2662

SECOND PUBLIC EXAMINATION

Honour School of Physics Part A: 3 and 4 Year Courses

A2: ELECTROMAGNETISM AND OPTICS

TRINITY TERM 2012

Saturday, 16 June, 9.30 am – 12.30 pm

Answer **all** *of Section A and* **three** *questions from Section B.*

For Section A start the answer to each question on a **fresh page.**
For Section B start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Write down an expression for the energy density stored in an electromagnetic wave, and show that the instantaneous rate of energy flow is given by the Poynting vector: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$. [4]

A plane sinusoidal wave in the form of a laser beam propagating in vacuum has power P_0 and diameter w. Obtain expressions for the amplitudes of the electric and magnetic fields, E_0 and B_0 respectively. Estimate the value of E_0 for a 100 gigawatt laser with w =1 mm. [3]

2. A transmission line consists of two circular coaxial cylinders (radii a, b, where $b > a$) separated by a dielectric material with permittivity ε_{m}. (The permeability can be taken to be 1.) Derive an expression for the characteristic impedance Z of the transmission line in terms of its capacitance C per unit length and its inductance L per unit length. [3]

Show that Z is given by

$$Z = \frac{1}{2\pi}\left(\frac{\mu_0}{\varepsilon_0\varepsilon_{\mathrm{m}}}\right)^{1/2} \ln\left(\frac{b}{a}\right) .$$

A typical coaxial cable has $Z = 50\,\Omega$. Suggest reasonable values of a, b and ε_{m} for this cable. [5]

3. Two functions $f(x)$ and $g(x)$ have Fourier transforms $F(k)$ and $G(k)$ respectively. Show that the Fourier transform of $h(x) = f(x) * g(x)$ (the convolution of $f(x)$ and $g(x)$), is given by

$$H(k) = F(k) \times G(k) .$$

Use this result to calculate the angular intensity distribution of light diffracted by a double-slit arrangement, each of width b and separation a, in the Fraunhofer limit. [6]

4. Write down the wave equations for **E** and **H** propagating in a highly conducting lossless medium, and show that high frequency waves propagate only a short distance δ, the skin depth. Estimate the impedance $|\mathbf{E}/\mathbf{H}|$ for copper at a frequency of 1 MHz and compare it with the free space impedance. [7]

[The conductivity of copper is $6 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$.]

5. Explain what is meant by *birefringence* in a uniaxial crystal, and illustrate your answer with a diagram showing propagation parallel and perpendicular to the optic axis. Show in a diagram how a quarter-wave plate and a linear polarizer may be used to produce circularly polarized light. [4]

Obtain an expression for the phase difference between ordinary and extraordinary rays, and estimate the thickness of a zero-order calcite plate for light of wavelength 589 nm ($n_{\mathrm{o}} = 1.658$ and $n_{\mathrm{e}} = 1.486$). Explain why a 100-order plate is more practically suitable. [4]

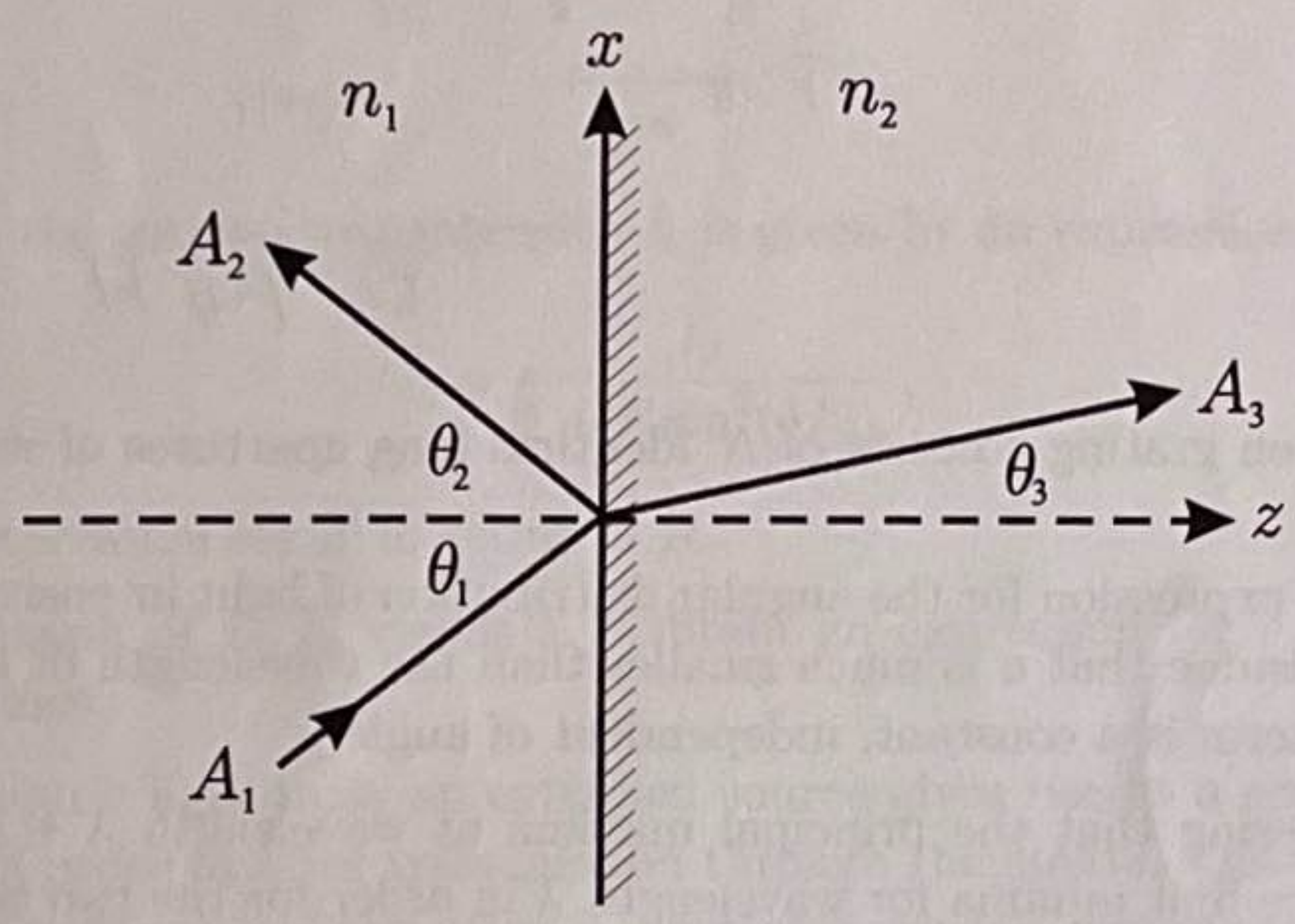
6. A Michelson interferometer is set up to analyse light from a laser with wavelength λ_0. It consists of circular mirrors placed in each arm separated by a path difference d. Show that the interference pattern consists of alternately bright and dark rings, and show that the angular diameter of the nth fringe is

$$\psi_n = \left(\frac{n\lambda_0}{d}\right)^{1/2} .$$

[4]

Section B

7. Light is incident on the interface at $z = 0$ between two non-conducting media with refractive index n_1 and n_2 respectively, as shown in the diagram, with the electric field of the wave being polarized in the plane of incidence. Give expressions for the space and time dependence of the components for the incident, reflected and transmitted **E** and **H** fields. [6]



State the appropriate boundary conditions, and obtain the following relationships between the amplitudes of the electric fields of the three waves:

$$\frac{A_2}{A_1} = \frac{\sin 2\theta_3 - \sin 2\theta_1}{\sin 2\theta_3 + \sin 2\theta_1} ,$$

$$\frac{A_3}{A_1} = \frac{4 \sin\theta_3 \cos\theta_1}{\sin 2\theta_3 + \sin 2\theta_1} .$$

[8]

Show that there is no reflected wave when the angle of incidence equals the Brewster angle, given by

$$\theta_1 = \tan^{-1}\left(\frac{n_2}{n_1}\right) .$$

Estimate the Brewster angle for air-water and water-air interfaces and comment on the results. [6]

[The refractive index of water is 1.33.]

8. A paramagnetic sphere of radius r_0 and relative magnetic permeability μ is inserted into a uniform magnetic field $\mathbf{H}$. Show that the magnetic scalar potential must satisfy the Laplace equation, and that the potentials

$$\begin{aligned}\phi_1 &= -c_1 r\cos\theta \quad (r \le r_0)\,,\\ \phi_2 &= -c_2 r\cos\theta + c_3 r^{-2}\cos\theta \quad (r \ge r_0)\,,\end{aligned}$$

give a suitable description of the magnetic field inside and outside the sphere, respectively. (The distance r is measured from the centre of the sphere, and the angle θ is measured from the direction of $\mathbf{H}$.) [6]

Determine the constants c_1, c_2 and c_3 in terms of $|\mathbf{H}|$ and μ. [8]

Show that ϕ_2 is the sum of two contributions: the potential arising from the external field and the field of a magnetic dipole. Obtain an expression for the dipole moment M_0 of the sphere. What is the effective magnetic permeability μ_{eff} of a composite material made up of N such spheres per unit volume, each separated by a distance much greater than r_0, suspended in a non-magnetic material? [6]

$-\frac{1}{r}\frac{\partial}{\partial\theta}$ $\frac{\partial\phi}{\partial r} = H_r$ $B = \mu_0(M+H)$

$\mu H_r =$ $\nabla \times$

$B = \mu_{eff} H =$ $NB = \mu_0 M + \mu_0 H$

9. A diffraction grating consists of N identical long apertures of width a separated by a distance d.

Obtain an expression for the angular distribution of light intensity diffracted from the grating. (Assume that a is much smaller than the wavelength of light, so that the single aperture term is a constant, independent of angle.) [8]

By considering that the principal maxima at wavelength $\lambda + \Delta\lambda$ occur at the same angle as the first minima for wavelength λ in order for the two wavelengths to be resolved, obtain an expression for the theoretical resolving power of the grating. [5]

Show that the range of wavelengths transmitted by a grating spectrometer (the bandpass) depends on the angular dispersion of the grating, the focal length of the collimating optics, and the width of the exit slit. Estimate the bandpass of a spectrometer which has a 100 mm wide grating with $d = (1/1800)$ mm. The spectrometer operates in first order, has 1 m focal length collimating optics, and a slit width of 100 μm. What value of slit width would yield a bandpass corresponding to the theoretical resolving power for $\lambda \approx 500$ nm? [7]

$\nabla \times E = 0$

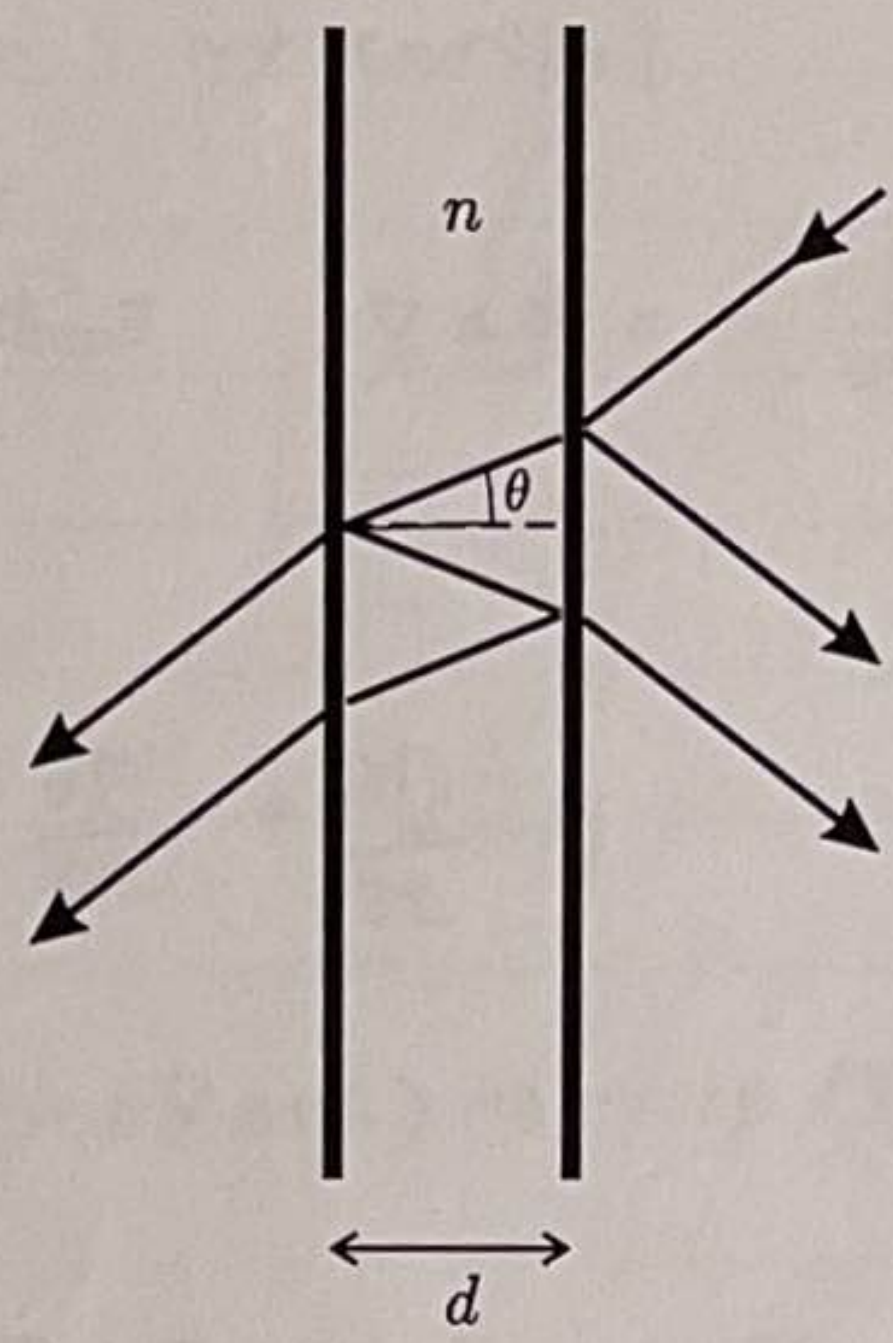
-1

$\frac{\mu-1}{\mu+2} - \frac{\mu+2}{\mu+1}$ $\int \underline{H} \cdot d\underline{r}$

