\underline{x}} = \frac{e}{\omega^2 m}\tilde{\underline{E}}$

Dipole moment $\tilde{\underline{p}}' = -e\tilde{\underline{x}} = -\frac{e^2}{\omega^2 m}\tilde{\underline{E}}$ of one electron

total dipole moment $\tilde{\underline{P}} = N\tilde{\underline{p}}' = -\frac{Ne^2}{\omega^2 m}\tilde{\underline{E}}$

~~Nu~~ Number density $N = n_e$ $\therefore \tilde{\underline{P}} = -\frac{n_e e^2}{\omega^2 m}\tilde{\underline{E}}$

$\tilde{\underline{P}} = \varepsilon_0 \chi_e \tilde{\underline{E}} \rightarrow \chi_e = -\frac{n_e e^2}{\varepsilon_0 \omega^2 m}$ ✓

$$\varepsilon_r = 1 + \chi_e = 1 - \left(\frac{n_e e^2}{\varepsilon_0 m}\right)\frac{1}{\omega^2}$$

$\because$ $\omega_p = \frac{n_e e^2}{\varepsilon_0 m}$ $\therefore$ $\boxed{\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}}$ ✓

Dispersion relation: $\tilde{k}^2 = \mu_0 \tilde{\varepsilon} \omega^2$.

$$k^2 = \mu_0 \varepsilon_0 \varepsilon_r \omega^2 = \mu_0 \varepsilon_0 \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right).$$

$$\therefore k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

For $\omega < \omega_p$, k is imaginary, let $k = ik''$ (k'' real).

~~Then the complex wave solution;~~

Assume $\mu = \mu_0$.

Define $\tilde{\underline{D}} = \varepsilon_0 \tilde{\underline{E}} + \tilde{\underline{P}} = \varepsilon_0 \tilde{\varepsilon}_r \tilde{\underline{E}}$, then

$$\nabla \cdot \tilde{\underline{D}} = 0$$
$$\nabla \cdot \tilde{\underline{B}} = 0$$
$$\nabla \times \tilde{\underline{E}} = -\frac{\partial \tilde{\underline{B}}}{\partial t}.$$
$$\nabla \times \tilde{\underline{B}} = \mu_0 \frac{\partial \tilde{\underline{D}}}{\partial t}.$$

This gives rise to the wave equation:

$$\nabla^2 \tilde{\underline{E}} = \mu_0 \tilde{\varepsilon} \frac{\partial^2 \tilde{\underline{E}}}{\partial t^2}.$$

More general solution is: $\tilde{\underline{E}}(z,t) = \tilde{E}_0 e^{i(\tilde{k}z - \omega t)}$.

$\because \tilde{k}$ is imaginary . $\tilde{k} = ik''$

$\therefore \quad \tilde{\underline{E}}(z,t) = \tilde{\underline{E}}_0 \, e^{i(ik''z - \omega t)}$

$\qquad\qquad = \tilde{\underline{E}}_0 \, e^{-k''z} \, e^{-i\omega t}$

$\because$ This is a standing wave.

$\therefore$ Waves cannot propagate in the plasma. ✓

(b) for $\omega > \omega_p$, $\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} > 0$

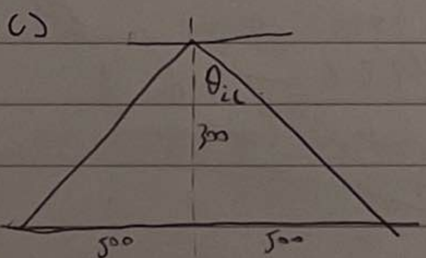
$\therefore \boxed{n = \sqrt{\varepsilon_r} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$ is real and $\boxed{0 < n < 1}$

Using result from previous question.

Total reflection occurs at incident angle larger than θ_{ic} where θ_{ic} is given by $\boxed{\sin\theta_{ic} = n}$. yes

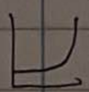
(assuming atmosphere below ionosphere has index of refraction $= 1$)

(c)



Total internal reflection occurs in this configuration because at distance shorter than ~~1000~~ 1000 km ~~the~~ signal is transmitted

There is partial reflection but curve shoots up



$\therefore \tan\theta_{ic} = \frac{500}{300} = \frac{5}{3}$. $\quad \sin\theta_{ic} = \frac{5}{\sqrt{5^2+3^2}} = 0.857 = n$. ✓

~~$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \Rightarrow \omega_p^2 = \omega^2(1-n^2)$ and $k = \frac{\omega}{cn}\sqrt{1-\frac{\omega_p^2}{\omega^2}}$~~

or partial reflection is very small!

$\therefore$ ~~$k = \frac{\omega}{c}$~~

$\because$ $n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ $\therefore$ $\omega_p^2 = \omega^2 (1 - n^2)$

kc

$\omega_p^2 = \left(\frac{kc}{n}\right)^2 (1 - n^2) = \left(\frac{2\pi c}{\lambda}\right)^2 \left(\frac{1}{n^2} - 1\right)$

$\checkmark$ $\times$ $kc = \frac{2\pi c}{\lambda}$, not

$n^2 = \frac{25}{34}$ $\therefore$ $\frac{n_e e^2}{\varepsilon_0 m} = \left(\frac{2\pi c}{\lambda}\right)^2 \left(\frac{1}{n^2} - 1\right)$

$\therefore$ $n_e = \frac{\varepsilon_0 m}{e^2} \left(\frac{2\pi c}{\lambda}\right)^2 \left(\frac{1}{n^2} - 1\right)$

$\frac{34}{25}$ $= \frac{(8.854 \times 10^{-12} \times 9.11 \times 10^{-31})}{(1.6 \times 10^{-19})^2} \left(\frac{2\pi \times 3.0 \times 10^8}{21}\right)^2 \left(\frac{1}{0.857^2} - 1\right)$

$= (0.000315)(8.08 \times 10^{15})(0.3616)$

$= 9.2 \times 10^5 \text{ cm}^{-3}$ $\checkmark$

$\omega = kc$ $\frac{kc}{n}$

n_e

This is between the day time value and night value.

