XPHC 2662 / XPHA 2662

SECOND PUBLIC EXAMINATION

Honour School of Physics Part A: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part A

A2: Electromagnetism and Optics

Thursday, 19 June 2003, 9.30 am – 12.30pm

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a fresh page.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight which the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Write down Maxwell's equations. Allowing for free surface charge density, derive the boundary conditions for **E** and **D** fields at a plane boundary between two homogeneous, isotropic, linear media. [6]

2. Estimate the radiation pressure at a distance 1 m from a 100 W light bulb. [6]

3. Viewed through a thin layer of cloud, the Moon sometimes appears to be covered by a translucent disk of light of angular diameter 2°. The disk is white at its centre but reddish at its periphery. Explain this phenomenon and estimate the size of water droplets in the cloud. [7]

4. A collimated beam from a white-light source is incident normally on a transmission grating with 500 lines per mm. The transmitted light then passes through a lens which is used to project the visible (380–780 nm) spectrum of the light source on to a strip of photographic film of length 35 mm. Calculate the focal length of the lens. [6]

5. Sketch carefully the transmitted flux density distribution for a Fabry–Perot interferometer as a function of the optical path-length difference between adjacent rays for two values of finesse $\mathcal{F}$:

(a) $\mathcal{F} = 30$;

(b) $\mathcal{F} = 1$. [7]

6. An elliptically polarized beam of light propagating along the z-axis has **E**-field components E_x and E_y which have equal amplitude E_0 and differ in phase by an angle ϕ. The endpoint of **E** traces out an ellipse given by

$$E_x^2 + E_y^2 - 2E_xE_y\cos\phi = E_0^2\sin^2\phi.$$

Use a method involving matrix diagonalization to find the angle this ellipse makes with the (E_x, E_y)-coordinate system. [8]

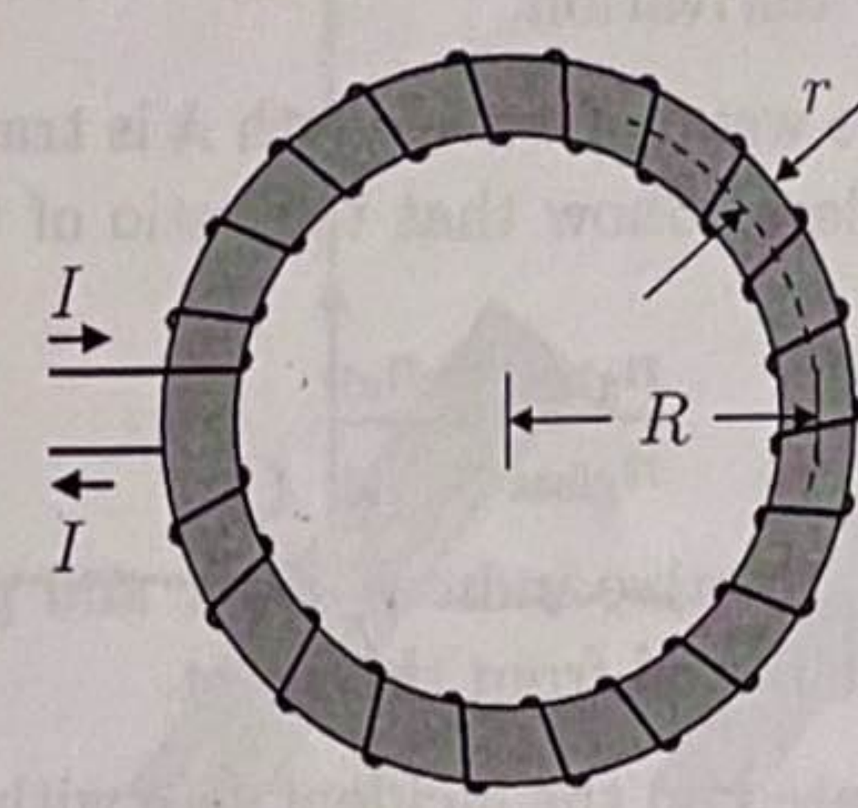
$\underline{H} = \frac{B}{\mu} - M.$ $\quad M \sim \chi \underline{H}$

