To: Robin Nicholas

~~E~~ Statistical Mechanics 1 — Ziyan Li

1) (a) ~~x cos~~ $x(r,\theta) = r\cos\theta$ $\quad y(r,\theta) = r\sin\theta$

$r(x,y) = (x^2+y^2)^{1/2}$ $\quad \theta(x,y) = \arctan(\frac{y}{x})$

$\therefore \left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta = \frac{x}{r}$

when ~~we~~ we do the same ~~t~~ operation to

$x^2+y^2=r^2$, ~~we~~ we should have

$$2x\left(\frac{\partial x}{\partial r}\right)_\theta + 2y\left(\frac{\partial y}{\partial r}\right)_\theta = 2r$$

$$\Rightarrow \cancel{2x\cos\theta}\ \frac{2x^2}{r} + \frac{2y^2}{r} = \frac{2r^2}{r} = 2r$$, which is consistent.

~~consistent~~

when we keep y constant in differentiation,

we then get

$$\frac{\partial}{\partial r}(x^2+y^2) = \frac{\partial}{\partial r} r^2$$

$$\Rightarrow 2x\left(\frac{\partial x}{\partial r}\right)_y = 2r$$

$\therefore \left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{x}$, this is different from $\left(\frac{\partial x}{\partial r}\right)_\theta = \frac{x}{r}$

$\therefore$ there is <u>no contradiction</u>.

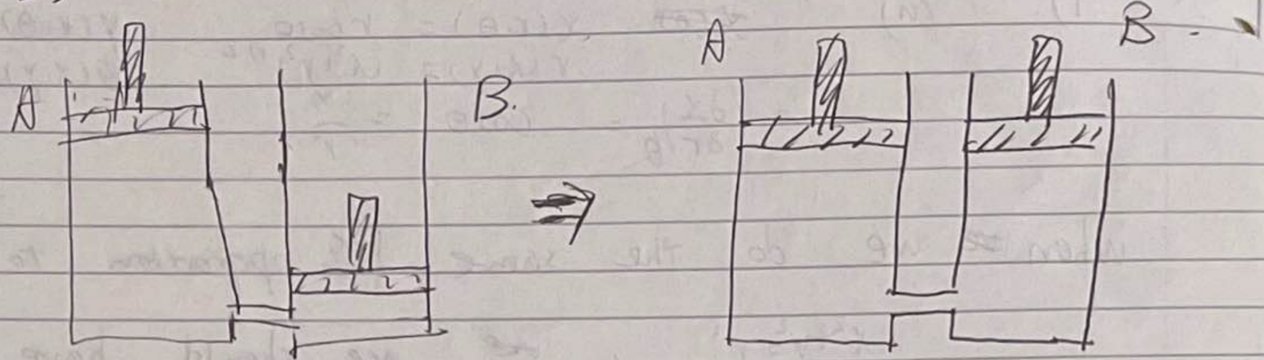
(b) $r^2 = x^2+y^2$ $\quad \left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{x}$

$\Rightarrow 2r\left(\frac{\partial r}{\partial y}\right)_x = 2y \Rightarrow \left(\frac{\partial r}{\partial y}\right)_x = \frac{y}{r}$

$0 = 2x + 2y\left(\frac{\partial y}{\partial x}\right)_r \Rightarrow \left(\frac{\partial y}{\partial x}\right)_r = -\frac{x}{y}$

$$\therefore \left(\frac{\partial x}{\partial r}\right)_y \left(\frac{\partial r}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_r = \left(\frac{r}{x}\right)\left(\frac{y}{r}\right)\left(-\frac{x}{y}\right) = \boxed{-1}$$

2)



Initial state : $V = V_i$, $T = T_i$

Final state : $V = 2V_i$, $T = T_f$

This is an ideal gas undergoing adiabatic process

$$\therefore \quad TV^{\gamma-1} = \text{constant.}$$

$$\gamma = \frac{5}{3} \Rightarrow \quad \cancel{T} \quad TV^{\frac{2}{3}} = \text{constant.}$$