$9.2 \times \frac{25}{34} = 6.7$

$6.7 \times 10^5 \text{ cm}^{-3}$

(d)

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad \text{when } \omega >> \omega_p, \ n \simeq 1$$

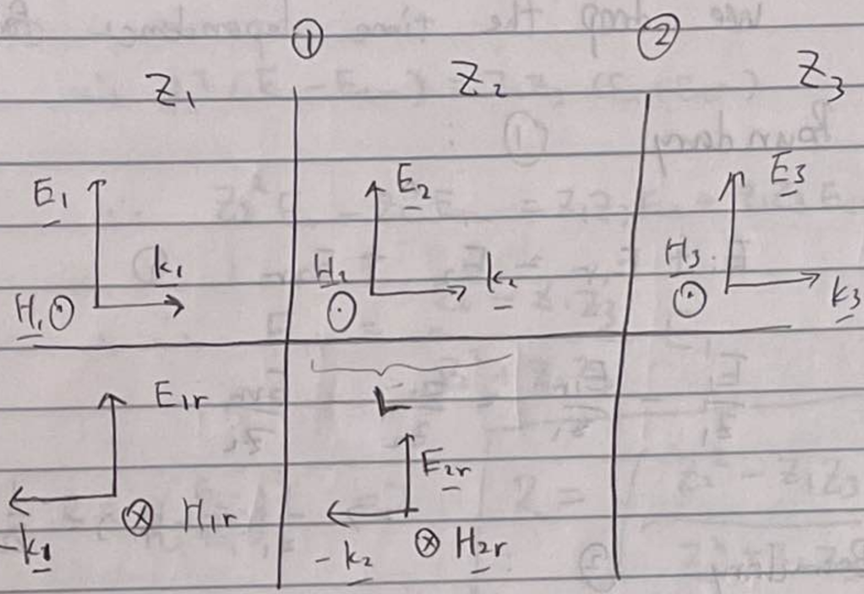
$$\sin\theta_{ic} \approx 1 \qquad \theta_{ic} \approx \pi/2 = 90°$$

∴ when high frequency waves hits the ionosphere they will ~~tran~~ be transmitted. ✓

AM signals have low frequency, which corresponds to large (small) ~~n~~ $n < 1$, so the ~~waves~~ signals are easily reflected even at small incident angle. Thus they are used in long-distance communication.

(e) At night there is no sunlight, the lower level of ionosphere ~~too~~ lose their ionization but the upper level remain ionized. So the ionosphere ~~is~~ ~~effective~~ effectively retracts to a higher altitude. This gives the AM signal a much greater ~~siz~~ distance on which to bounce. Hence ~~AM~~ radio signals travel further at night. yes

5. a)



Between (1) and (2)

Boundary conditions gives

$$E_1 + E_{1r} = E_2$$

$$H_1 - H_{1r} = H_2 \quad \cancel{\Rightarrow \frac{E_1}{Z_1} - \frac{H_{1r}}{Z_1} = H_2}$$

$$\Rightarrow \frac{E_1}{Z_1} - \frac{E_{1r}}{Z_1} = \frac{E_2}{Z_2}.$$

$\therefore$ ~~R~~ ~~E~~ $E_{1r} = \dfrac{Z_2 - Z_1}{Z_1 + Z_2} E_1$

$$\therefore R_{12} = \left(\frac{E_{1r}}{E_1}\right)^2 = \boxed{\left(\frac{Z_2 - Z_1}{Z_1 + Z_2}\right)^2} \checkmark$$

$$\text{Similarly } R_{23} = \boxed{\left(\frac{Z_3 - Z_2}{Z_2 + Z_3}\right)^2} \checkmark$$

b) we drop the time dependence for simplicity.

Boundary ①:

$$E_1 + E_{1r} = E_2 + E_{2r} \quad ①$$

$$\frac{E_1}{Z_1} - \frac{E_{1r}}{Z_1} = \frac{E_2}{Z_2} - \frac{E_{2r}}{Z_2}$$

$$\Rightarrow \frac{1}{Z_1}(E_1 - E_{1r}) = \frac{1}{Z_2}(E_2 - E_{2r}) \quad ②$$

Boundary ②:

$$E_2 e^{ik_2L} + E_{2r} e^{-ik_2L} = E_3 e^{ik_3L}$$

$$\frac{E_2}{Z_2} e^{ik_2L} - \frac{E_{2r}}{Z_2} e^{-ik_2L} = \frac{E_3}{Z_3} e^{ik_3L}.$$

$\rightarrow \quad L = \frac{\lambda_2}{4} \qquad k_2 = \frac{2\pi}{\lambda_2} \qquad \therefore k_2 L = \frac{2\pi}{\lambda_2}\frac{\lambda_2}{4} = \pi/2$

$$\therefore e^{ik_2L} = e^{i\pi/2} = i$$
$$e^{-ik_2L} = e^{-i\pi/2} = -i$$

$$\therefore \quad i(E_2 - E_{2r}) = E_3 e^{ik_3L}. \quad ③$$

$$\frac{i}{Z_2}(E_2 + E_{2r}) = \frac{E_3}{Z_3} e^{ik_3L}. \quad ④$$

$\therefore \quad ③ \rightarrow ② \Rightarrow \quad E_1 - E_{1r} = \frac{Z_1}{Z_2}(E_2 - E_{2r}) = \frac{Z_1}{Z_2}\frac{E_3}{i} e^{ik_3L}.$

$④ \rightarrow ① \Rightarrow \quad E_1 + E_{1r} = E_2 + E_{2r} = \frac{Z_2}{Z_3}\frac{E_3}{i} e^{ik_3L}.$

$$\therefore \quad \frac{E_1 - E_{1r}}{E_1 + E_{1r}} = \frac{Z_1/Z_2}{Z_2/Z_3} = \frac{Z_1 Z_3}{Z_2^2}.$$

$\therefore Z_2^2 (E_1 - E_{1r}) = Z_1 Z_3 (E_1 + E_{1r})$

$\therefore Z_2^2 E_1 - Z_2^2 E_{1r} = Z_1 Z_3 E_1 + Z_1 Z_3 E_{1r}$

$$\therefore E_{1r} = \frac{Z_2^2 - Z_1 Z_3}{Z_2^2 + Z_1 Z_3} E_1$$

$$R \equiv \left(\frac{E_{1r}}{E_1}\right)^2 \Rightarrow \boxed{R = \left(\frac{Z_2^2 - Z_1 Z_3}{Z_2^2 + Z_1 Z_3}\right)^2}$$