$B = \mu H + M.$

$= 1 + \mu$

Section B

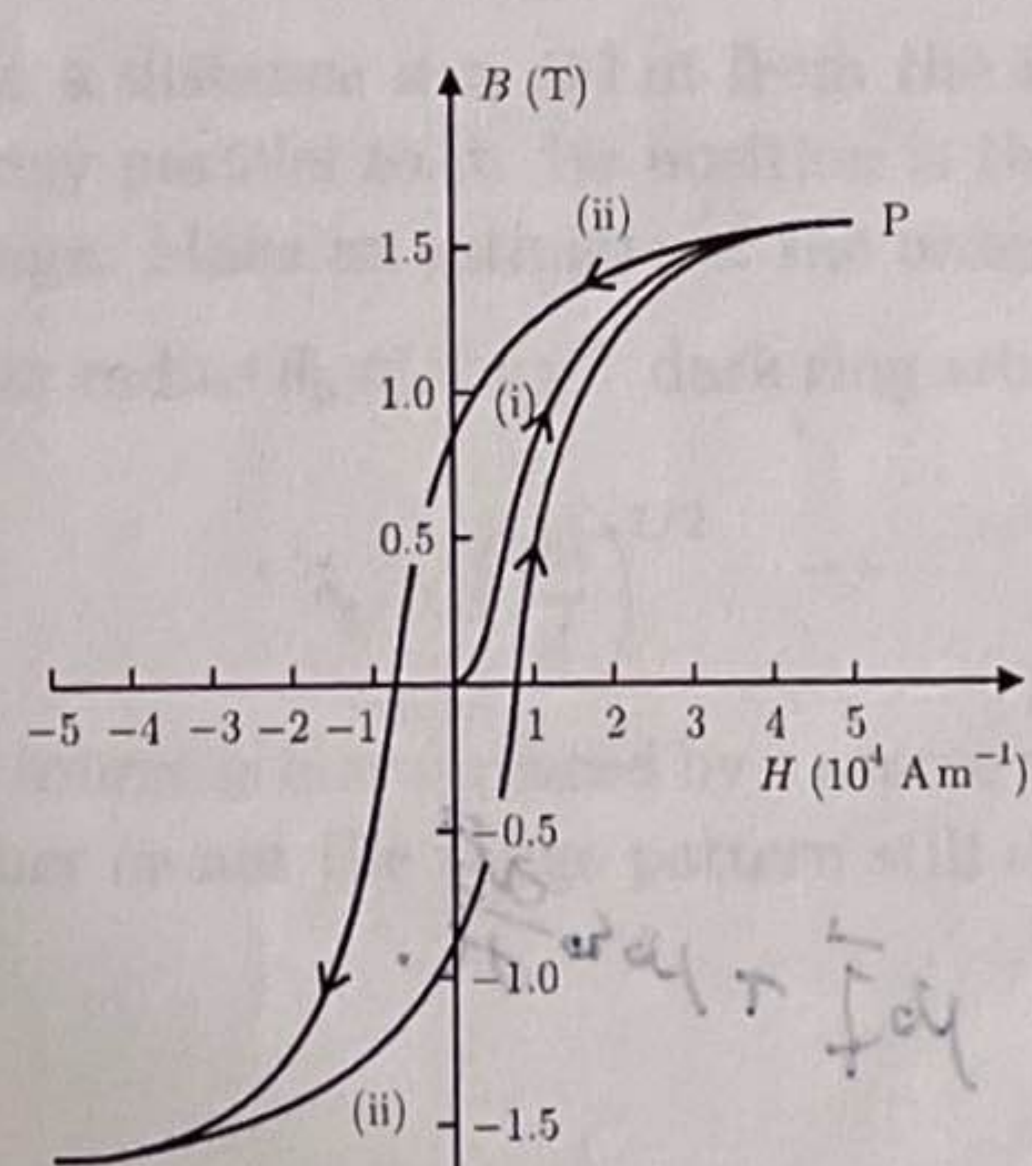
7. What is meant by *magnetic permeability* and *magnetic susceptibility*? [4]



The figure shows a coil of copper wire wrapped around a torus of material. The copper wire makes N turns and carries a current I. The torus has a major radius R and a minor radius r, where $r \ll R$. Assume that the magnetic field is entirely within the material.

(a) The material has a small, constant, positive magnetic susceptibility χ. Obtain expressions for the H and B fields. [5]

(b) The material is ferromagnetic but owing to heat treatment it is initially unmagnetized.



The graph shows the experimentally measured relationship between H and B as the material is first magnetized along curve (i) to point P, and then taken around the hysteresis loop (ii). Given that $R = 0{\cdot}2$ m and $r = 0{\cdot}02$ m estimate the energy needed to magnetize the material and the energy needed to take the material around the hysteresis loop. [7]

(c) The material is ferromagnetic and has been prepared for use in a permanent magnet. Sketch the hysteresis loop for such a material. What is the asymptotic value of B/H at high H? [4]

8. Consider the propagation of an electromagnetic wave in a homogeneous, isotropic, linear, non-conducting, non-magnetic medium of relative permittivity ϵ_r. Give expressions for the velocity of propagation and for the refractive index of the medium in terms of ϵ_r and outline their derivation. [4]

A linearly polarized plane wave of wavelength λ is travelling in air and is incident normally on a plane sheet of glass. Show that the ratio of the reflected to the incident electric field amplitude is

$$\frac{n_{\text{glass}} - n_{\text{air}}}{n_{\text{glass}} + n_{\text{air}}},$$

where n_{air} and n_{glass} are the refractive indices of air and glass respectively. Estimate what fraction of the power is reflected from the sheet. [6]