$\nabla \times H = 0$

$= \frac{3}{\mu+2}$ $\int \underline{E} \cdot d\underline{r}$

10. A Fabry-Perot etalon has parallel flat surfaces with reflectivity R separated by a non-absorbing layer of thickness d and refractive index n. Light of wavelength λ_1 and intensity I_0 is incident on the etalon from the right (as shown in the diagram), making an internal angle θ with the normal to the plates, and undergoes multiple beam interference.



Show that the transmitted intensity I_t is given by an expression of the form

$$I_t = \frac{I_0}{1 + F\sin^2(\delta/2)},$$

and obtain an expression for F in terms of R. [9]

Sketch a graph of I_t/I_0 versus δ. Obtain an expression for δ, and explain its physical significance. [5]

Monochromatic light from an extended source gives rise to a series of concentric circular fringes of order m after transmission through the etalon. Obtain an expression for the width of a fringe. If the source contains a second wavelength λ_2 with equal intensity, find the order at which the fringes can be resolved. (Apply the resolution criterion that the fringe separation equals the fringe width.) [6]

$$m = \frac{2(1-R)}{\sqrt{R}} \frac{\lambda}{\delta\lambda} \frac{1}{2\pi}$$

$$\Rightarrow \frac{4}{\sqrt{F}} \frac{\lambda}{\delta\lambda} \frac{1}{2\pi} = m$$

$$F = \frac{4R}{(1-R)^2}$$

$$m = \frac{2\lambda}{\pi\,\delta\lambda\sqrt{F}}$$

$$\sqrt{F} = \frac{2\sqrt{R}}{1-R}.$$

$$\frac{1-R}{\sqrt{R}} = \frac{2}{\sqrt{F}}$$

A2 2012 First Attempt

1. $u = \cancel{\frac{1}{2}\varepsilon_0}\ \frac{1}{2}\underline{E}\cdot\underline{D} + \frac{1}{2}\underline{B}\cdot\underline{H}$

Assuming linear material $\underline{D} = \varepsilon\underline{E}$ $\underline{B} = \mu\underline{H}$

no current

$\cancel{\vec{\nabla}\times\vec{E}} = \quad \underline{\nabla}\times\underline{E} = -\frac{\partial B}{\partial t}$ $\cancel{\underline{\nabla}\times\underline{H} =}$

$\underline{\nabla}\times\underline{H} = \frac{\partial D}{\partial t}$

$\therefore \quad \frac{\partial u}{\partial t} = \underline{E}\cdot\frac{\partial D}{\partial t} + \frac{\partial B}{\partial t}\cdot\underline{H}$

$= \underline{E}\cdot(\underline{\nabla}\times\underline{H}) - \underline{H}\cdot(\underline{\nabla}\times\underline{E})$

$= -\underline{\nabla}\cdot(\underbrace{\underline{E}\times\underline{H}}_{\underline{S}}) = -\underline{\nabla}\cdot\underline{S}$

-8

divergence theorem $\rightarrow$ $\cancel{\oiint \underline{\nabla}\cdot\underline{S}\,d\tau = -\frac{\partial}{\partial t}\int}$

$\int_V \underline{\nabla}\cdot\underline{S}\,d\tau = -\frac{\partial}{\partial t}\int_V u\,d\tau$

$\rightarrow \quad \oiint_\Sigma \underline{S}\cdot d\underline{A} = -\frac{\partial U}{\partial t}$

energy ~~flu~~ flux

rate of change of energy stored in volume

$\therefore$ $\underline{S}$ is the rate of energy flow.

$\cancel{E = \frac{k \times B}{v}}$ $\quad H_0 = \frac{E}{Z}$ where $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$

$\therefore$ $\cancel{\frac{1}{2}\varepsilon E_0^2}$ $\frac{1}{2}\mu H_0^2 = \frac{1}{2}\mu \frac{E_0^2}{Z^2} = \frac{1}{2}\mu \frac{E_0^2}{\frac{\mu}{\varepsilon}} = \frac{1}{2}\varepsilon E_0^2$

$\rightarrow \quad u = \varepsilon E_0^2$

$\because$ E ~~varies~~ varies like ~~cos~~ $\cos\theta$ and the time average of $\cos^2\theta$ is $\frac{1}{2}$

$\therefore \langle u \rangle = \frac{1}{2}\varepsilon E_0^2$

the poynting vector $S = EH \quad (E \perp H)$

$\therefore \langle S \rangle = \frac{1}{2} E_0 H = \frac{1}{2} E_0^2 \frac{1}{Z} = \frac{1}{2} E_0^2 \sqrt{\frac{\varepsilon}{\mu}} = \frac{1}{2}\varepsilon E_0^2 \frac{1}{\sqrt{\varepsilon\mu}}$

$= \frac{1}{2}\varepsilon v E_0^2$

$\cancel{\rightarrow P_0\left(\frac{\pi}{4}w^2\right) = \frac{1}{2}\varepsilon v E_0^2 \rightarrow E_0 =}$

$\cancel{\therefore \quad E_0 = \left(\frac{\pi P_0 w^2}{2\varepsilon v}\right)^{\frac{1}{2}}}$

$\cancel{B_0 = \mu H_0 = \mu\sqrt{\frac{\varepsilon}{\mu}}\, E_0 = \frac{E_0}{v}}$

$\cancel{\therefore \quad B_0 = \frac{1}{v}\left(\frac{\pi P_0 w^2}{2\varepsilon v}\right)^{\frac{1}{2}}}$

$\rightarrow \quad P_0 = \frac{1}{2}(\varepsilon v E_0^2)(\frac{\pi}{4} W^2)$

$\therefore \quad E_0 = \left(\frac{8P_0}{\pi \varepsilon v W^2}\right)^{\frac{1}{2}}$

$B_0 = \frac{E_0}{v} = \frac{1}{v}\left(\frac{8P_0}{\pi \varepsilon_0 v W^2}\right)^{\frac{1}{2}}$

For $\varepsilon \simeq \varepsilon_0$ & $v \simeq c$ $\quad P = 100 \times 10^9 \, W$

$W = (\cancel{\ } 1 \times 10^{-3} m)$

$\rightarrow \quad E_0 = 9.8 \times 10^9 \, N/C$

2. Wave equation for transmission line

(-8)

$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial x^2} \qquad \frac{\partial^2 I}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 I}{\partial x^2}$

Substitute in $V = V_0 \cos(\omega t - kx)$

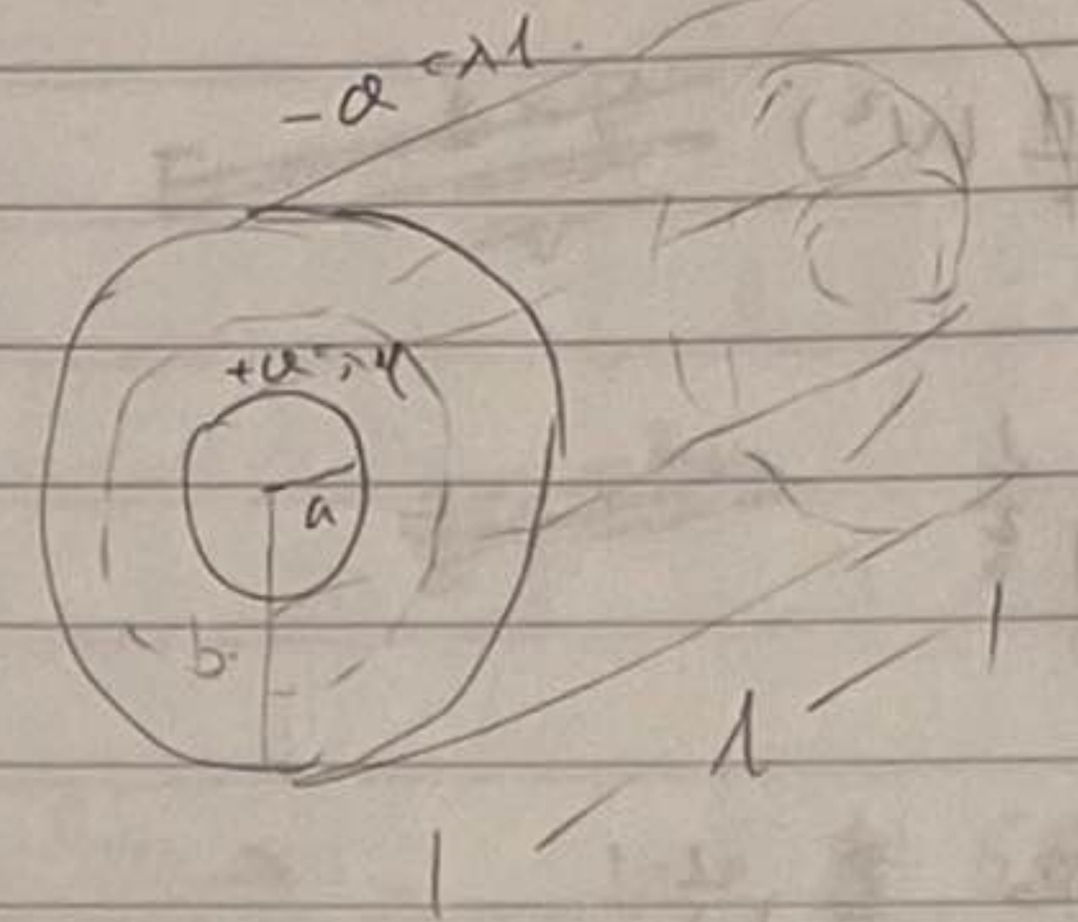
$I = I_0 \cos(\omega t - kx)$

gives $\omega^2 = \frac{1}{LC} k^2 \rightarrow \quad k = \omega\sqrt{LC}$

Also $\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$, $\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$

$-kV_0 = -\omega L I_0 \quad \therefore \quad \cancel{\omega}\sqrt{LC}\, V_0 = \cancel{\omega} L I_0$