c) If we choose $\boxed{Z_2^2 = Z_1 Z_3}$ then

we have $R = 0$

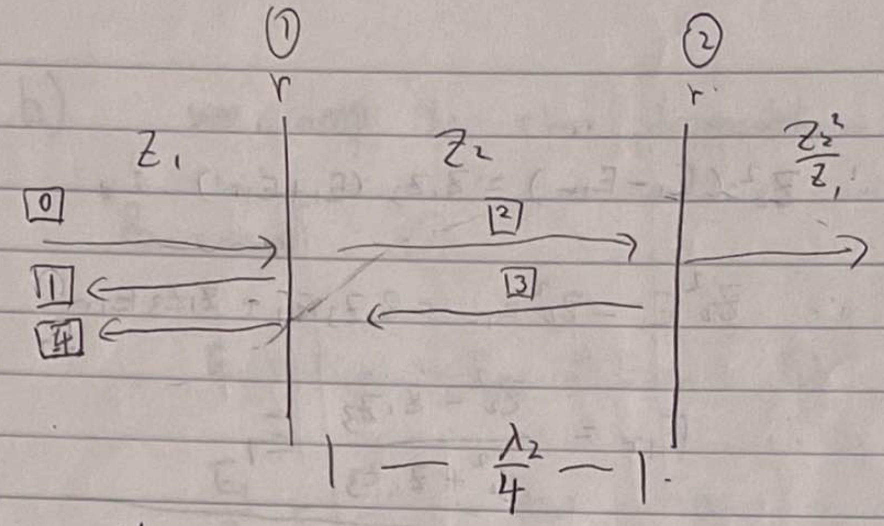
In this case $Z_3 = \frac{Z_2^2}{Z_1}$

$$R_{12} = \left(\frac{Z_2 - Z_1}{Z_1 + Z_2}\right)^2 \qquad R_{23} = \left(\frac{Z_3 - Z_2}{Z_2 + Z_3}\right)^2$$

$$= \left(\frac{\frac{Z_2^2}{Z_1} - Z_2}{Z_2 + \frac{Z_2^2}{Z_1}}\right)^2 = \left(\frac{Z_2^2 - Z_1 Z_2}{Z_2^2 + Z_1 Z_2}\right)^2$$

$$= \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

$\therefore$ In this case $\boxed{R_{12} = R_{23}}$



When R_{12} and R_{23} are small, ~~almost all~~ most waves are transmitted at boundary. when incident wave hits boundary ① it is reflected as $\boxed{1}$ and transmitted as $\boxed{2}$. $\boxed{2}$ is then reflected as $\boxed{3}$ at ② and then transmitted ~~but~~ back through ① as $\boxed{4}$

For small R_{12} and ~~R_{23}~~ R_{23}. r, $(r=r, t=1)$ amplitude of $\boxed{4}$ is ~~equal~~ essentially equal to ~~[illegible]~~ that of $\boxed{3}$ ~~and that of $\boxed{2}$ and that of $\boxed{1}$~~ ∴ $\boxed{4} = \boxed{3}$

~~[illegible]~~ $\boxed{3} = r\boxed{2}$, $\boxed{2} = \boxed{0}$, $\boxed{1} = r\boxed{0}$, $\boxed{4} = r\boxed{0} \Rightarrow \boxed{\boxed{1} = \boxed{4}}$

If $Z_2 > Z_1$ then $\frac{Z_2^2}{Z_1} > Z_2$ if $Z_2 < Z_1$ then $\frac{Z_2^2}{Z_1} < Z_2$

∴ Net phase shift at 2 boundaries are always 0

Path difference between $\boxed{1}$ and $\boxed{4}$ is $2 \times L = \frac{\lambda_2}{2}$

∴ phase difference is $k(2L) = \frac{2\pi}{\lambda_2}\frac{\lambda_2}{2} = \boxed{\pi}$ between $\boxed{1}$ and $\boxed{4}$

∵ Amplitude $\boxed{1} = \boxed{4}$

∴ Interference is completely distructive

∴ we get $R=0$ yes

To: Caroline Terquem

Ziyan Li

6. a) for conductors $\vec{J}_f = \sigma \vec{E}$

~~∵ Material is assumed to be have no polari~~

∴ Maxwell's equation becomes

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\varepsilon_0} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_0 \sigma \vec{E} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

wait for time $t \gg \tau = \frac{\varepsilon}{\sigma}$, ρ_f relaxes to 0

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0}) - \vec{\nabla}^2 E = -\vec{\nabla}^2\vec{E}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = -\frac{\partial}{\partial t}\left(\mu_0\sigma\vec{E} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}\right)$$

$$= -\mu_0\varepsilon_0\frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0\sigma\frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \quad \vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} + \mu_0\sigma\frac{\partial\vec{E}}{\partial t}$$

Similarly $\vec{\nabla}^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} + \mu_0\sigma\frac{\partial\vec{B}}{\partial t}$

If we consider the equivalent dielectric constant $\tilde{\varepsilon}$ then we have wave equations

$$\vec{\nabla}^2\vec{E} = \mu_0\tilde{\varepsilon}\frac{\partial^2\vec{E}}{\partial t^2}, \quad \vec{\nabla}^2\vec{B} = \mu_0\tilde{\varepsilon}\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\therefore \quad \mu_0\tilde{\varepsilon}\frac{\partial^2\vec{E}}{\partial t^2} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} + \mu_0\sigma\frac{\partial\vec{E}}{\partial t}$$

Substitute ~~in~~ solution $\underline{\tilde{E}} = \underline{\tilde{E}_0}\, e^{i(\tilde{k}z-\omega t)}$ in to this

we get $\quad -\omega^2\tilde{\varepsilon} = -\omega^2\varepsilon_0 - i\omega\sigma$

$$\therefore \quad \tilde{\varepsilon} = \varepsilon_0 + i\frac{\sigma}{\omega}$$

$$\tilde{\varepsilon}_r = \frac{\tilde{\varepsilon}}{\varepsilon_0} \quad \Rightarrow \quad \boxed{\tilde{\varepsilon}_r = 1 + i\frac{\sigma}{\omega\varepsilon_0}}$$

$\because \quad \nabla^2 \tilde{\underline{E}} = \mu_0 \varepsilon_0 \tilde{\varepsilon}_r \frac{\partial^2 \tilde{\underline{E}}}{\partial t^2}$