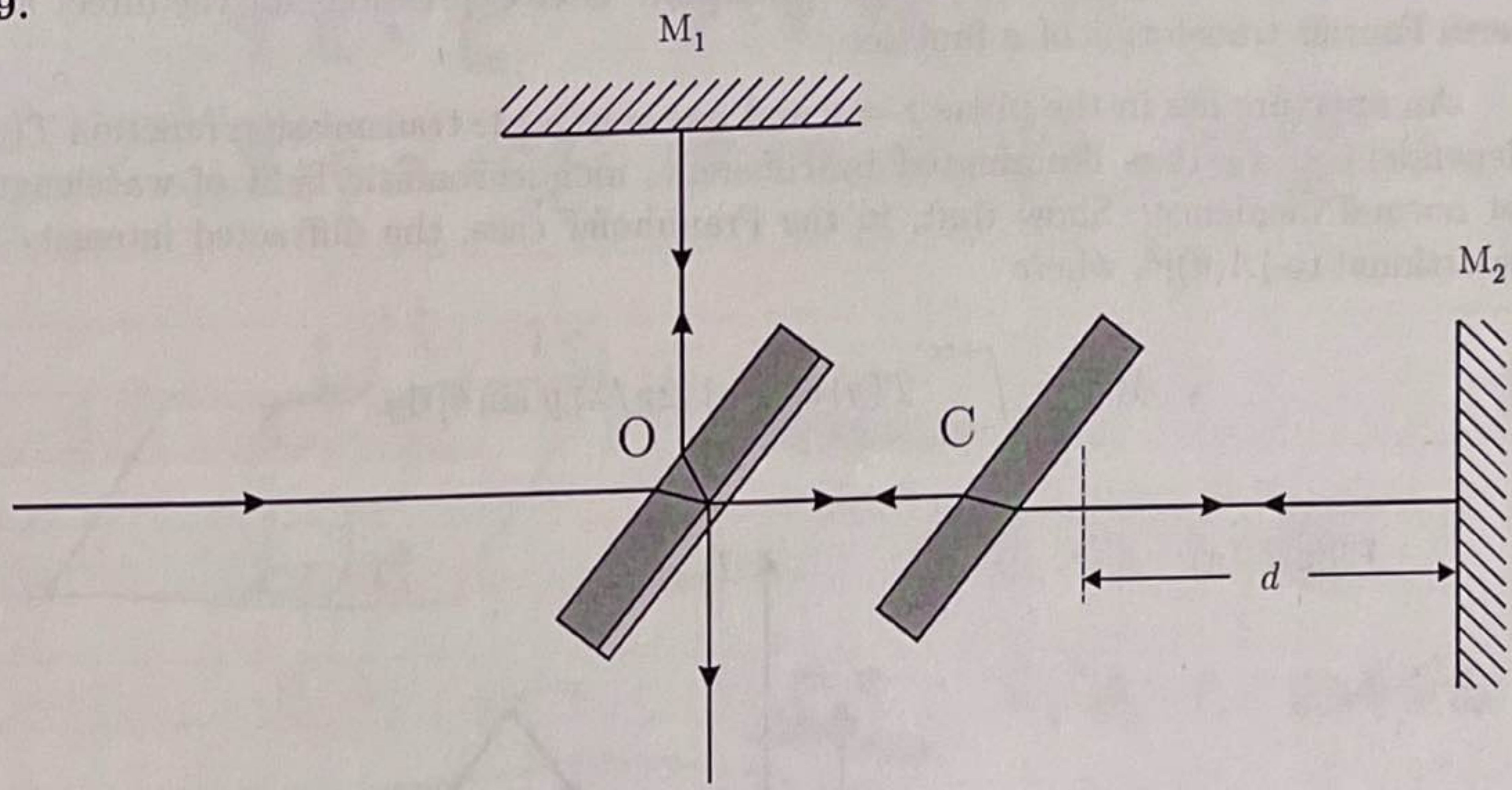
The glass sheet is now coated on the incident side with a material with a refractive index $n_{\text{coat}} = (n_{\text{air}} n_{\text{glass}})^{1/2}$ and a thickness $d = \lambda/4$. Prove that there is now, in principle, no reflected power. Give one reason why, in practice, some reflected power is inevitable. [5]

Simple materials with $n_{\text{coat}} < 1{\cdot}35$ are not readily available. Anti-reflection coatings often employ porous materials. Explain why very low values of power reflection can be achieved with such coatings. [5]

[Assume $n_{\text{air}} = 1{\cdot}0$ and $n_{\text{glass}} = 1{\cdot}5$.]

$\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$.

9.