$\rightarrow \quad Z = \frac{V_0}{I_0} = \sqrt{\frac{L}{C}}$



Gauss's Law $\oiint \underline{D}\cdot \underline{ds} = Q_f$

$(D=\varepsilon E)$

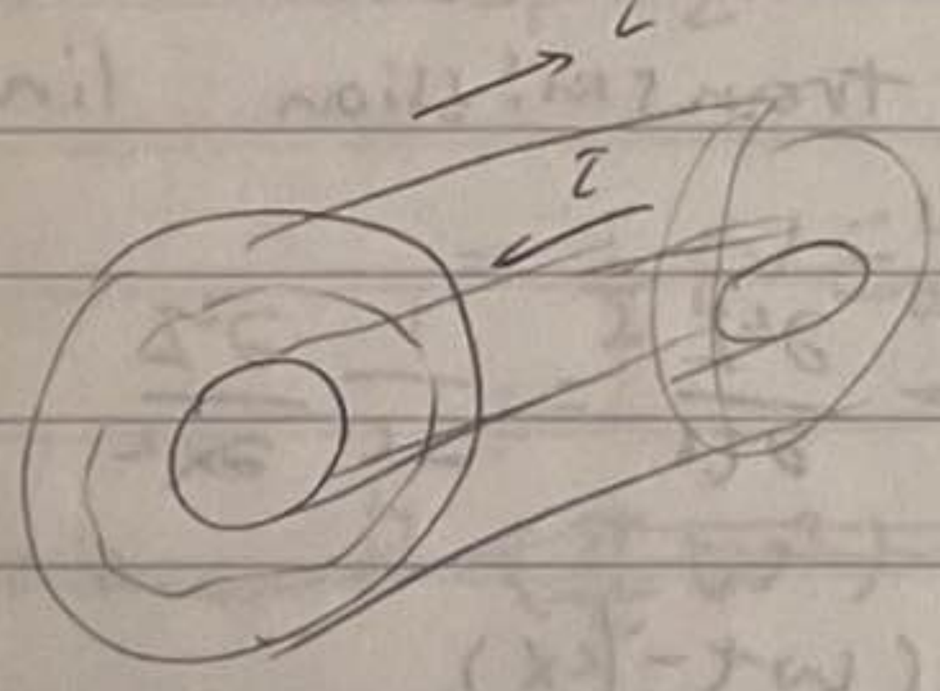
$$\cancel{\varepsilon_0}\,(2\pi r l)\,E = \frac{\lambda l}{\cancel{\varepsilon_0}\,\varepsilon_0\varepsilon_m}$$

$$\therefore\ E = \frac{\lambda}{2\pi\varepsilon_0 r\varepsilon_m}$$

$$V = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_m}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{Vl} = \frac{\lambda l}{\frac{\lambda l}{2\pi\varepsilon_0}\ln(b/a)} = \frac{2\pi\varepsilon_0\varepsilon_m}{\ln(b/a)}$$

magnetic energy density $u_B = \frac{1}{2\mu_0}B^2$



Ampere's Law

$$(2\pi r)\,B = \mu_0 I$$

$$\therefore\ B = \frac{\mu_0 I}{2\pi r}$$

magnetic energy $W_B = \int \frac{1}{2\mu_0}B^2\,d\tau$

$$= \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi}\right)^2\int_a^b \frac{1}{r^{\cancel{2}}}(2\pi \cancel{r} l)\,dr$$

$$= \frac{1}{2\cancel{\mu_0}}\,\frac{\mu_0^{\cancel{2}} I^2}{\cancel{4}\pi^{\cancel{2}}}\cdot \cancel{2}\pi l\left(\ln\frac{b}{a}\right)$$

$$= \frac{1}{2}I^2\left(\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)\right)l = \frac{1}{2}I^2 L l.$$

$\rightarrow \quad \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$

$\therefore \quad \cancel{z_c}\ z = \sqrt{\frac{L}{C}} = \left(\frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \frac{\ln(b/a)}{2\pi \varepsilon_0 \varepsilon_m} \right)^{\frac{1}{2}}$

$$= \frac{1}{2\pi} \left(\frac{\mu_0}{\cancel{2\pi}\varepsilon_0 \varepsilon_m} \right)^{\frac{1}{2}} \ln\left(\frac{b}{a}\right)$$

$\rightarrow \quad \ln\left(\frac{b}{a}\right) \frac{1}{\sqrt{\varepsilon_m}} = 2\pi z \left(\frac{\varepsilon_0}{\mu_0}\right)^{\frac{1}{2}} = 0.8339.$

a) $\varepsilon_m \simeq 2$

$b \simeq 2.5\,cm$ $\quad \frac{b}{a} = 5.3$

$a = 0.47\,cm$

3. $F(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx \qquad G(k) = \int_{-\infty}^{\infty} g(x) e^{-ikx}\, dx$

$\ast \quad h(x) = f(x) * g(x) = \int_{-\infty}^{\infty} f(x-x') g(x')\, dx'$

$H(k) = \int_{-\infty}^{\infty} h(x) e^{-ikx}\, dx \quad \ast$

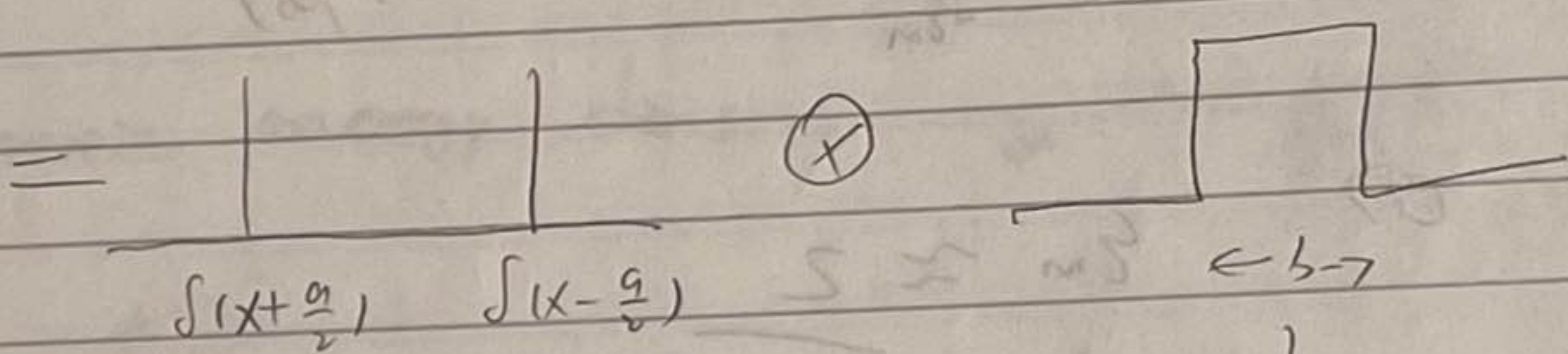
$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' f(x-x') g(x') e^{-ikx}$$

$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' f(x-x') g(x') e^{-ik(x-x')} e^{-ik\cancel{x}'}$$

$$= \int_{-\infty}^{\infty} d(x-x') f(x-x') e^{-ik(x-x')} \int_{-\infty}^{\infty} dx' g(x') e^{-ikx'}$$

$$= F(k) \times G(k)$$

Double slit

$=$ 

$F(\beta)$ $\qquad$ $G(\beta)$ $\qquad$ $\beta = k\sin\theta$

$$F(\beta) \propto \int_{-\infty}^{\infty} [\delta(x+\tfrac{a}{2}) + \delta(x-\tfrac{a}{2})] e^{i\beta x} dx$$

$$= 2 \times \frac{e^{i\beta\frac{a}{2}} + e^{-i\beta\frac{a}{2}}}{2} = \cancel{2\cos} \; 2\cos(\tfrac{1}{2}\beta a)$$

$$\neq G(\beta) \propto \int_{-\infty}^{\infty} \mathrm{rect}(b) e^{i\beta x} dx = \int_{-b/2}^{b/2} e^{i\beta x} dx$$

$$= \frac{1}{i\beta}(e^{i\beta b/2} - e^{-i\beta b/2}) = b \frac{\cancel{2i}\sin(\frac{1}{2}\beta b)}{\cancel{i}\frac{\beta d}{2}}$$

$\therefore$ ~~Ac~~ $H(\beta) = F(\beta)\, G(\beta)$

$$\propto \cos\left(\tfrac{1}{2}\beta a\right) \frac{\sin\left(\frac{\beta d}{2}\right)}{\frac{\beta d}{2}} \qquad \beta = k\sin\theta = \frac{2\pi}{\lambda}\sin\theta$$

$$\therefore \ I(\theta) = I_0 \cos^2\left(\frac{\pi a}{\lambda}\sin\theta\right) \frac{\sin^2\left(\frac{\pi b \sin\theta}{\lambda}\right)}{\left(\frac{\pi b \sin\theta}{\lambda}\right)^2}$$

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4 Highly conducting $\rightarrow \sigma$ large

Lossless $\rightarrow$ ε ~~real~~. $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

$\rightarrow \ \nabla\cdot\underline{E} = 0 \qquad \nabla\cdot\underline{B} = 0$

$\nabla\times\underline{E} = -\frac{\partial B}{\partial t} \qquad \nabla\times\underline{B} = \mu\sigma\underline{E} + \mu\varepsilon\frac{\partial \underline{E}}{\partial t}$

small $\rightarrow$ ignore

$$\therefore \ \nabla\times(\nabla\times\underline{E}) = \nabla(\underbrace{\nabla\cdot\underline{E}}_{0}) - \nabla^2\underline{E}$$

$$\nabla\times(\nabla\times\underline{E}) = -\frac{\partial}{\partial t}(\nabla\times\underline{B}) = -\frac{\partial}{\partial t}(\mu\sigma\underline{E})$$

$$\therefore \ \nabla^2\underline{E} = \mu\sigma\frac{\partial \underline{E}}{\partial t}$$

Similarly $\nabla^2\underline{B} = \mu\sigma\frac{\partial B}{\partial t}$

try $\underline{\tilde{E}} = \tilde{E}_0 e^{i(kz-\omega t)}$

$\underline{\tilde{B}} = \tilde{B}_0 e^{i(kz-\omega t)}$

we get $-k^2 = \mu\sigma(-i\omega) \rightarrow k^2 = i\mu\sigma\omega$

$\therefore$ $\tilde{k} = \cancel{\left(\frac{1\pm i}{\sqrt{2}}\right)\sqrt{\mu\sigma\omega}}$ $\tilde{k} = \pm \frac{(1+i)}{\sqrt{2}}\sqrt{\mu\sigma\omega}$

If $\tilde{k} = -\frac{1-i}{\sqrt{2}}\sqrt{\mu\sigma\omega}$ then the real exponential

term $e^{\sqrt{\frac{\mu\sigma\omega}{2}}z}$ ~~gets~~ $\to \infty$ as $z \to \infty$

$\therefore$ we take $\tilde{k} = \cancel{+i}(1+i)\sqrt{\frac{\mu\sigma\omega}{2}}$

$$\therefore \tilde{\underline{E}} = \tilde{\underline{E}}_0 \exp\left(i\left[(1+i)\sqrt{\frac{\mu\sigma\omega}{2}}z - \omega t\right]\right)$$

$$= \tilde{\underline{E}}_0 \exp\left(-\sqrt{\frac{\mu\sigma\omega}{2}}z\right)\exp\left(i\left(\sqrt{\frac{\mu\sigma\omega}{2}}z - \omega t\right)\right)$$

the amplitudes decays as $\exp(-z/\delta)$

where $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$ is the skin depth.

is the characteristic distance for wave to propagate.

$\frac{\partial B}{\partial t} = -\nabla \times \underline{E} = -i\underline{k} \times \underline{E}$ let $\tilde{\underline{k}} = \tilde{k}\hat{\underline{z}}$ and $\tilde{k} = |\tilde{k}|e^{i\phi}$

$$\to \frac{\partial \underline{B}}{\partial t} = -i|\tilde{k}|(\hat{\underline{z}} \times \tilde{\underline{E}}_0)e^{-z/\delta}\exp(i(z/\delta - \omega t))$$

$$\therefore \underline{B} = \frac{|\tilde{k}|}{\omega}(\hat{\underline{z}} \times \tilde{\underline{E}}_0)e^{-z/\delta}\exp(i(z/\delta - \omega t))$$

$\because$ $\cancel{\nabla \times \underline{E} = i\underline{k}\times}$ $\nabla \cdot \underline{E} = i\underline{k} \cdot \underline{E} = 0$ $\therefore$ $\hat{\underline{z}} \cdot \tilde{\underline{E}} = 0$