Substitute in the wave solution ~~gives~~ $\tilde{\underline{E}} = \tilde{\underline{E}}_0 e^{i(\tilde{k}z - \omega t)}$

gives $\quad -\tilde{k}^2 = \mu_0 \varepsilon_0 \left(1 + \frac{i\sigma}{\omega \varepsilon_0}\right)(-\omega^2)$

$$\Rightarrow \quad \boxed{\tilde{k}^2 = \mu_0 \varepsilon_0 \omega^2 + i \mu_0 \sigma \omega}$$

which is the dispersion relation

let $\quad \tilde{k} = k' + ik'' \quad (k', k'' \in \mathbb{R})$

then $\quad k'^2 - k''^2 = \mu_0 \varepsilon_0 \omega^2 \quad$ and $\quad 2k'k'' = \mu_0 \sigma \omega$

$$\Rightarrow \quad k' = \omega \left[\frac{\mu_0 \varepsilon_0}{2} \left(1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon_0 \omega}\right)^2} \right) \right]^{1/2}$$

$$k'' = \omega \left[\frac{\mu_0 \varepsilon_0}{2} \left(-1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon_0 \omega}\right)^2} \right) \right]^{1/2}$$

Conductor is excellent $\Rightarrow \sigma \gg \omega \varepsilon_0 \Rightarrow \frac{\sigma}{\varepsilon_0 \omega} \gg 1$

~~$\therefore k' = \omega[$~~ $\quad \therefore k' = k'' = \omega \left[\frac{\mu_0 \varepsilon_0}{2} \left(\frac{\sigma}{\varepsilon_0 \omega} \right) \right]^{1/2}$

$$= \sqrt{\omega \sigma \mu_0 / 2} = \frac{1}{\delta}$$

$$\therefore \tilde{k} = \boxed{\frac{1}{\delta} + i\frac{1}{\delta}}$$

Substitute $\tilde{k} = k' + ik''$ into $\underline{\tilde{E}} = \underline{\hat{E}_0} e^{i(\hat{k}z - wt)}$

gives $\underline{\tilde{E}} = \underline{\tilde{E}_0} e^{-k''z} e^{i(k'z - wt)}$

$\underline{\tilde{B}} = \underline{\tilde{B}_0} e^{-k''z} e^{i(k'z - wt)}$

$\therefore$ phase velocity $V_\varphi = \frac{\omega}{k'}$ ~~= [illegible]~~

$= \omega \sqrt{\frac{2}{\mu_0 \sigma \omega}} = \boxed{\sqrt{\frac{2\omega}{\mu_0 \sigma}}}$

$V_g = \frac{d\omega}{dk'}$ (k' is the wave factor in $\hat{k}$).

$\because$ $k' = \sqrt{\omega \mu_0 \sigma / 2} \Rightarrow k'^2 = \frac{1}{2} \omega \mu_0 \sigma$

$\therefore$ $2k' dk' = \frac{1}{2} \mu_0 \sigma \, d\omega$

$\therefore$ $V_g = \frac{d\omega}{dk'} = \cancel{\frac{1}{2}\mu_0\sigma} \frac{2k'}{\frac{1}{2}\mu_0 \sigma} = 4 \frac{k'}{\mu_0 \sigma}$

$= 4 \frac{1}{\mu_0 \sigma} \sqrt{\frac{\omega \mu_0 \sigma}{2}} = \boxed{2 \sqrt{\frac{2\omega}{\mu_0 \sigma}}}$

$\therefore$ $\boxed{V_g = 2 V_\varphi}$

$$\tilde{\nabla}^2 \tilde{\underline{E}} = \mu_0 \tilde{\varepsilon} \frac{\partial^2 \tilde{\underline{E}}}{\partial t^2} \Rightarrow \tilde{k}^2 = \mu_0 \tilde{\varepsilon} \omega^2$$

$$\therefore \quad \tilde{k} = \frac{1}{\sqrt{\mu_0 \tilde{\varepsilon}}} \omega \Rightarrow \tilde{k} = \tilde{n} \frac{\omega}{c}$$

if $n = n' + in''$ $\quad (n', n'' \in \mathbb{R})$ then

$$n' = \left(\frac{\omega}{c}\right)^{-1} k' = \left(\frac{\omega}{c}\right)^{-1} \sqrt{\omega \mu_0 \sigma / 2}$$

and $v_\varphi = \dfrac{c}{n'}$

$\therefore$ n' ~~increases~~ decreases with ω

$\therefore$ This is $\boxed{\text{~~normal~~ anomalous dispersion}}$

The impedence Z is given by

$$Z = \underbrace{\frac{E_{ph}}{H_{ph}} = \mu_0 \frac{E_{ph}}{B_{ph}}}_{\underline{\tilde{B}} = \frac{\tilde{k} \times \underline{\tilde{E}}}{\omega}, \ \tilde{H} = \tilde{B}/\mu_0} = \mu_0 \frac{\omega}{\tilde{k}} = \mu_0 \frac{\omega}{|\tilde{k}|^2} (\tilde{k}^*)$$

$$= \mu_0 \frac{\omega}{k'^2 + k''^2} (k' - ik'') = \frac{\mu_0 \omega}{2\left(\frac{\omega \mu_0 \sigma}{2}\right)} \left(\sqrt{\frac{\omega \mu_0 \sigma}{2}}\right)(1-i)$$

$$= \sqrt{\frac{\omega \mu_0}{2\sigma}} (1-i) = \frac{1}{\sigma} \sqrt{\frac{\omega \mu_0 \sigma}{2}} (1-i)$$

$$= \boxed{\frac{1-i}{\sigma \delta}}$$

b)

Poynting vector is ~~$\tilde{S}$~~

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$ (we use real $\vec{E}$ and $\vec{B}$ for energy)

$$\Rightarrow \vec{E} = \vec{E}_0 e^{-k''z}\cos(k'z - \omega t + \varphi)$$

~~$\vec{B} = \frac{1}{\omega}(\hat{z}\times\vec{E}_0) e^{-k''z}\cos($~~

$$\tilde{B} = \frac{\tilde{k}\hat{z}\times\tilde{E}}{\omega} = \frac{\tilde{k}}{\omega}(\hat{z}\times\tilde{E}_0) e^{-k''z} e^{i(k'z-\omega t)}.$$

$$\therefore \vec{B} = \frac{1}{\omega}(\hat{z}\times\vec{E}_0) e^{-k''z}[k'\cos(k'z-\omega t+\varphi) - k''\sin(k'z-\omega t+\varphi)]$$

$$\therefore \boxed{\vec{S} = \hat{z}\frac{1}{\mu_0\omega}E_0^2 e^{-2k''z}[k'\cos^2(k'z-\omega t+\varphi) - k''\sin(k'z-\omega t+\varphi)\cos(k'z-\omega t+\varphi)]}$$

$\because \langle\cos^2\theta\rangle = \frac{1}{2}$, $\langle\sin\theta\cos\theta\rangle = 0$

$$\therefore \boxed{\langle\vec{S}\rangle = \hat{z}\frac{k'E_0^2}{2\mu_0\omega} e^{-2k''z}}$$