The figure shows the elements of a Michelson interferometer: O, a half-silvered glass plate; C, a glass plate; M_1, a fixed mirror; and M_2, an adjustable mirror which can be moved towards or away from O. Also shown are paths of rays in a collimated beam from a coherent, monochromatic light source of wavelength $\lambda = 500\,\text{nm}$. Explain the functions of elements O and C. Explain why interference fringes are formed and describe how they might be observed. [6]

M_2 is initially set at a distance $d \simeq 0{\cdot}1\,\text{m}$ from the image (in the half-silvered mirror) of M_1 and accurately parallel to it. Its position is then finely adjusted so as to produce a central dark fringe. Make an estimate of the order of this fringe. [3]

Show that the angular radius θ_p of the p^{th} dark ring around the central dark fringe is given by

$$\theta_p \simeq \left(\frac{p\lambda}{d}\right)^{1/2} .$$ [6]

The monochromatic source is now replaced by a source with a range of wavelengths $\Delta\lambda = 1\,\text{nm}$. Discuss whether or not the fringe pattern still exists. [5]

10. What is meant by the *Fraunhofer condition*? Give expressions for the direct and inverse Fourier transforms of a function. [4]

An aperture lies in the plane $z = 0$ and has amplitude transmission function $T(y)$ independent of x. It is illuminated by coherent, monochromatic light of wavelength λ at normal incidence. Show that, in the Fraunhofer case, the diffracted intensity is proportional to $|A(\theta)|^2$, where

$$A(\theta) = \int_{-\infty}^{+\infty} T(y) \exp[-\mathrm{i}\,(2\pi/\lambda)\, y \,\sin\theta]\, \mathrm{d}y.$$ [5]

Figure (a)

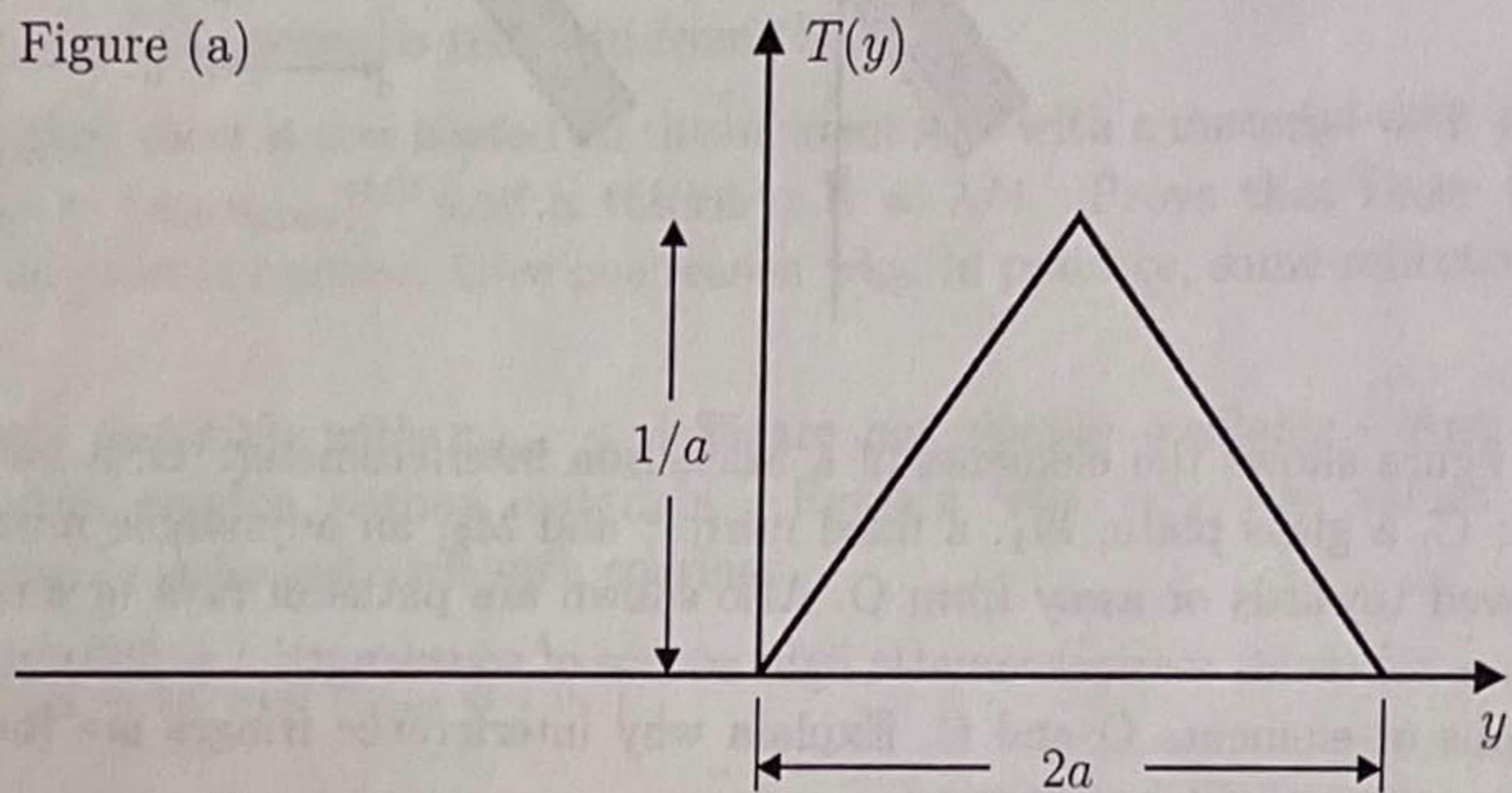


Figure (a) shows a particular transmission function. Calculate and sketch the diffracted intensity as a function of $\sin\theta$. [6]

Figure (b)