$$\therefore \quad T_i V_i^{2/3} = T_f (2V_i)^{2/3}$$

$$\therefore \quad T_f = T_i \left(\frac{1}{2}\right)^{2/3}$$

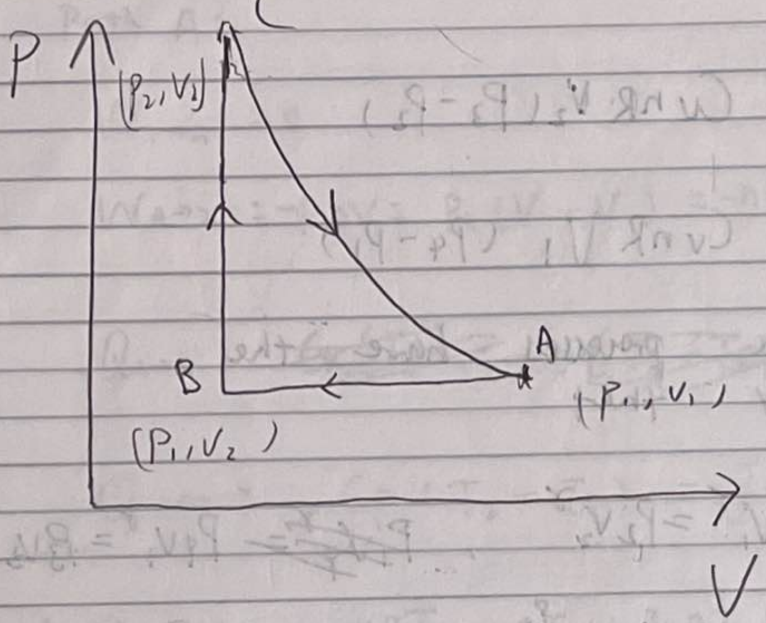
$$\Rightarrow \quad \boxed{T_f = \frac{T_i}{2^{2/3}}}$$

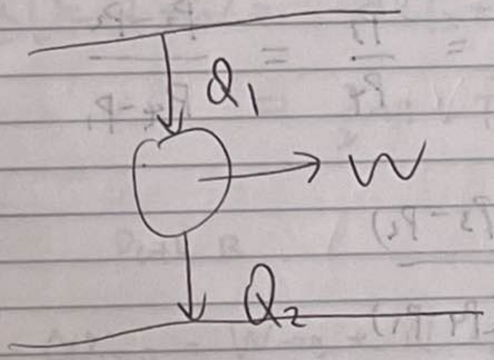
Slow process → reversible.

thermally isolated → ~~Adiabatic~~. adia thermo. + $\cancel{\theta}$

= adiabatic

3)





The efficiency of a heat engine is

$$\eta = 1 - \frac{Q_2}{Q_1} \qquad PV = nRT.$$

Q_1 = heat put in $= C_v \Delta T_1 = C_v V_2 (P_2 - P_1)/nR$

Q_2 = heat give out. $= C_p \Delta T_2 = C_p P_1 (V_1 - V_2)/nR.$

$$\therefore \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p P_1 (V_1 - V_2)/\cancel{nR}}{C_v V_2 (P_2 - P_1)/\cancel{nR}}$$

$$= 1 - \underbrace{\left(\frac{C_p}{C_v}\right)}_{\gamma} \frac{\left(\frac{V_1}{V_2} - 1\right)}{\left(\frac{P_2}{P_1} - 1\right)}$$

$$= 1 - \gamma \frac{(V_1/V_2) - 1}{(P_2/P_1) - 1}$$

4)

$$Q_1 = C_V nR V_2 (P_3 - P_2)$$

$$Q_2 = C_V nR V_1 (P_4 - P_1)$$

Adiabatic processes have the property that

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad , \quad \cancel{P_1 V_4^{\gamma}} = P_4 V_1^{\gamma} = P_3 V_2^{\gamma}.$$

$$\therefore \left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{P_2}{P_1} = \frac{P_3}{P_4} = \frac{P_3 - P_2}{P_4 - P_1}$$

$$\therefore \frac{Q_2}{Q_1} = \frac{\cancel{C_V nR} V_2 (P_3 - P_2)}{\cancel{C_V nR} V_1 (P_4 - P_1)}$$

$$= \left(\frac{V_1}{V_2}\right)^{-1} \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$= \left(\frac{V_2}{V_1}\right)^{1-\gamma} = r^{1-\gamma}$$

$$\therefore \eta = 1 - \frac{Q_2}{Q_1} = \underline{1 - r^{1-\gamma}}$$

5) Path A:

$W_{1\to A} = 0$

$W_{A\to 2} = -P\Delta V = P_2(V_2 - V_1) = -nR(T_2 - T_A)$

$Q_{1\to A} = \cancel{C_V(T_2} \cancel{C_V(T_A - T_A)}\ C_V(T_A - T_1) = \frac{3}{2}nR(T_A - T_1)$

$Q_{A\to 2} = C_P(T_2 - T_A) = \frac{5}{2}nR(T_2 - T_A)$

$\Delta U = Q + W = -nRT_2 + nRT_A + \frac{3}{2}nRT_A - \frac{3}{2}nRT_1 + \frac{5}{2}nRT_2 - \frac{5}{2}nRT_A$

$= \frac{3}{2}nR(T_2 - T_1) = C_V(T_2 - T_1)$

Path B:

$\Delta U_{1\to B} = W_{1\to B} + Q_{1\to B} = 0 \Leftarrow$ isothermal expansion

$W_{B\to 2} = 0$

$Q_{B\to 2} = C_V(T_2 - T_B) = C_V(T_2 - T_1)$

$\underbrace{\quad}_{T_B = T_1}$

Path C:

$Q_{1\to C} = 0$

$W_{1\to C} = -\int \underbrace{P\,dV}_{PV^\gamma = \text{const}} = -\int_{V_1}^{V_2} \frac{P_1V_1^\gamma}{V^\gamma}\,dV$

$\cancel{= -P_1V_1^\gamma\left(\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right)(}$

$= \cancel{= P_1V_1^\gamma(}$

$$= -P_1 V_1^{\gamma} \int_{V_1}^{V_2} V^{-\gamma}\, dV$$

$$= -P_1 V_1^{\gamma} \frac{1}{1-\gamma} \left(V_2^{1-\gamma} - V_1^{1-\gamma} \right).$$

$$\because \gamma = \frac{5/2}{3/2} = \frac{5}{3} \qquad \therefore 1-\gamma = -\frac{2}{3}$$

$$\cancel{W = \frac{3}{2} P_1 V_1 \left(V_2^{-\frac{2}{3}} - V_1^{-\frac{2}{3}} \right)}$$

$$\therefore W_{1\to c} = \frac{3}{2} P_1 V_1^{\frac{5}{3}} \left(V_2^{-\frac{2}{3}} - V_1^{-\frac{2}{3}} \right).$$

$$= \frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right] = \frac{3}{2} nRT_1 \left[\left(\frac{V_1}{V_2} \right)^{\frac{2}{3}} - 1 \right]$$

$$W_{c\to 2} = 0$$

$$Q_{c\to 2} = C_V (T_2 - T_c)$$

$$T_c V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\therefore T_c V_2^{\frac{2}{3}} = T_1 V_1^{\frac{2}{3}} \qquad \therefore \frac{T_c}{T_1} = \left(\frac{V_1}{V_2} \right)^{\frac{2}{3}}$$

$$\therefore \Delta U = W_{1\to c} + Q_{c\to 2}$$

$$= C_V T_1 \left[\frac{T_c}{T_1} - 1 \right] + C_V \left[T_2 - T_c \right].$$

$$= C_V \left[T_2 - T_1 \right]$$

6)

The efficiency of ideal heat pump is

$$\eta = \frac{T}{T-T_0}$$

$\therefore$ power ~~~~ input to the house

$$= \frac{TW}{T-T_0}$$

This is equal to the rate at which the building lose heat for the building to keep constant ~~temp~~ temperature.

$$\therefore \quad \alpha(T-T_0) = \frac{TW}{T-T_0}$$

$$\therefore \alpha(T-T_0)^2 - WT = 0$$

$$\therefore \alpha(T-T_0)^2 - W(T-T_0) - WT_0 = 0$$

$$\therefore T-T_0 = \frac{1}{2\alpha}\left[W \pm \sqrt{W^2 + 4\alpha T_0 W}\right]$$

$$\therefore T-T_0 = \frac{W}{2\alpha}\left(1 + \sqrt{1 + 4\alpha T_0/W}\right).$$

$$\therefore \boxed{T = T_0 + \frac{W}{2\alpha}\left(1 + \sqrt{1 + 4\alpha T_0/W}\right)}$$