Lorentz force : $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

work ~~dx~~ $dw = \vec{F}\cdot d\vec{x}$

$$\therefore \frac{dw}{dt} = \vec{F}\cdot\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{v} = q(\vec{E} + \underbrace{\vec{v}\times\vec{B})\cdot\vec{v}}_{0} = q\vec{E}\cdot\vec{v}$$

on one charge

Rate of Total work done is

$$\frac{dW}{dt} = \iiint \frac{dW'}{dt} \, n \, d\tau$$

$\uparrow$ number density

$$= \iiint nq\vec{v}\cdot\vec{E}\, d\tau = \iiint \vec{J}_f \cdot \vec{E}'\, d\tau$$

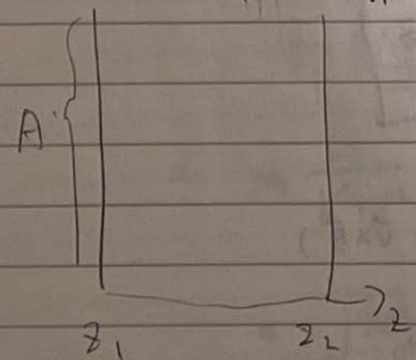
$$= \iiint \sigma \vec{E}\cdot\vec{E}\, d\tau = \iiint \sigma E^2 \, d\tau$$

$$\vec{E} = \vec{E}_0 e^{-k''z} \cos(k'z - \omega t + \psi)$$

$$\langle \sigma E^2 \rangle = \sigma E_0^2 e^{-2k''z} \underbrace{\langle \cos^2(k'z - \omega t + \psi) \rangle}_{\frac{1}{2}}$$

$$= \frac{1}{2} E_0^2 e^{-2k''z}$$

let $A =$ cross sectional area



$$\left\langle \frac{dW}{dt} \right\rangle = \iiint \langle \sigma E^2 \rangle \underbrace{d\tau}_{A\,dz}$$

$$= \int_{z_1}^{z_2} \sigma \frac{1}{2} E_0^2 e^{-2k''z} A \, dz$$

$$= \cancel{-\sigma} - \frac{1}{4} \frac{\sigma E_0^2}{k''} \left[e^{-2k''z} \right]_{z_1}^{z_2} A$$

$$= \frac{1}{4} \frac{\sigma E_0^2}{k''} \left[e^{-2k''z_1} - e^{-2k''z_2} \right] \cdot A$$

$$= \frac{1}{4} A\sigma E_0^2 \sqrt{\frac{2}{\omega\mu_0\sigma}} [e^{-2k''z_1} - e^{-2k''z_2}]$$

$$= \cancel{\frac{\sqrt{2}}{4} A\sigma E_0^2} \boxed{\frac{\sqrt{2}}{4} A E_0^2 \sqrt{\frac{\sigma}{\omega\mu_0}} [e^{-2k''z_1} - e^{-2k''z_2}]}.$$

Time average of flux of Poynting vector

is $\langle\Phi\rangle = \langle\Phi_{z_1}\rangle - \langle\Phi_{z_2}\rangle$.

$$= \langle S\rangle_{z_1} A - \langle S\rangle_{z_2} A$$

$$= \frac{k' E_0^2 A}{2\mu_0\omega} [e^{-2k''z_1} - e^{-2k''z_2}].$$

$$= \frac{1}{2} \frac{E_0^2 A}{\mu_0\omega} \frac{\sqrt{\omega\mu_0\sigma}}{\sqrt{2}} [e^{-2k''z_1} - e^{-2k''z_2}].$$

$$= \frac{1}{2\sqrt{2}} E_0^2 A \sqrt{\frac{\sigma}{\omega\mu_0}} [e^{-2k''z_1} - e^{-2k''z_2}].$$

$$= \frac{\sqrt{2}}{4} E_0^2 A \sqrt{\frac{\sigma}{\omega\mu_0}} [e^{-2k''z_1} - e^{-2k''z_2}].$$

$$\therefore \langle\Phi\rangle = \left\langle \frac{dW}{dt} \right\rangle$$

c) Energy density

$$u = \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{u_E} + \underbrace{\frac{1}{2\mu_0}B^2}_{u_m}$$

$$\langle u_E \rangle = \frac{1}{2}\varepsilon_0 \langle E^2 \rangle$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 e^{-2k''z} \underbrace{\langle \cos^2(k'z - \omega t + \varphi) \rangle}_{1/2}$$

$$= \boxed{\frac{1}{4}\varepsilon_0 E_0^2 e^{-2k''z}}$$

$$\langle u_m \rangle = \frac{1}{2\mu_0}\langle B^2 \rangle$$

$$= \frac{1}{2\mu_0}\frac{1}{\omega^2}E_0^2 e^{-2k''z}\Big[k'^2 \overbrace{\langle \cos^2(k'z-\omega t+\varphi)\rangle}^{1/2} + k''^2 \overbrace{\langle \sin^2(k'z-\omega t+\varphi)\rangle}^{1/2}$$

$$- 2k'k'' \underbrace{\langle \sin(k'z-\omega t+\varphi)\cos(k'z-\omega t+\varphi)\rangle}_{0}\Big].$$

$$= \frac{1}{\mu_0}\frac{1}{4\omega^2}E_0^2 e^{-2k''z}\left[k'^2 + k''^2\right].$$

$$= \frac{\cancel{\omega\mu_0\sigma}\ \omega\mu_0\sigma}{4\mu_0\omega^2}E_0^2 e^{-2k''z}$$

$$= \boxed{\frac{\sigma}{4\omega}E_0^2 e^{-2k''z}}$$

$$\therefore \boxed{\langle u \rangle = \frac{1}{4}\varepsilon_0 E_0^2 e^{-2k''z} + \frac{\sigma}{4\omega}E_0^2 e^{-2k''z}}$$

$\because \ \sigma \gg \omega \varepsilon_0 \quad \therefore \ \frac{\sigma}{\omega} \gg \varepsilon_0$

$\therefore \ u_m \gg u_E$

$\therefore$ Energy is mainly carried by the <u>magnetic</u> field.

$$\cancel{u_E = \frac{1}{4}\varepsilon_0 \frac{1}{2}\varepsilon_0 E_0^2 e^{-2k''z} \cos^2(k'z - \omega t + \varphi)}$$

$$\cancel{= \frac{1}{2}\varepsilon_0 E_0^2 e^{-2k''z} \left(\frac{1}{2} + \frac{1}{2}\cos\right.}$$

if The speed at which energy travels is V_E.

$$\cancel{V_E = \frac{\langle S\rangle}{\langle u\rangle} \approx \frac{\langle S\rangle}{\langle u_m\rangle} =}$$

then during time dt the flux of energy is $u(V_E\, dt)A$.