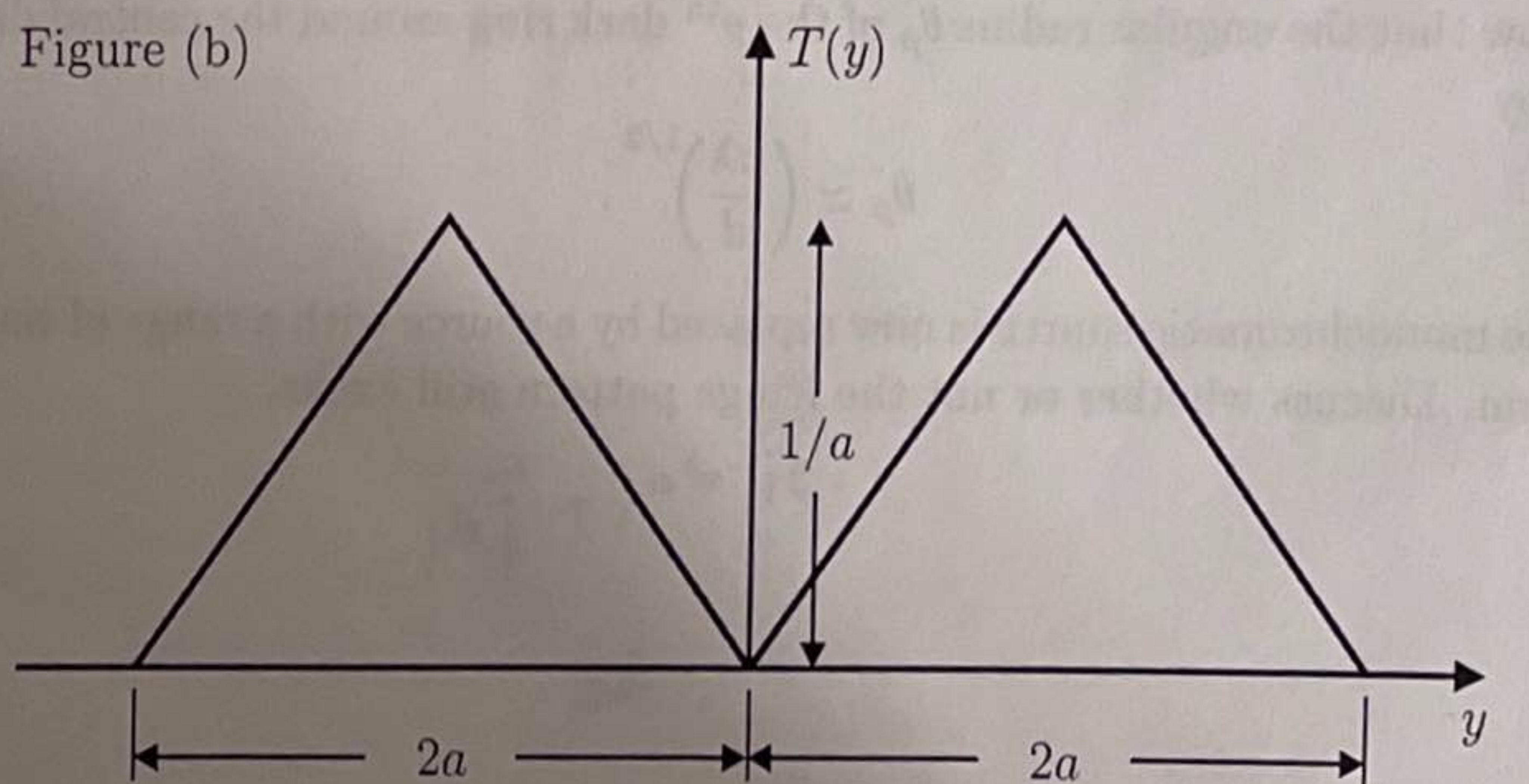


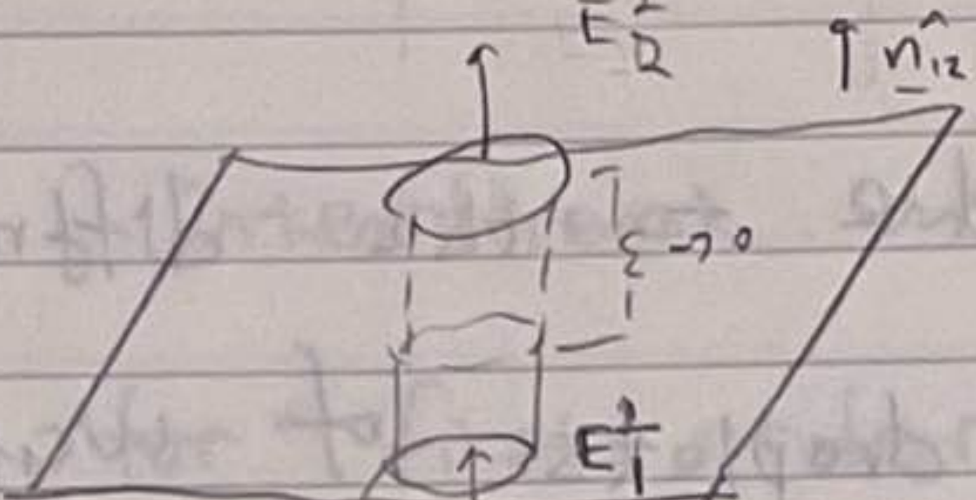
Figure (b) shows another transmission function. Calculate and sketch the diffracted intensity as a function of $\sin\theta$. Comment on the location of the first zero of the diffracted intensity with regard to the spatial-frequency content of $T(y)$. [5]