$\therefore |\hat{\underline{z}} \times \tilde{\underline{E}}_0| = |\tilde{E}_0|$

$$\therefore Z = \left|\frac{E}{H}\right| = \mu\left|\frac{E}{B}\right| = \frac{\omega}{|k|}\mu$$

$$= \frac{\omega}{\sqrt{\sigma\mu\omega}}\mu = \sqrt{\frac{\mu\omega}{\sigma}}$$

For copper assume $\mu = \mu_0 = 4\pi \times 10^{-7}$

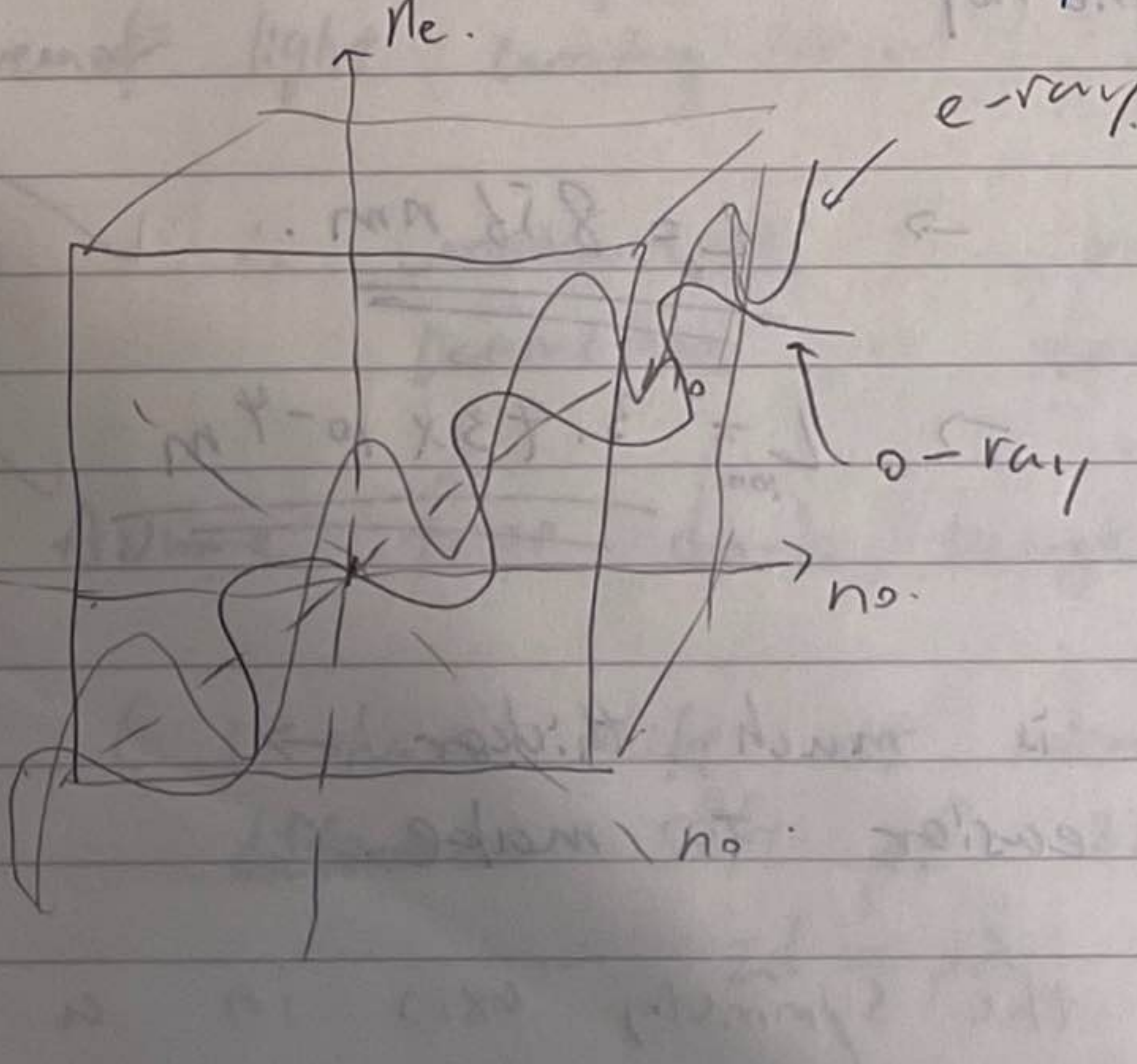
$\sigma = 6 \times 10^{7}\ \Omega^{-1} m^{-1}$ $\qquad \omega = 2\pi f = 2\pi \times 10^{6}\ Hz$

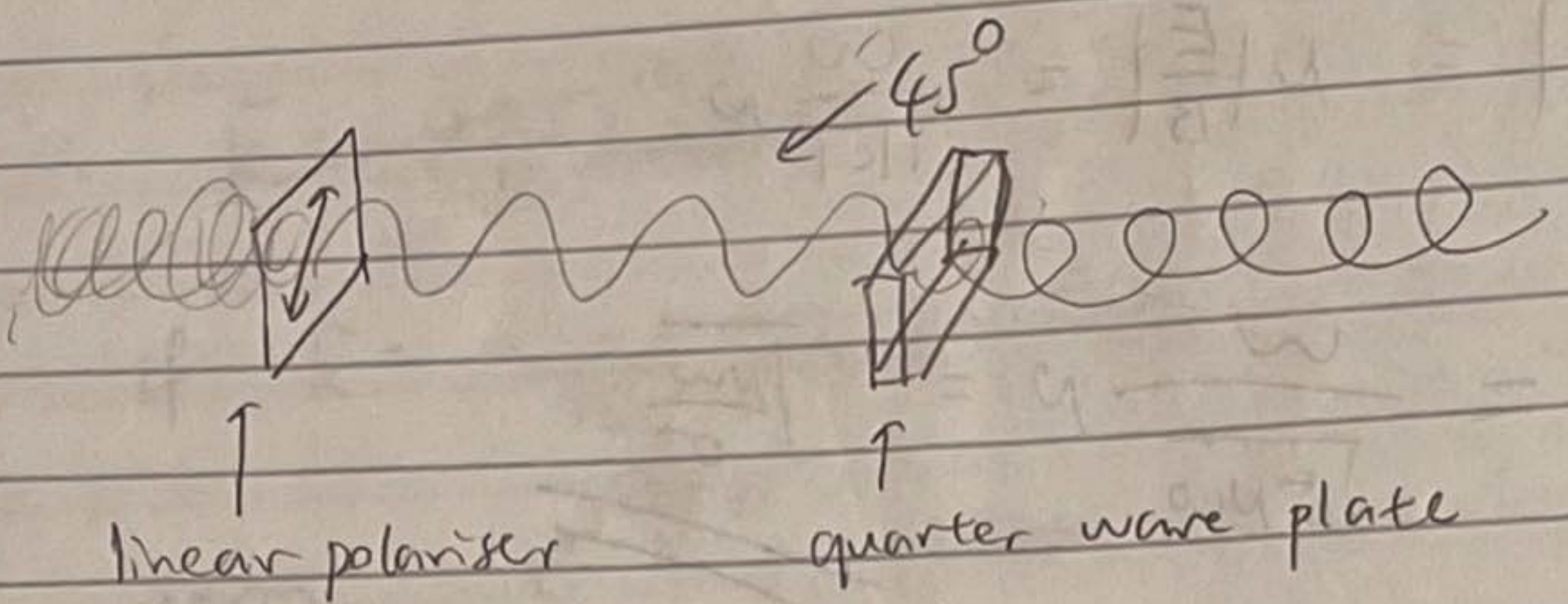
$$\therefore Z = \left(\frac{4\pi \times 10^{-7} \times 2\pi \times 10^{6}}{6 \times 10^{7}}\right)^{\frac{1}{2}} = 3.6 \times 10^{-4}\ \Omega \ \cancel{\ll 376}$$

$\ll 376\ \Omega$

$= Z_{\text{free space}}$

5. Birefringence ~~is~~ describes that the refractive index of a material is different for wave ~~....~~ polarised in the ordinary axis ~~and~~ (a plane) and in the extraordinary axis.





-4

Orient the linear polariser 45° with respect to the optical axes of ~~+~~ the quarter wave plate

→ produce circularly polarized light.

The phase difference $\Delta\phi = k\Delta L = \frac{2\pi}{\lambda} L |n_e - n_o|$

For n^{th} order quarter wave plate

$\cancel{\frac{\pi}{2}}$ $(2n + \frac{1}{2})\pi = \frac{2\pi}{\lambda_0} L |n_e - n_o|$.

$\rightarrow L = \frac{(n + \frac{1}{4})\lambda_0}{|n_e - n_o|}$

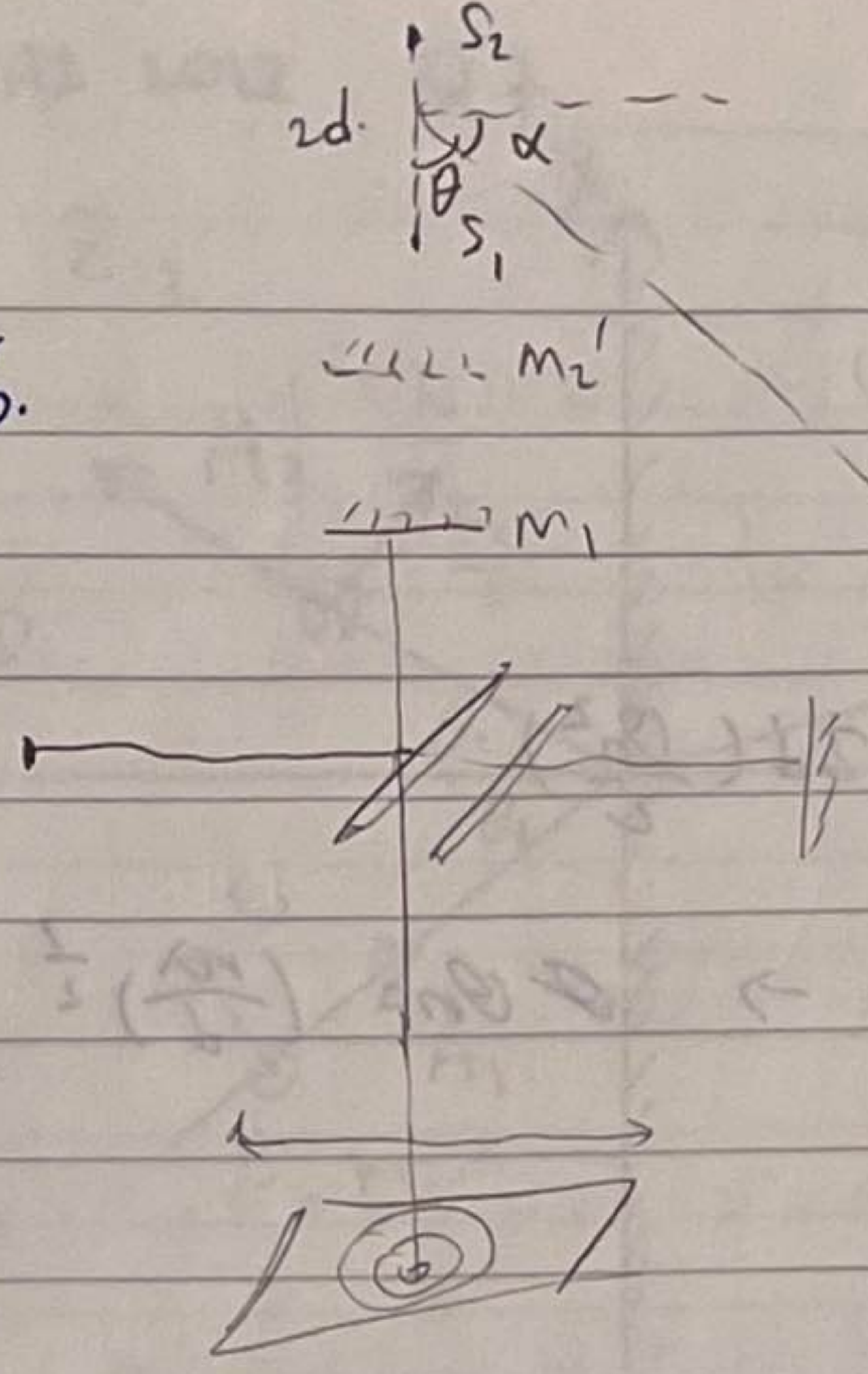
For $n = 0$ → $L_0 = 856$ nm.

For $n = 100$ → $L_{100} = 3.43 \times 10^{-4}$ m

∴ 100^{th} order is much thicker → practically easier to make.

optical axis : the symmetry axis in a uniaxial crystal.

6.



→ The images of source S formed by the ~~b~~ mirrors are S_1 and S_2

→ They act as coherent sources and produce interference patterns at the screen.

→ The configuration is axis-symmetric with respect to the central fringe, so the fringe patterns are circular.