$V_E dt$

$\therefore \ u(V_E\, dt)A = SA\,dt$

$\therefore \ V_E = \frac{S}{u} = \frac{\langle S\rangle}{\langle u\rangle}$

$$V_E = \underbrace{\frac{\langle S\rangle}{\langle u\rangle}}_{u \approx u_m} = \frac{\langle S\rangle}{\langle u_m\rangle} = \frac{\frac{k' \cancel{E_0^2}}{2\mu_0\omega} \cancel{e^{-2k''z}}}{\frac{\sigma}{4\omega} \cancel{E_0^2} \cancel{e^{-2k''z}}} = \boxed{\frac{2k'}{\mu_0 \sigma}}$$

$\cancel{\therefore k'^2 = \frac{1}{2}\omega\mu\sigma}$

Hence we have

$$V_E = V_\varphi = \frac{1}{2} V_g$$

$\therefore V_E < V_g$ $\therefore$ Energy velocity is smaller than the group velocity.

In dissipative media the energy velocity is smaller than the group velocity because the wave is damped.

7. $\vec{\nabla}\cdot\tilde{\vec{E}}=0 \qquad \vec{\nabla}\cdot\tilde{\vec{B}}=0$

$$\vec{\nabla}\times\tilde{\vec{E}}=-\frac{\partial\tilde{\vec{B}}}{\partial t} \qquad \vec{\nabla}\times\tilde{\vec{B}}=\mu_0\tilde{\varepsilon}\frac{\partial\tilde{\vec{E}}}{\partial t}$$

let $\tilde{E}=\vec{E}'+i\vec{E}'' \qquad \tilde{B}=\vec{B}'+i\vec{B}'' \qquad \tilde{\varepsilon}=\varepsilon'+i\varepsilon''$

Real part:

$$\cancel{\vec{\nabla}\cdot\vec{E}_R=0 \qquad \vec{\nabla}\cdot\vec{B}_R=0}$$
$$\cancel{\vec{\nabla}\times\vec{E}_R=-\frac{\partial\vec{B}_R}{\partial t} \qquad \vec{\nabla}\times\vec{B}_R=\mu_0}$$

$$\vec{\nabla}\cdot\vec{E}'=0 \qquad \vec{\nabla}\cdot\vec{B}'=0$$
$$\vec{\nabla}\times\vec{E}'=-\frac{\partial\vec{B}'}{\partial t} \qquad \vec{\nabla}\times\vec{B}'=\mu_0\varepsilon'\frac{\partial\vec{E}'}{\partial t}-\mu_0\varepsilon''\frac{\partial\vec{E}''}{\partial t}$$

Imaginary part:

$$\vec{\nabla}\cdot\vec{E}''=0 \qquad \vec{\nabla}\cdot\vec{B}''=0$$
$$\vec{\nabla}\times\vec{E}''=-\frac{\partial\vec{B}''}{\partial t} \qquad \vec{\nabla}\times\vec{B}''=\mu_0\varepsilon'\frac{\partial\vec{E}''}{\partial t}+\mu_0\varepsilon''\frac{\partial\vec{E}'}{\partial t}$$

Physical Quantity is the real part:

$$\vec{E}'\cdot(\vec{\nabla}\times\vec{B}')=\mu_0\varepsilon'\,\vec{E}'\cdot\frac{\partial\vec{E}'}{\partial t}-\mu_0\varepsilon''\,\vec{E}'\cdot\frac{\partial\vec{E}''}{\partial t}=\frac{1}{2}\mu_0\varepsilon'\frac{\partial E'^2}{\partial t}-\mu_0\varepsilon''\vec{E}'\cdot\frac{\partial\vec{E}''}{\partial t}$$

$$\vec{B}'\cdot(\vec{\nabla}\times\vec{E}')=-\vec{B}'\cdot\frac{\partial\vec{B}'}{\partial t}=-\frac{1}{2}\frac{\partial B^2}{\partial t}$$

$$\therefore \frac{1}{\mu_0}\big[\underbrace{\vec{E}'\cdot(\vec{\nabla}\times\vec{B}')-\vec{B}'\cdot(\vec{\nabla}\times\vec{E}')}_{-\vec{\nabla}\cdot(\vec{E}'\times\vec{B}')}\big]=\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon'E'^2+\frac{1}{2}\mu_0 B^2\right)-\varepsilon''\vec{E}'\cdot\frac{\partial\vec{E}''}{\partial t}$$

$$\Rightarrow \quad -\vec{\nabla}\cdot\left(\frac{\vec{E}'\times\vec{B}'}{\mu_0}\right)=\frac{\partial u}{\partial t}-\mu_0\varepsilon''\vec{E}'\cdot\frac{\partial\vec{E}''}{\partial t}$$

$$\therefore \quad \frac{\partial u}{\partial t}=\varepsilon''\vec{E}'\cdot\frac{\partial\vec{E}''}{\partial t}-\vec{\nabla}\cdot\vec{S}$$

[redacted]

Wrong!

$\therefore$ The rate at which energy is dissipated

is $\boxed{-\varepsilon'' \vec{E}' \cdot \frac{\partial \vec{E}''}{\partial t}} = \boxed{+\omega \varepsilon'' E^2}$

b) $\langle \vec{S} \rangle = \langle \frac{\vec{E} \times \vec{B}}{\mu_0} \rangle = \hat{z} c n' \frac{\varepsilon_0 E_0^2}{2} e^{-2k'' z}$

Flux into the volume

$$\Phi = \Phi_{z_1} - \Phi_{z_2} = A c n' \frac{\varepsilon_0 E_0^2}{2} \left[e^{-2k'' z_1} - e^{-2k'' z_2} \right].$$

Total energy dissipated in the volume ΔE :

plane wave ~~substitute~~ solution : $\tilde{\underline{E}} = \tilde{\underline{E}}_0 e^{i(\tilde{k} z - \omega t)}$