A2 June 2003 First Attempt

1. Maxwell's equations:

$$\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t} \qquad \nabla \times \underline{B} = \mu_0 \underline{j} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

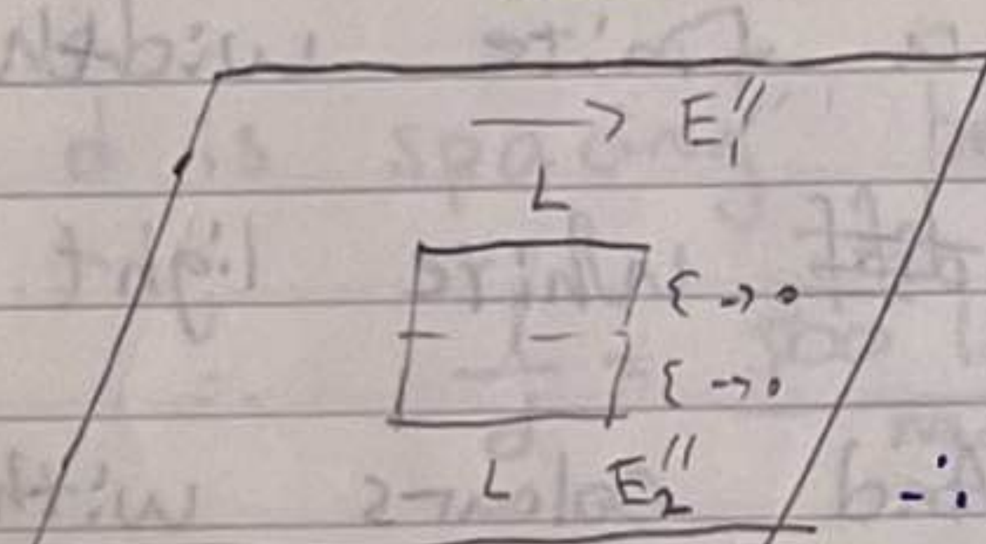


Gauss's Law:

$$\oiint \underline{E} \cdot d\underline{s} = \frac{Q}{\varepsilon_0}$$

$$\rightarrow E_2^{\perp} A - E_1^{\perp} A = \sigma A / \varepsilon_0$$

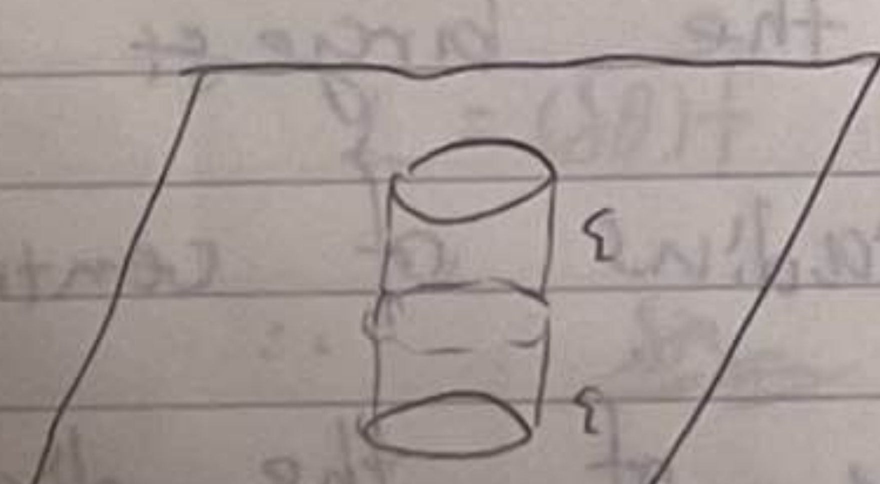
$$\therefore \boxed{\underline{E_2^{\perp}} - \underline{E_1^{\perp}} = \frac{\sigma}{\varepsilon_0} \hat{n}_{12}}$$



$$\oint \underline{E} \cdot d\underline{l} = 0$$

$$\therefore E_1^{\parallel} L - E_2^{\parallel} L = 0$$

$$\rightarrow \boxed{\underline{E_1^{\parallel}} = \underline{E_2^{\parallel}}}$$



$$\oiint \underline{D} \cdot d\underline{S} = \sigma_{free} = \sigma_f$$

$$\therefore D_2^{\perp} A - D_1^{\perp} A = \sigma_f$$

$$\rightarrow \boxed{\underline{D_2^{\perp}} - \underline{D_1^{\perp}} = \sigma_f \hat{\underline{n}}_{12}}$$

$$\because \underline{E_1^{\parallel}} = \underline{E_2^{\parallel}} \quad \text{and} \quad \cancel{\underline{D} = \varepsilon_0 \underline{E}} \quad \underline{D} = \varepsilon_0 \underline{E} + \underline{P}$$

$$\therefore \boxed{\underline{D_2^{\parallel}} - \underline{D_1^{\parallel}} = \underline{P_2^{\parallel}} - \underline{P_1^{\parallel}}}$$