→ Different angle means different path difference between ~~of~~ light coming from S_1 and S_2

∴ Constructive → bright fringe
Destructive → dark fringe

Assume first dark fringe is at centre

(1-phase shift at mirror gives a ~~path~~ phase ~~difference~~ shift of 2π

$\therefore \; 2d = p\lambda$ (p = order of dark fringe)

$$2d = p\lambda$$

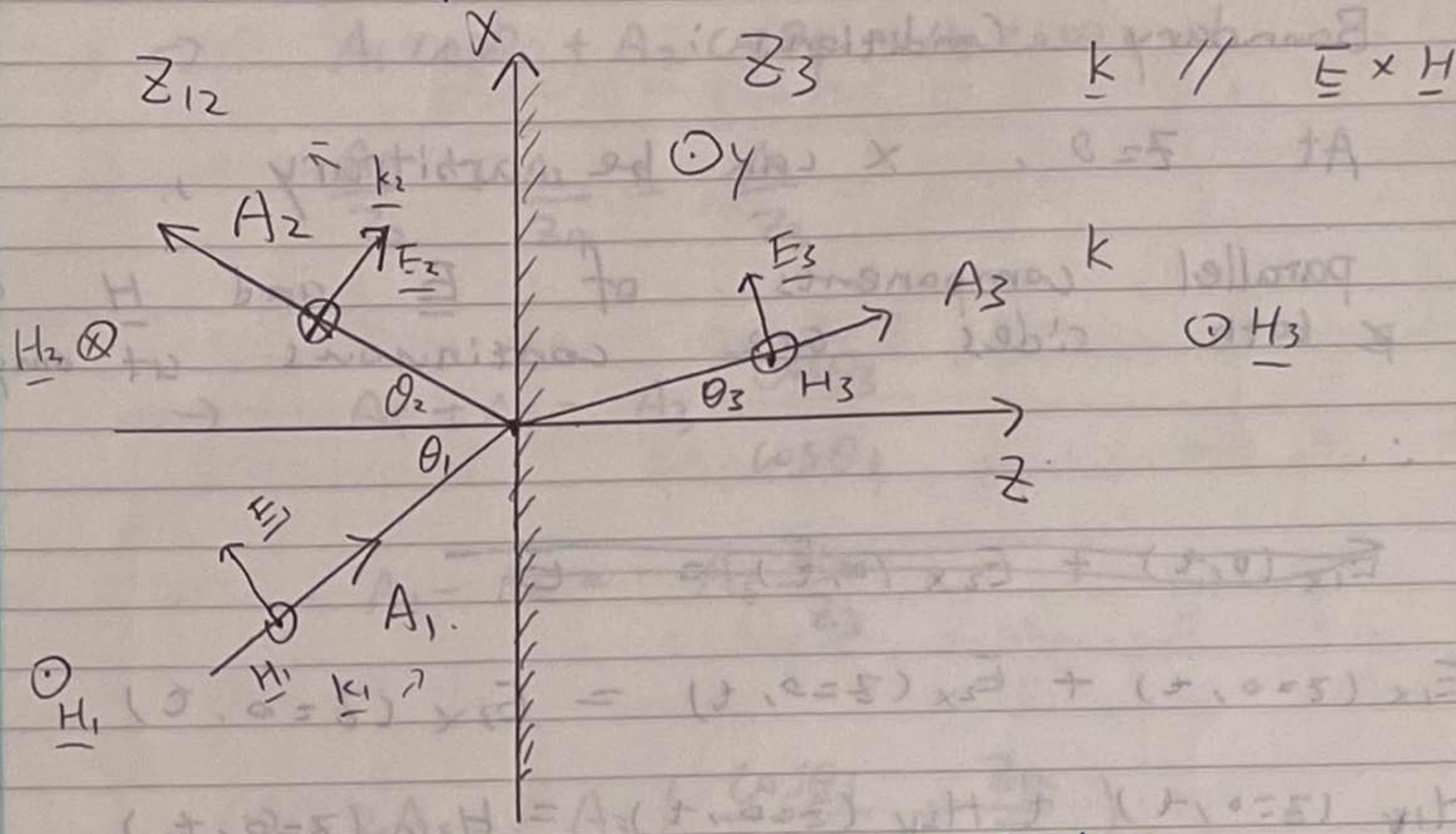
$$2d\cos\theta_n = (p-n)\lambda$$

$$2d(1-\cos\theta_n) = n\lambda = 2d\left(\frac{\theta_n^2}{2}\right)$$

$$\rightarrow \quad \theta_n^2 = \frac{n\lambda}{d} \quad \rightarrow \quad \theta_n = \left(\frac{n\lambda}{d}\right)^{\frac{1}{2}}$$

$$\varphi_n = \left(\frac{n\lambda}{d}\right)^{\frac{1}{2}}$$

A2 2012 Q7



$\cancel{E_x =}$ $\cancel{E_{1x} = A_1 \cos\theta_1 \exp(i k_1 \cdot r}$

$$E_{1x} = A_1 \cos\theta_1 \exp(i(k_1 \cos\theta_1 z + k_1 \sin\theta_1 x - \omega t))$$

$$E_{1z} = -A_1 \sin\theta_1 \exp(i(k_1 \cos\theta_1 z + k_1 \sin\theta_1 x - \omega t))$$

$\cancel{H_{1x} = 0}$

$\cancel{H_{1z} =}$

$$H_{1y} = \frac{A_1}{Z_{12}} \exp(i(k_1 \cos\theta_1 z + k_1 \sin\theta_1 x - \omega t))$$

$$E_{2x} = \cancel{A_2}\ A_2 \cos\theta_2 \exp(i(-k_2 \cos\theta_2 z + k_2 \sin\theta_2 x - \omega t))$$

$$E_{2z} = A_2 \sin\theta_2 \exp(i(-k_2 \cos\theta_2 z + k_2 \sin\theta_2 x - \omega t))$$

$$H_{2y} = -\frac{A_2}{Z_{12}} \exp(i(-k_2 \cos\theta_2 z + k_2 \sin\theta_2 x - \omega t))$$

$$E_{3x} = A_3 \cos\theta_3 \exp(i(k_3 \cos\theta_3 z + k_3 \sin\theta_3 x - \omega t))$$

$$E_{3z} = -A_3 \sin\theta_3 \exp(i(k_3 \cos\theta_3 z + k_3 \sin\theta_3 x - \omega t))$$

$$H_{3y} = \frac{A_3}{Z_3} \exp(i(k_3 \cos\theta_3 z + k_3 \sin\theta_3 x - \omega t))$$

$\cancel{Z_{12} =}$ $\because \mu = \mu_0$ $\therefore$ $Z = \mu v_\varphi = \mu_0 v_\varphi = \mu_0 c / n$

$$\therefore Z_{12} = \frac{\mu_0 c}{n_1} \qquad Z_3 = \frac{\mu_0 c}{n_2} \qquad \therefore \frac{Z_{12}}{Z_3} = \frac{n_2}{n_1}$$

Boundary Conditions :

At $z=0$, x can be arbitrary,

parallel components of $\underline{E}$ and $\underline{H}$ on both sides are continuous at any time

$\therefore$

$\cancel{E_{1x}(0,t) + E_{2x}(0,t) = E}$

$E_{1x}(z=0,t) + E_{2x}(z=0,t) = E_{3x}(z=0,t)$

$H_{1y}(z=0,t) + H_{2y}(z=0,t) = H_{3y}(z=0,t)$

dispersion relation $k^2 = \mu\varepsilon\omega^2$ $\therefore$ $k_1 = k_2$ (same side, same μ, ε)

$$\therefore A_1 \cos\theta_1 \exp(i(k_1 \sin\theta_1 x - \omega t))$$

$$+ A_2 \cos\theta_2 \exp(i(k_1 \sin\theta_2 x - \omega t))$$

$$= A_3 \cos\theta_3 \exp(i(k_3 \sin\theta_3 x - \omega t)) \quad ①$$

$$\frac{A_1}{Z_{12}} \exp(i(k_1 \sin\theta_1 x - \omega t))$$

$$- \frac{A_2}{Z_{12}} \exp(i(k_1 \sin\theta_2 x - \omega t))$$

$$= \frac{A_3}{Z_3} \exp(i(k_3 \sin\theta_3 x - \omega t)) \quad ②$$

$\because$ ①, ② true for all x

$\therefore$ $\sin\theta_1 = \sin\theta_2 \rightarrow \theta_1 = \theta_2$

$k_1 \sin\theta_1 = k_3 \sin\theta_3$

$$\therefore \frac{\sin\theta_1}{\sin\theta_3} = \frac{k_3}{k_1} = \frac{\omega\sqrt{\mu_3\varepsilon_3}}{\omega\sqrt{\mu_1\varepsilon_1}} = \frac{v_{\varphi_1}}{v_{\varphi_3}} = \frac{c/v_{\varphi_3}}{c/v_{\varphi_1}} = \frac{n_2}{n_1}$$

$$\rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_3 \rightarrow \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\rightarrow \quad A_1 \cos\theta_1 + A_2 \cos\theta_1 = A_3 \cos\theta_3$$

$$\frac{A_1}{Z_{12}} - \frac{A_2}{Z_{12}} = \frac{A_3}{Z_3}$$

$$\rightarrow \quad A_1 + A_2 = A_3 \frac{\cos\theta_3}{\cos\theta_1}$$

$$A_1 - A_2 = A_3 \frac{Z_{12}}{Z_3}$$

$$\rightarrow \quad 2A_1 = A_3 \left(\frac{\cos\theta_3}{\cos\theta_1} + \frac{Z_{12}}{Z_3} \right)$$

$$\therefore \quad \frac{A_3}{A_1} = \frac{2}{\frac{\cos\theta_3}{\cos\theta_1} + \frac{Z_{12}}{Z_3}}$$

$$= \frac{2}{\frac{\cos\theta_3}{\cos\theta_1} + \frac{n_2}{n_1}} = \frac{2}{\frac{\cos\theta_3}{\cos\theta_1} + \frac{\sin\theta_1}{\sin\theta_3}}$$

$$= \frac{4\cos\theta_1 \sin\theta_3}{2\cos\theta_3 \sin\theta_3 + 2\sin\theta_1 \cos\theta_1}$$

$$= \frac{4\sin\theta_3 \cos\theta_1}{\cancel{2\sin\theta} \sin 2\theta_3 + \sin 2\theta_1}$$

$$\frac{A_2}{A_1} = \frac{A_3}{A_1} \cdot \frac{\cos\theta_3}{\cos\theta_1} - 1 \cancel{A}$$

$$= \frac{4\sin\theta_3 \cos\theta_1 \cdot \frac{\cos\theta_3}{\cos\theta_1} - \sin 2\theta_3 - \sin 2\theta_1}{\sin 2\theta_3 + \sin 2\theta_1}$$

$$= \frac{2\sin 2\theta_3 - \sin 2\theta_3 - \sin 2\theta_1}{\sin 2\theta_3 + \sin 2\theta_1}$$

$$= \frac{\sin 2\theta_3 - \sin 2\theta_1}{\sin 2\theta_3 + \sin 2\theta_1}$$

At Browster angle :

$$\sin 2\theta_3 = \sin 2\theta_1$$

$$\therefore \quad \sin\theta_3 \cos\theta_3 = \sin\theta_1 \cos\theta_1$$

$$\because \quad \sin\theta_3\, n_2 = \sin\theta_1\, n_1$$

$$\therefore \quad \frac{\cos\theta_3}{n_2} = \frac{\cos\theta_1}{n_1}$$

$$\cos\theta_3 = \sqrt{1-\sin^2\theta_3} = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1}$$