$\Rightarrow \tilde{k} = \mu_0 \tilde{\varepsilon} \omega^2$

$\tilde{k} = k' + i k''$

$\tilde{\underline{E}} = \tilde{E}_0 e^{-k'' z} e^{i(k' z - \omega t)}.$

$\therefore$ $\vec{E}' = \hat{z} E_0 e^{-k'' z} \cos(k' z - \omega t + \phi)$

$\vec{E}'' = \hat{z} E_0 e^{-k'' z} \sin(k' z - \omega t + \phi).$

$$\langle \Delta E \rangle = \iiint_{\tau} \langle \varepsilon'' \vec{E}' \cdot \frac{\partial \vec{E}''}{\partial t} \rangle d\tau = \int_{z_1}^{z_2} + \omega E_0^2 \varepsilon'' e^{-2k'' z} \underbrace{\langle \cos^2(k' z - \omega t + \phi) \rangle}_{1/2} A \, dz$$

$$= + \frac{1}{4} \frac{\omega E_0^2}{k''} \varepsilon'' \left[e^{-2k'' z_1} - e^{-2k'' z_2} \right].$$

$\hat{k} = \frac{\omega}{c}\hat{n} \quad \therefore k' = \frac{\omega}{c}n' \quad k'' = \frac{\omega}{c}n''$

$\hat{k}^2 \stackrel{?}{=} \mu_0 \hat{\varepsilon}\omega^2, \ \hat{k} = k' + ik'', \ \hat{\varepsilon} = \varepsilon' + i\varepsilon''$

$\Rightarrow \quad k'^2 - k''^2 = \mu_0\omega^2\varepsilon', \quad 2k'k'' = \mu_0\omega^2\varepsilon''$

$$\therefore \underbrace{\frac{\varepsilon''}{k''} = \frac{2k'}{\mu_0\omega}, \quad n' = \frac{ck'}{\omega^2}}$$

$\therefore \frac{1}{2}cn'\varepsilon_0 = \frac{1}{2}c^2\frac{k'}{\omega}\varepsilon_0 = \frac{1}{2\mu_0\cancel{\varepsilon_0}}\frac{k'}{\omega}\cancel{\varepsilon_0} = \frac{k'}{2\mu_0\omega}$

$\cancel{= \frac{1}{4}\omega\varepsilon_0^2}$

$\frac{1}{4}\frac{\omega\varepsilon''}{k''} = \frac{1}{4}\omega\left(\frac{2k'}{\mu_0\omega^2}\right) = \frac{k'}{2\mu_0\omega}$

$\therefore \frac{1}{2}cn'\varepsilon_0 = \frac{1}{4}\frac{\omega\varepsilon''}{k''}\cancel{\varepsilon_0}$

$\Rightarrow \cancel{\Phi + \langle \Delta E \rangle = 0}$

$\therefore \Phi = \langle \Delta E \rangle$

$\therefore$ flux equals energy dissipated.

c) $\vec{\nabla}\cdot\tilde{\underline{E}} = 0$ $\quad \vec{\nabla}\cdot\hat{\underline{B}} = 0$ $\quad \vec{J} = \sigma\vec{E}$

$$\vec{\nabla}\times\tilde{\underline{E}} = -\frac{\partial\hat{B}}{\partial t} \qquad \vec{\nabla}\times\tilde{\underline{B}} = \mu_0\sigma\tilde{\underline{E}} + \mu_0\tilde{\varepsilon}\frac{\partial\tilde{E}}{\partial t}.$$

take real part:

$$\vec{\nabla}\cdot\vec{E}' = 0 \qquad \vec{\nabla}\cdot\vec{B}' = 0$$

$$\vec{\nabla}\times\vec{E}' = -\frac{\partial\vec{B}'}{\partial t}$$

$$\vec{\nabla}\times\vec{B}' = \mu_0\sigma E' + \mu_0\varepsilon'\frac{\partial\vec{E}'}{\partial t} - \mu_0\varepsilon''\frac{\partial\vec{E}''}{\partial t}.$$

plane wave solution

$$\tilde{\underline{E}} = \tilde{E}_0 e^{i(\tilde{k}z-\omega t)}$$

$$\vec{E}' = E_0 e^{-k''z}\cos(k'z-\omega t+\phi)\ \hat{z}$$

$$\hat{E}'' = E_0 e^{-k''z}\sin(k'z-\omega t+\phi)\ \hat{z}$$

$$\therefore \frac{\partial\hat{E}''}{\partial t} = -\omega\vec{E}'$$

$$\therefore \vec{\nabla}\times\vec{B}' = \mu_0\underbrace{(\sigma+\varepsilon''\omega)}_{\sigma'}\vec{E}' + \mu_0\varepsilon'\frac{\partial\vec{E}}{\partial t}$$

$\therefore \boxed{\sigma' = \sigma+\varepsilon''\omega}$ is the effective conductivity.

Drawing Analogy to ohmic dissipation:

Ohmic dissipation the rate of energy loss is

$$\vec{J}\cdot\vec{E} = \sigma E^2$$

$\therefore$ In this ~~case~~ case (lossy dielectric)

the rate of energy loss is

$$\sigma' E^2 = \boxed{(\sigma + \varepsilon'' \omega) E^2}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\cancel{dW = \vec{F} \cdot d\vec{l}}$$

$$dw = \vec{F} \cdot d\vec{l} = q(\vec{E} \cdot d\vec{l} + (\vec{v} \times \vec{B}) \cdot d\vec{l})$$

$$= q(\vec{E} \cdot \vec{v}dt + \underbrace{(\vec{v} \times \vec{B}) \cdot \vec{v}dt}_{0})$$

$$= q\vec{E} \cdot \vec{v}dt.$$

$$\therefore \frac{dw}{dt} = q\vec{E} \cdot \vec{v}$$

$$\frac{dW}{dt} = \iiint nq\vec{E} \cdot \vec{v}\, d\tau$$

$$\vec{J} = nq\vec{v} \implies \frac{dW}{dt} = \iiint \vec{J} \cdot \vec{E}\, d\tau$$

Energy transfer from the wave

to the charges.

Wave is losing energy.