2. power P and intensity I is related by

$$I = \frac{P}{4\pi r^2} = \frac{100}{4\pi (1)^2} = 7.69\ W/m^2$$

radiation pressure $P_{rad} = \frac{I}{c} = \frac{7.96}{3\times10^8} = 2.65\times10^{-8}$ pa

3. This phenomenon is due to the diffraction of of moonlight by small droplets of water in the atmosphere. Because of diffraction the image of moon cannot be completely well ~~resolve~~ resolved. There must be a finite width of the central maximum. Also ~~diff~~ white light consists of light of different colours. And colours with different wavelengths ~~has~~ has different width of central maximum. Red light have the largest wavelength and thus the largest radius of central maximum. So the periphery of the disk is red.

$\because$ Angular diameter $= 2^\circ$

$\therefore$ angular resolution

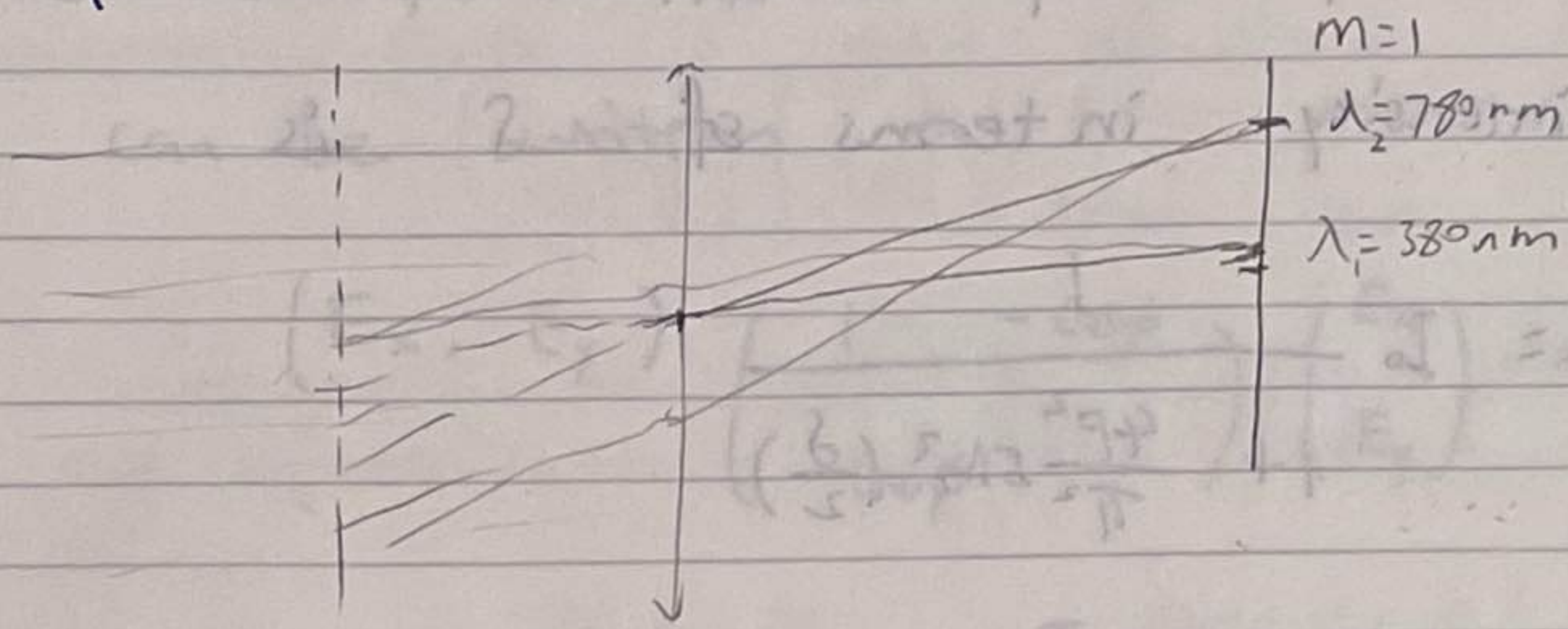
$$\delta\theta = \frac{2^\circ}{2} = 1^\circ = \frac{1}{180}\times\pi \text{ rad}$$

$$\rightarrow \frac{\pi}{180} = 1.22\frac{\lambda}{D}$$

let $\lambda = 700$ nm for red light. $D = \frac{1.22\times700\text{nm}}{\pi}\times180$

$$\rightarrow D = \boxed{4.89\times10^{-5}\text{ m}}$$

4.