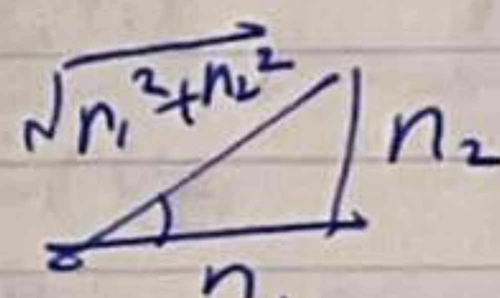
$$\therefore \quad \frac{n_1}{n_2}\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1} = \sqrt{1-\sin^2\theta_1}$$

$$\therefore \quad \frac{n_1^2}{n_2^2}\left(1-\frac{n_1^2}{n_2^2}\sin^2\theta_1\right) = 1-\sin^2\theta_1$$

$$\therefore \quad \frac{1-\frac{n_1^2}{n_2^2}}{1-\frac{n_1^4}{n_2^4}} = \sin^2\theta_1$$

$$\rightarrow \sin^2\theta_1 = \cancel{\frac{(1+\frac{n_1}{n_2})(1-\frac{n_1}{n_2})}{}} \quad \frac{1}{1+\frac{n_1^2}{n_2^2}} = \frac{n_2^2}{n_1^2+n_2^2}$$

$$\sin\theta_1 = \frac{n_2}{\sqrt{n_1^2+n_2^2}}$$



$$\rightarrow \tan\theta_1 = \frac{n_2}{n_1} \rightarrow \underline{\theta_1 = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

air - water : ~~tan~~ $\theta_b = \tan^{-1}(1.33) = \underline{53.06^\circ}$

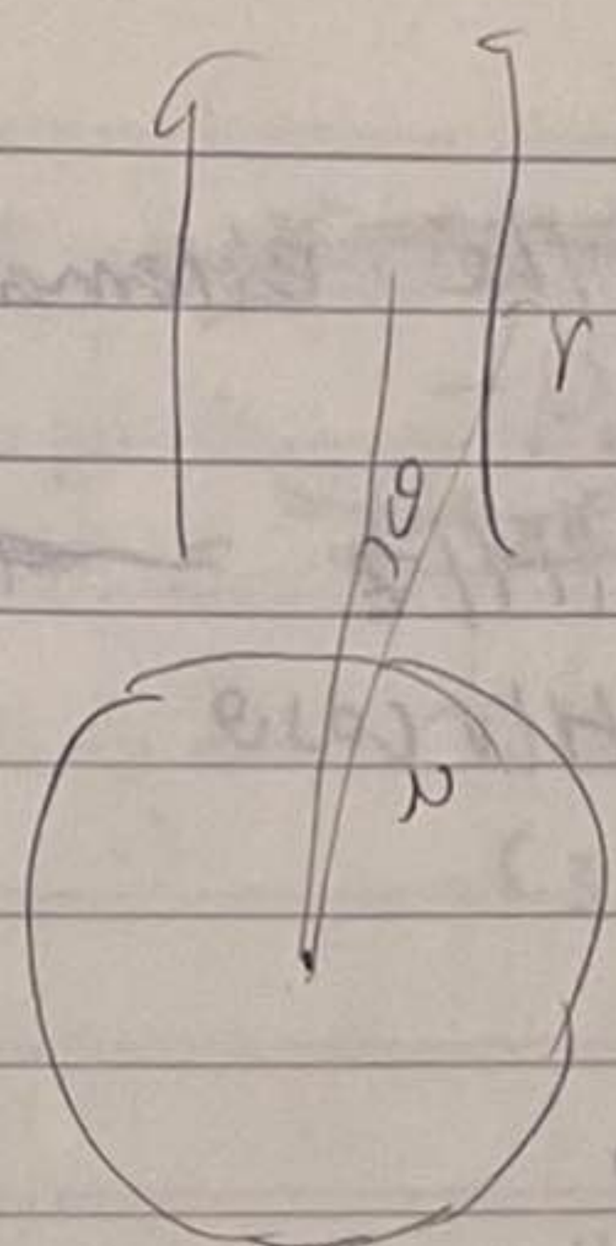
water - air : $\theta_b = \tan^{-1}\left(\frac{1}{1.33}\right) = \underline{36.94^\circ}$

→ The Brewster angles going forwards and ~~back wa~~ backwards between ~~two ma~~ two media are complementary (sum to 90°)

→ going to denser material, brewster angle $> 45^\circ$

going to less dense material, brewster angle $< 45^\circ$

8.



No free current , No varying electric field

$$\underline{\nabla} \times \underline{H} = J_f + \frac{\partial D}{\partial t} = 0 + 0 = 0$$

$\because \underline{\nabla} \cdot \underline{B} = 0$ inside sphere $\underline{B} = \mu \underline{H}$

outside sphere $\underline{B} = \mu_0 \underline{H}$

Both cases $\underline{\nabla} \cdot \underline{H} = 0$

$\underline{\nabla} \times \underline{H} = 0 \rightarrow \exists\ H = -\underline{\nabla}\phi$

$\underline{\nabla} \cdot \underline{H} = 0 \rightarrow \underline{\underline{\nabla^2 \phi = 0}}$

satisfies Laplace's equation.

System has azimuthal symmetry.

$\rightarrow$ General solution

$$\phi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

($P_l(x)$ = l^{th} Legendre ~~poly~~ Polynomial).

indeed ϕ_1 and ϕ_2 given in this question are of this form.

If we can show ϕ_1 ϕ_2 satisfy all the Boundary conditions then by uniqueness theorem they are "the" solutions.

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At $r \to \infty$ $\underline{H} \to |H|\hat{\underline{z}}$ = the external field.

$\therefore$ $\because \underline{H} = -\underline{\nabla}\phi$ $\therefore$ $\phi = -|H|z$ ~~= |H|zc~~

$= -|H| r\cos\theta$

As $r \to \infty$ $\phi_2 \to -C_2 r\cos\theta$

$\to$ $\underline{C_2 = |H|}$

~~ϕ is continuous at the Boundary because no free current any where~~

~~$\therefore -C_1 r_0 = \frac{C_3}{r_0^2} - |H| r_0$~~

~~$\therefore C_3 = (|H| - C_1) r_0^3$~~

$\to$ ϕ is continuous at boundary because no free current. any where $\underline{H} = -\underline{\nabla}\phi$ always hold.

$\therefore$ $-C_1 r_0 = \frac{C_3}{r_0^2} - |H| r_0$

$\to$ $C_3 = (|H| - C_1) r_0^3$

$\to$ radial component of B is continuous

$\therefore$ $\mu_r \frac{\partial \phi_1}{\partial r}\Big|_{r=r_0} = \frac{\partial \phi_2}{\partial r}\Big|_{r=r_0}.$

$\rightarrow$ $\cancel{\mu_r C_1 r_0}$

$$-\mu_r C_1 = -|H| - 2C_3 r_0^{-3}$$

$\cancel{= \mu_r C_1 r \cos\theta}$

$$\therefore \quad C_3 = \frac{1}{2}\left(-|H| + \mu_r C_1\right) r_0^3$$

$$\therefore \quad |H| - C_1 = \frac{1}{2}\mu_r C_1 - \frac{1}{2}|H|$$

$$\therefore \quad \cancel{\frac{3}{2}C_1} \quad \frac{3}{2}|H| = \frac{\mu_r+2}{2}C_1$$

$$\rightarrow \quad C_1 = \frac{3|H|}{\mu_r+2}$$

$$C_3 = (|H| - C_1) r_0^3 = \left(\frac{\mu_r+2}{\mu_r+2} - \frac{3}{\mu_r+2}\right)|H| r_0^3$$

$$= \frac{\mu_r-1}{\mu_r+2}|H| r_0^3$$

$$\therefore \quad \phi = \begin{cases} \phi_1 = -\dfrac{3|H|}{\mu_r+2} r\cos\theta & (r<r_0) \\ \phi_2 = -|H| r\cos\theta + \dfrac{\mu_r-1}{\mu_r+2}|H|\dfrac{r_0^3}{r^2} & (r>r_0) \end{cases}$$

In ϕ_2, $-|H| r\cos\theta$ is the potential of external field

$\frac{\mu_r-1}{\mu_r+2}|H|\frac{r_0^3}{r^2}$ is the dipole potential.

because it is $\propto \frac{1}{r^2}$

$$\underline{M}_0 = \frac{\underline{B}}{\mu_0} - \underline{H} = \cancel{(\frac{1}{\mu})} \left(\frac{\mu}{\mu_0} - 1\right)\underline{H} \quad \text{for } (r < r_0)$$

$$\rightarrow \quad \underline{M}_0 = \left(\frac{\mu}{\mu_0} - 1\right)\left(\nabla\left(C_1 z\right)\right)$$

$$= \left(\frac{\mu}{\mu_0} - 1\right) C_1 \hat{\underline{z}}$$

$$= \left(\frac{\mu}{\mu_0} - 1\right)\left(\frac{3|H|}{\mu_r + 2}\right)\hat{\underline{z}}$$

$$= \frac{3(\mu_r - 1)}{(\mu_r + 2)} |H| \hat{\underline{z}}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

~~Total dipole moment.~~

dipole moment $\underline{m}_0 = \underline{M}_0 V$

$$= \underline{M}_0 \left(\frac{4}{3}\pi r_0^3\right) = \frac{4\pi(\mu_r - 1)}{\mu_r + 2} |H| r_0^3 \hat{\underline{z}}$$

Total dipole moment ~~is~~ per volume

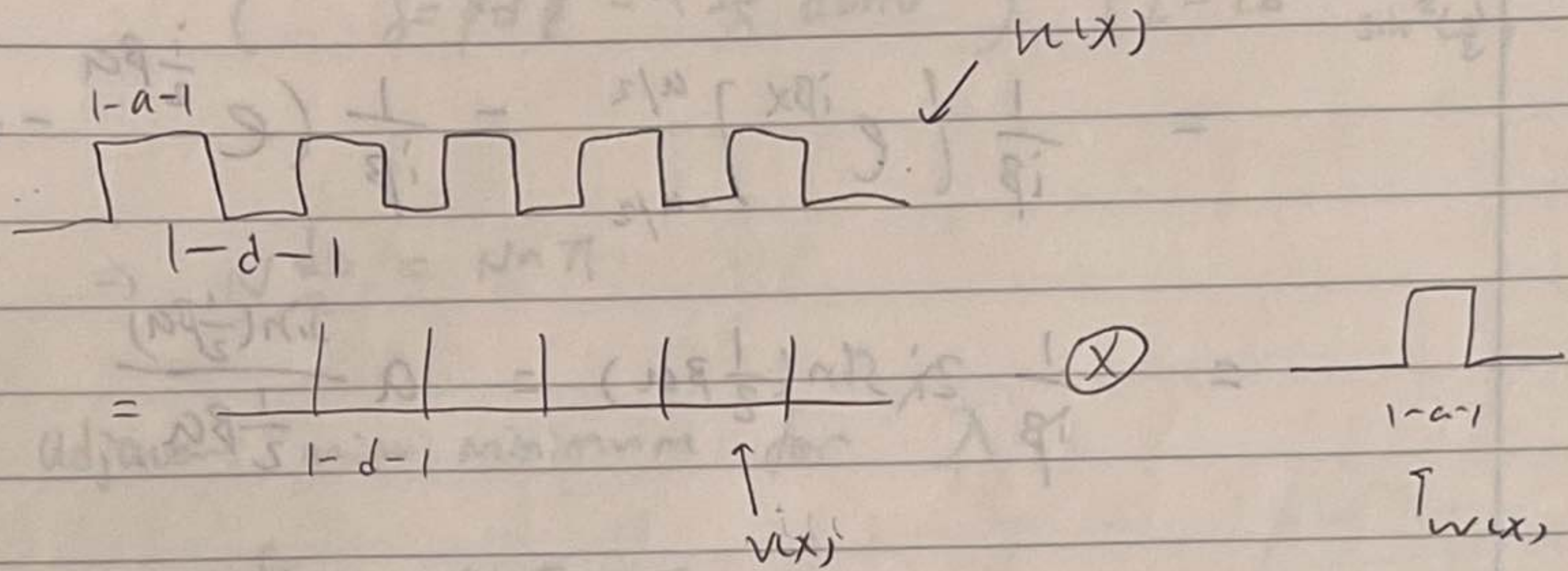
$$\underline{M}' = \underline{m}_0 N$$

$$\underline{B} = \mu_0(\underline{M} + \underline{H}) \rightarrow \mu\underline{H} = \mu_0\underline{M} + \mu_0\underline{H} \quad \therefore$$

$$\therefore \quad \mu_{eff} = \frac{\mu_0(|M| + |H|)}{|H|} = \mu_0\left(\frac{|M|}{|H|} + 1\right)$$

$$= \mu_0\left(1 + \frac{4\pi(\mu_r - 1)}{\mu_r + 2} r_0^3 N\right)$$

9. N finite slit is the convolution of N delta peaks and a top hat function



Convolution theorem If $u(x) = v(x) \otimes w(x)$

then their Fourier transforms

$$\tilde{u}(\beta) = \tilde{V}(\beta) \times \tilde{w}(\beta)$$

$$\rightarrow \quad \tilde{V}(\beta) = \int_{-\infty}^{\infty} \sum_{m=0}^{N-1} \delta(x-md)\, e^{i\beta x}\, dx$$

$$= \sum_{m=1}^{\infty} e^{i\beta m d} = 1 + e^{i\beta d} + e^{2i\beta d} + \dots$$

$$= \frac{1-e^{Ni\beta d}}{1-e^{i\beta d}} = \frac{e^{\frac{N}{2}i\beta d}}{e^{\frac{1}{2}i\beta d}} \frac{e^{\frac{N}{2}i\beta d} - e^{-\frac{N}{2}i\beta d}}{e^{\frac{1}{2}i\beta d} - e^{-\frac{1}{2}i\beta d}}$$

$$= e^{\frac{N-1}{2}i\beta d} \frac{2i}{2i} \frac{\sin(\frac{N}{2}\beta d)}{\sin(\frac{1}{2}\beta d)}$$

$$= e^{\frac{N-1}{2}i\beta d} \frac{\sin(\frac{N}{2}\beta d)}{\sin(\frac{1}{2}\beta d)}$$

$$\tilde{w}(\beta) = \int_{-\infty}^{\infty} w(x) e^{i\beta x} dx = \int_{-a/2}^{a/2} e^{i\beta x} dx$$

$$= \frac{1}{i\beta}\left[e^{i\beta x} \right]_{-a/2}^{a/2} = \frac{1}{i\beta}\left(e^{\frac{i}{2}\beta a} - e^{-\frac{i}{2}\beta a} \right)$$

$$= \frac{1}{i\beta} 2i \sin\left(\frac{1}{2}\beta a\right) = a \frac{\sin(\frac{1}{2}\beta a)}{\frac{1}{2}\beta a}$$

$$\therefore \quad \tilde{u}(\beta) = \tilde{v}(\beta)\tilde{w}(\beta) \propto \frac{\sin(\frac{1}{2}N\beta d)}{\sin(\frac{1}{2}\beta d)} \frac{\sin(\frac{1}{2}\beta a)}{\frac{1}{2}\beta a}$$