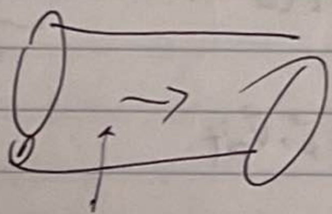
Charge is gaining energy

Q1

$\vec{E}$ is constant.

$$\frac{dW}{dt} = \left[\iiint_V \vec{J}\, d\tau \right] \cdot \vec{E}$$

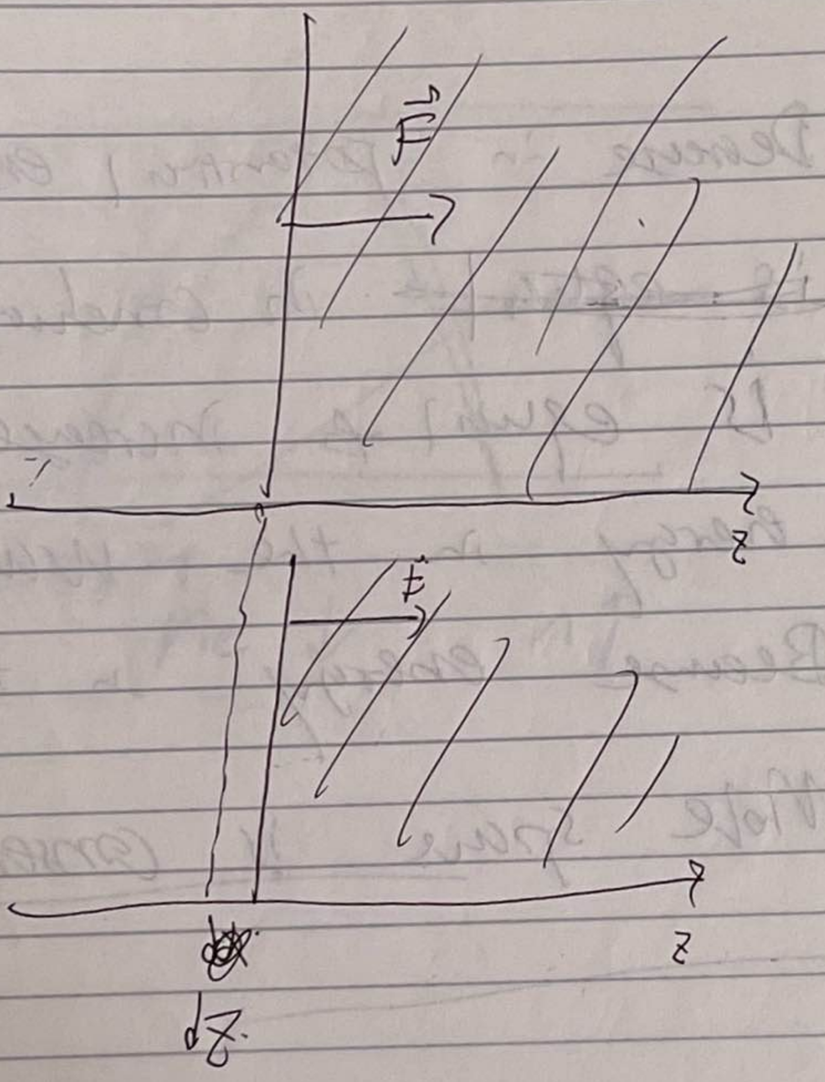
$$= (IL)E = I(LE) = IV$$



Inside conductor $\vec{E}$ is constant.

surface contributions only works for

outside and on surface



If you hold the conductor you effectlely keep ~~the~~ a constant potential energy ~~for~~ in the conductor.

When you release the conductor potential energy stored in conductor Decrease, because work is done by the radiation pressure (bottle analogy.)

Decrease in potential energy

~~is equal~~ in conductor

is equal to increase in

EM energy in the wave.

Because energy in the

whole space is conserved.

6

anomalous dispersion

$k^2 = n^2 \frac{\omega}{c}$

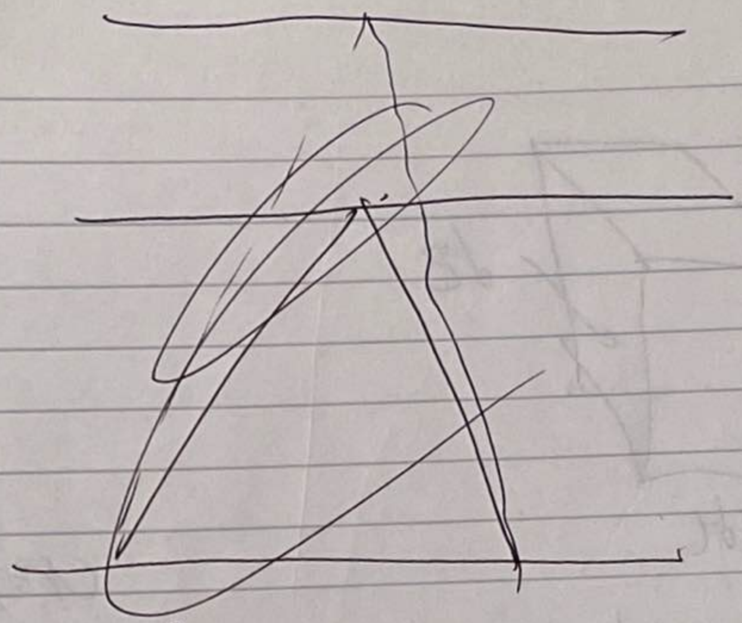
SB!

$n' \frac{\omega}{c} = k'$

$\therefore n' = \frac{c}{\omega} k' = \frac{c}{\omega}\sqrt{\omega\mu_0\sigma/2}$

$n' \downarrow$ if $\omega \uparrow$

$\therefore$ anomalous dispersion



at night

$n_e \downarrow$ $n \uparrow$

transport until

~~一直传播直至~~ the condition

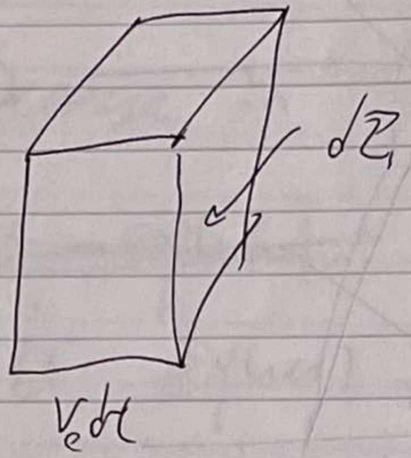
$\sin\theta_i = n$ is ~~mee~~ met

n is a variable with

altitude.

higher you need to

reach at night.



cross surface

$c\langle u\rangle V_e\, dt\, dZ$ = energy ~~transported~~ during ~~at~~ a the dt

$= \langle S\rangle dt\, dZ$

$\therefore \quad V_e = \dfrac{\langle S\rangle}{c\langle u\rangle} < V_g$

If you are dissipating energy

∴ It makes no sense to talk about group velocity

$\delta = \dfrac{\lambda}{2\pi}$

You cannot even travel longer than a wavelength before you dissipate most the energy