$\therefore$ ~~Intensity $I(\beta) =$~~ $\beta = k\sin\theta = \frac{2\pi}{\lambda}\sin\theta$

$$\therefore \quad I(\theta) \propto |u(\beta)|^2$$

$$\rightarrow \quad I(\theta) = I_0 \left(\frac{\sin(\frac{N\pi d}{\lambda}\sin\theta)}{\sin(\frac{\pi d}{\lambda}\sin\theta)} \right)^2 \left(\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi a}{\lambda}\sin\theta} \right)^3$$

If $a \ll \lambda$, then ignore the envelope

$$I(\theta) = I_0 \left(\frac{\sin(\frac{N\pi d}{\lambda}\sin\theta)}{\sin(\frac{\pi d}{\lambda}\sin\theta)} \right)^2$$

principle maximum: $\cancel{\text{let}}$ $\frac{\delta_\lambda}{2} = n\pi$
for λ

$$\left(\delta = \beta d = \frac{2\pi}{\lambda} d\sin\theta \right). \quad \left(I = I_0 \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \right)$$

$$\rightarrow N\frac{\delta_\lambda}{2} = Nn\pi$$

adjacent ~~major~~ minimum for λ:

$$N\frac{\delta_\lambda}{2} = Nn\pi + \pi$$

If this coincides with the nth principle maximum

$$\rightarrow \frac{\delta_{\lambda+\Delta\lambda}}{2} = n\pi$$

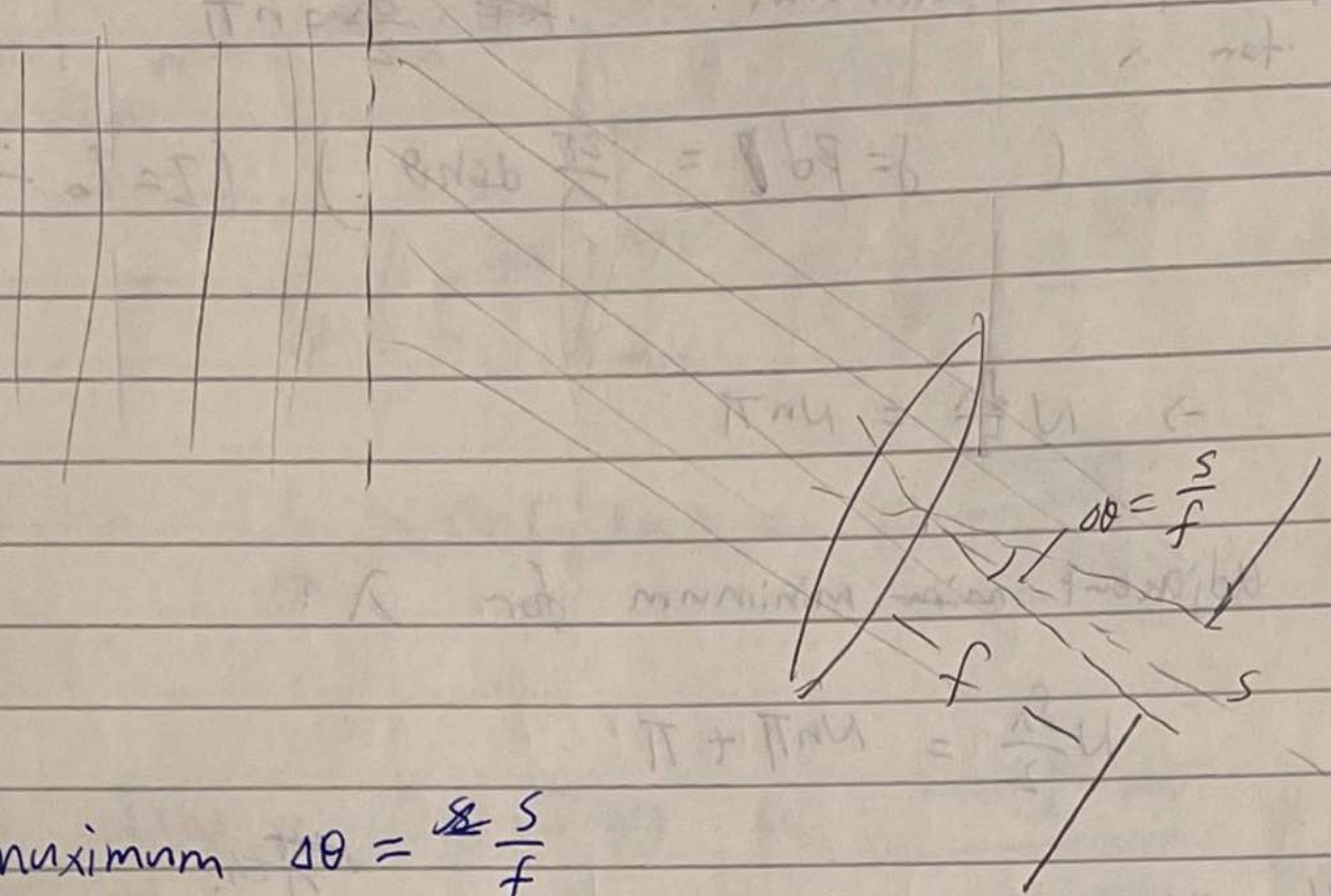
$$\therefore N\frac{\cancel{\pi}}{\lambda} d\sin\theta = Nn\cancel{\pi} + \cancel{\pi}\cdot 1$$

$$\frac{\cancel{\pi}}{\lambda+\Delta\lambda} d\sin\theta = n\cancel{\pi}$$

$$\therefore N d\sin\theta = \cancel{Nn\lambda} + \lambda = \cancel{Nn}(\lambda+\Delta\lambda)$$

$$\rightarrow \lambda = Nn\Delta\lambda$$

$$\rightarrow \frac{\lambda}{\Delta\lambda} = Nn = \text{Resolving power.}$$



maximum $\Delta\theta = \cancel{\delta}\frac{s}{f}$

Angular dispersion : $\cancel{d\theta}$ $\frac{d\theta}{d\lambda} \approx \frac{\delta\theta}{\delta\lambda}$

For a ~~max~~ principle maximum

$$d\sin\theta = n\lambda \quad \therefore \quad d\cos\theta \frac{\delta\theta}{\delta\lambda} = n$$

$$\rightarrow \frac{\delta\theta}{\delta\lambda} = \frac{n}{d\cos\theta} \rightarrow \cancel{\frac{\delta\lambda}{\delta\theta} = n\cos\theta}$$

$$\cancel{\Delta\lambda = \frac{\delta\theta}{\delta\lambda}\Delta\theta = \frac{n}{d\cos\theta}} \qquad \frac{\delta\lambda}{\delta\theta} = \frac{d\cos\theta}{n}$$

$$\rightarrow \Delta\lambda = \cancel{\delta\theta}\frac{\delta\lambda}{\delta\theta}\Delta\theta = \frac{sd\cos\theta}{nf}$$

Assuming work in first order $n=1$

Assuming ~~cos~~ small $\theta \rightarrow \cos\theta \approx 1$

$$\therefore \Delta\lambda \approx \frac{sd}{f} = \frac{(100\times10^{-6}\,m)(\frac{1}{1800}\times10^{-3}\,m)}{(1\,m)}$$

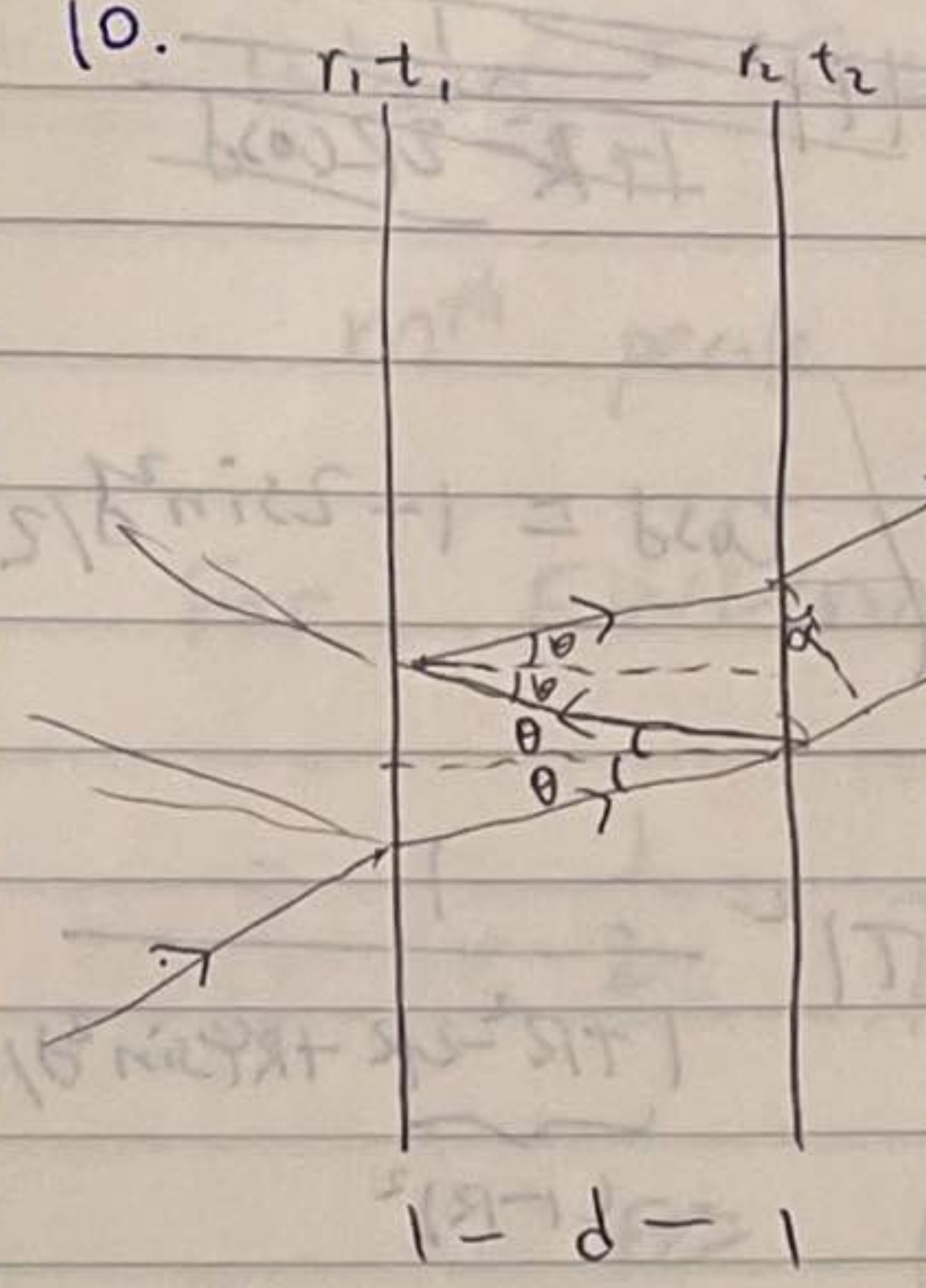
$$= 5.6\times10^{-11}\,m$$

10. Band path of theoretical resolving power

$$\frac{\lambda}{\delta\lambda} = nN \rightarrow \qquad \Delta\lambda = \frac{\lambda}{nN}$$

$$\rightarrow \quad \frac{\lambda}{nN} = \frac{sd\cos\theta}{nf}$$

$$S = \frac{\lambda f}{\underbrace{Nd}_{\sim w}\underbrace{\cos\theta}_{\approx 1}} \approx \frac{\lambda f}{w} = \underline{\underline{5 \times 10^{-6}\,m}}$$

10. 

$\sin\alpha = n\sin\theta$

phase difference : δ

$$\delta = k\frac{2nd}{\cos\theta} - k2d\tan\theta\sin\alpha$$

$$= 2ndk\left(\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}\right)$$

$$= 2knd\cos\theta = \frac{4\pi}{\lambda}nd\cos\theta$$

Multiple beam interference :

$$u(\delta) = \cancel{+} \; t_1t_2 + t_1r_2r_1t_2e^{i\delta} + t_1r_2r_1r_2r_1t_2e^{2i\delta}$$

$$+ \dots$$

$$= t_1t_2\left(1 + r_1r_2e^{i\delta} + (r_1r_2e^{i\delta})^2 + \dots\right)$$

$$= T\frac{1}{1-Re^{i\delta}}$$

$T = t_1t_2$

$R = r_1r_2$

$\therefore$ intensity $I(\delta) \propto |u(\delta)|^2$

$$= |T|^2\frac{1}{|1-Re^{i\delta}|^2} = \cancel{|T|^2\frac{1}{|(1-R\cos\delta) - R\sin\delta|}}$$

$$= |T|^2\frac{1}{|(1-R\cos\delta) - iR\sin\delta|^2}$$

$$= |T|^2 \frac{1}{(1-R\cos\delta)^2 + \sin^2\delta\, R^2} \; \cancel{= |T|^2 \frac{1}{1+R^2-2R\cos\delta}}$$

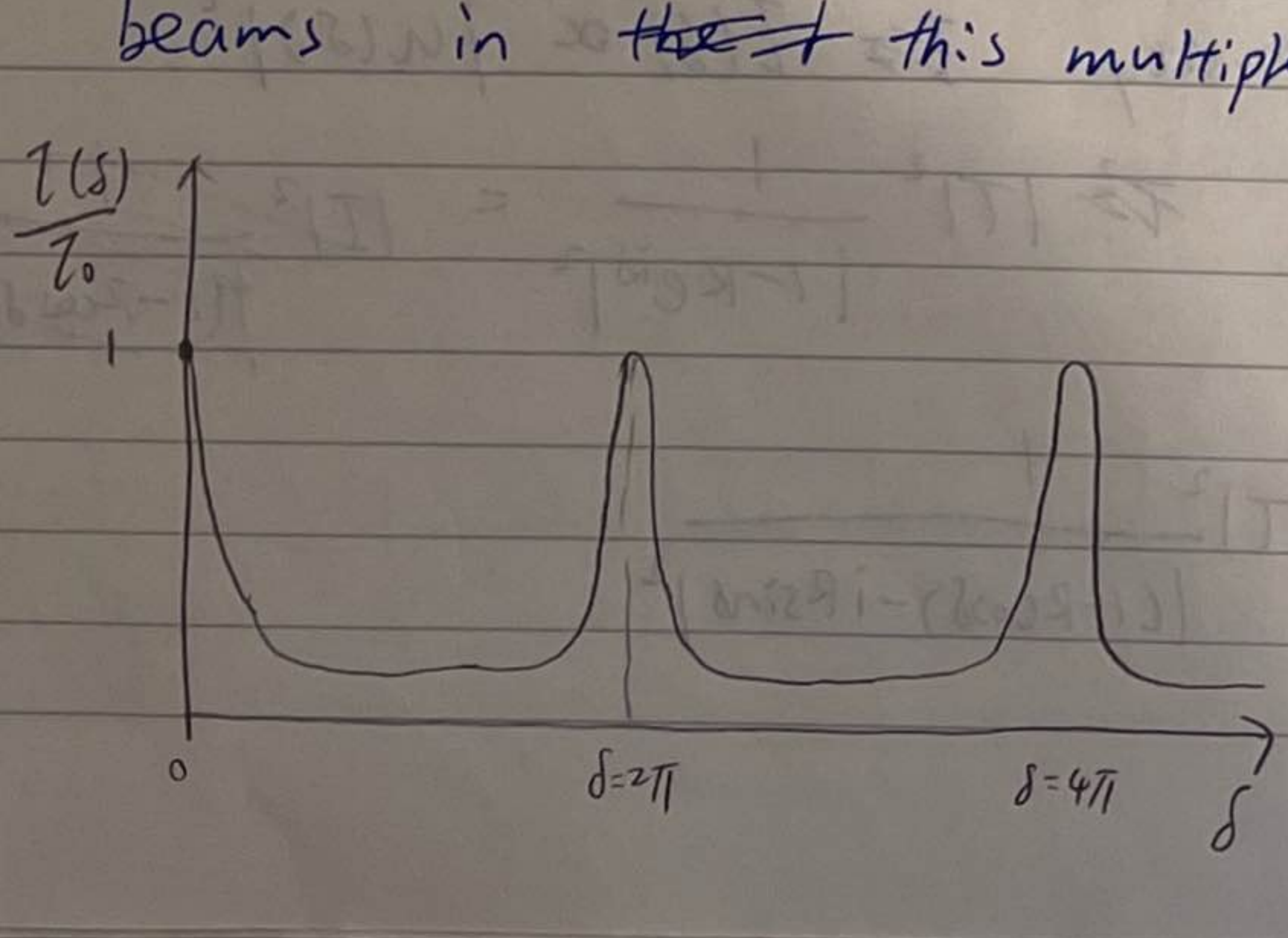
$$= |T|^2 \frac{1}{1+R^2-2R\cos\delta} \qquad \Big/ \; \cos\delta = 1-2\sin^2\delta/2$$

$$= |T|^2 \frac{1}{1+R^2-2R(1-2\sin^2\delta)} = |T|^2 \frac{1}{\underbrace{1+R^2-2R}_{(1-R)^2} + R4\sin^2\delta/2}$$

$$= \left|\frac{T}{1-R}\right|^2 \frac{1}{1+\underbrace{\frac{4R}{(1-R)^2}}_{F}\sin^2\delta/2} \propto \frac{1}{1+F\sin^2\delta/2}$$

$$\rightarrow I(\delta) = \frac{I_0}{1+F\sin^2(\delta/2)}$$

$\delta = \frac{4\pi}{\lambda} nd\cos\theta$ as derived before, it is the phase difference between successive beams in ~~the~~ this multiple beam interference



Widen = Full widen half maximum = FWHM

m^{th} peak $\delta/2 = m\pi \rightarrow \delta_m = 2m\pi$

For FWHM let $\delta = \cancel{2\pi}\, 2m\pi + \Delta\delta$.

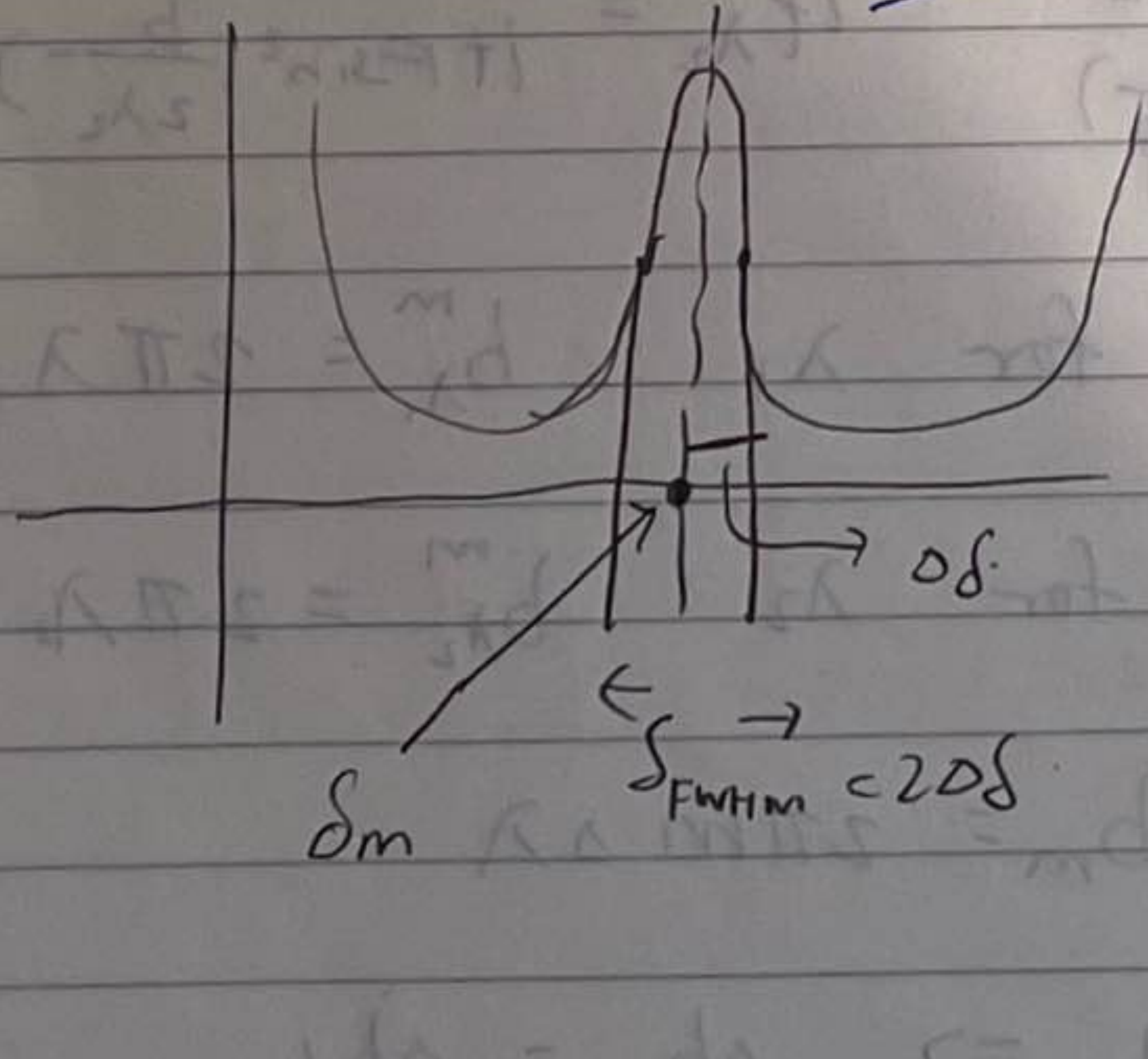
$$\rightarrow \frac{1}{2} = \frac{1}{1+F\sin^2(m\pi + \frac{\Delta\delta}{2})}$$

$$\therefore \cancel{F} \quad F\sin^2(m\pi + \frac{\Delta\delta}{2}) = 1$$

$$\therefore \left(\frac{\Delta\delta}{2}\right)^2 F = 1 \quad \text{for small } \Delta\delta$$

$$\rightarrow \Delta\delta^2 = \frac{4}{F} \rightarrow \Delta\delta = \frac{2}{\sqrt{F}}$$

$$\delta_{FWHM} = 2\Delta\delta = \frac{4}{\sqrt{F}}$$



~~Fringe separation between λ_1, λ_2 for the m^{th} order.~~

$$\cancel{\Delta\delta_m = \delta_{m,\lambda_1} - \delta_{m,\lambda_2}}$$

$$\cancel{= 2\pi}$$

$\because \quad \delta = \frac{4\pi n}{\lambda} d\cos\theta \quad$ let $\quad b = 4\pi n d\cos\theta$

$$\rightarrow \quad \delta = \frac{b}{\lambda}$$

FWHM $\quad \Delta\delta_{\frac{1}{2}} = \frac{4}{\sqrt{F}} \quad \rightarrow \quad \Delta b_{\frac{1}{2}} = \frac{4\lambda}{\sqrt{F}}$

Fringe separation between λ_1 and λ_2 in the unit of b for the m^{th} order :

$$I_{t_{\lambda_1}} = \frac{I_0}{1 + F\sin^2\left(\frac{b}{2\lambda_1}\right)} \qquad I_{t\,\lambda_2} = \frac{I_0}{1 + F\sin^2\left(\frac{b}{2\lambda_2}\right)}$$

$\therefore$ m^{th} order for λ_1 : $b^m_{\lambda_1} = 2\pi\lambda_1 m$

m^{th} order for λ_2 : $b^m_{\lambda_2} = 2\pi\lambda_2 m$

separation $\quad \Delta b_m = 2\pi m \Delta\lambda$

Just resolved $\Rightarrow \quad \Delta b_m = \Delta b_{\frac{1}{2}}$

(fringe separation = fringe width)

$\rightarrow \quad \frac{4\lambda}{\sqrt{F}} = 2\pi m \Delta\lambda$

$\therefore \quad m = \frac{2\lambda}{\pi\sqrt{F}\Delta\lambda}$ (λ can be λ_1, λ_2 or $\frac{\lambda_1+\lambda_2}{2}$)