Interference maximum : $d\sin\theta = m\lambda$

for first order $m=1$, $d\sin\theta = \lambda$

$\therefore \quad d\cos\theta\,\delta\theta = \delta\lambda$

$\rightarrow \quad \delta\theta = \dfrac{\delta\lambda}{d\cos\theta}$ for small θ, $\cos\theta \simeq 1$

$\therefore \quad \delta\theta \simeq (\delta\lambda)(\frac{1}{d})$

d is spacing between slits in the grating

$\therefore \quad \dfrac{1}{d} = \dfrac{500 \text{ lines}}{mm}$

the length of spectrum projected onto the screen is

$L = (\delta\theta) f$

$\therefore \quad \delta\lambda(\frac{1}{d}) f = L$

$$\therefore \quad f = \frac{L}{\delta\lambda(\frac{1}{d})} = \frac{35\,mm}{(780-380)\,nm \times (\frac{500}{mm})}$$

$$= \frac{35 \times 10^{-3}}{(780-380)\times 10^{-9} \times 500 \times 10^{3}}$$

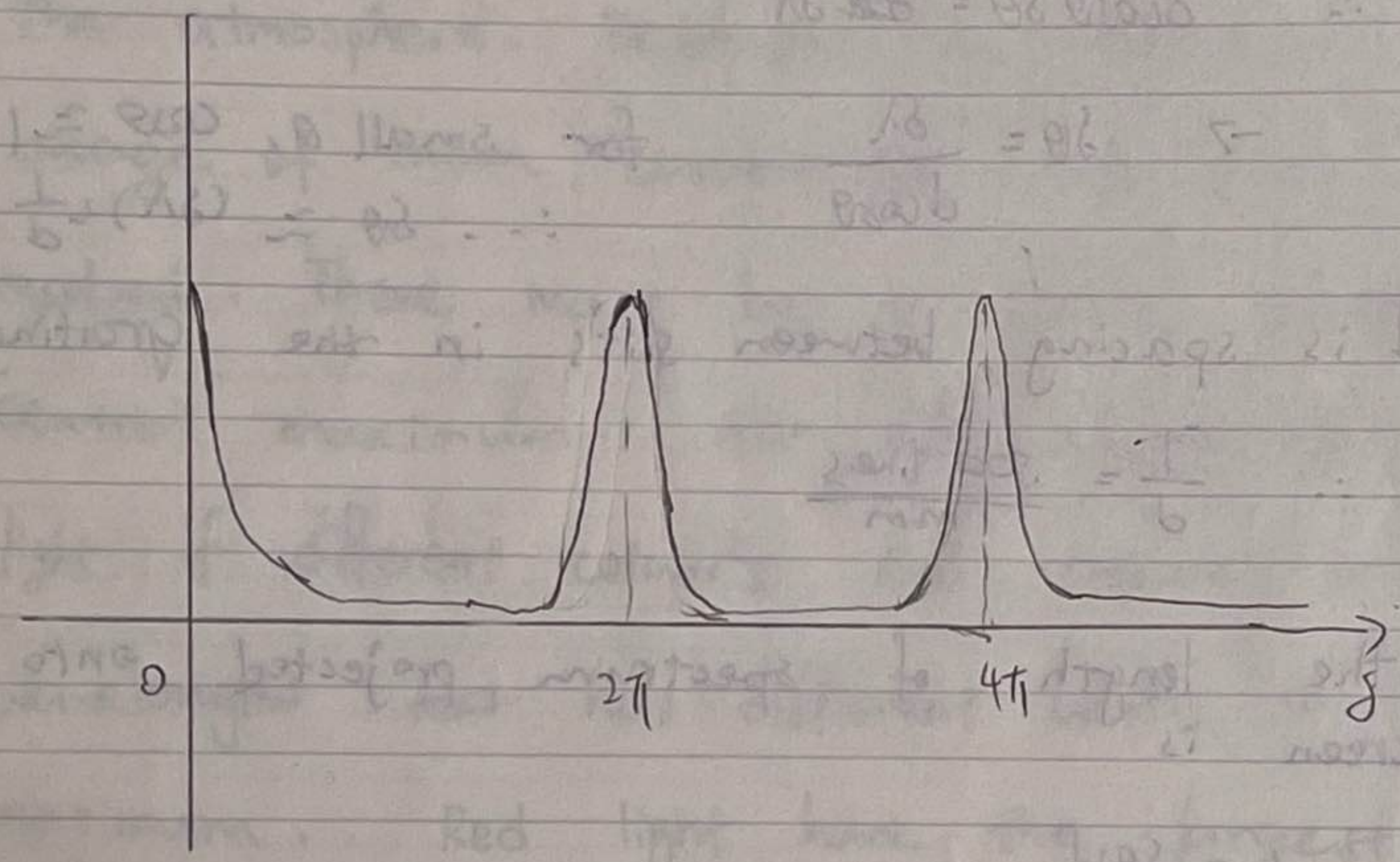
$$= \boxed{0.175\,m}$$

5. Let the optical path difference be δ, the the intensity in terms of δ is

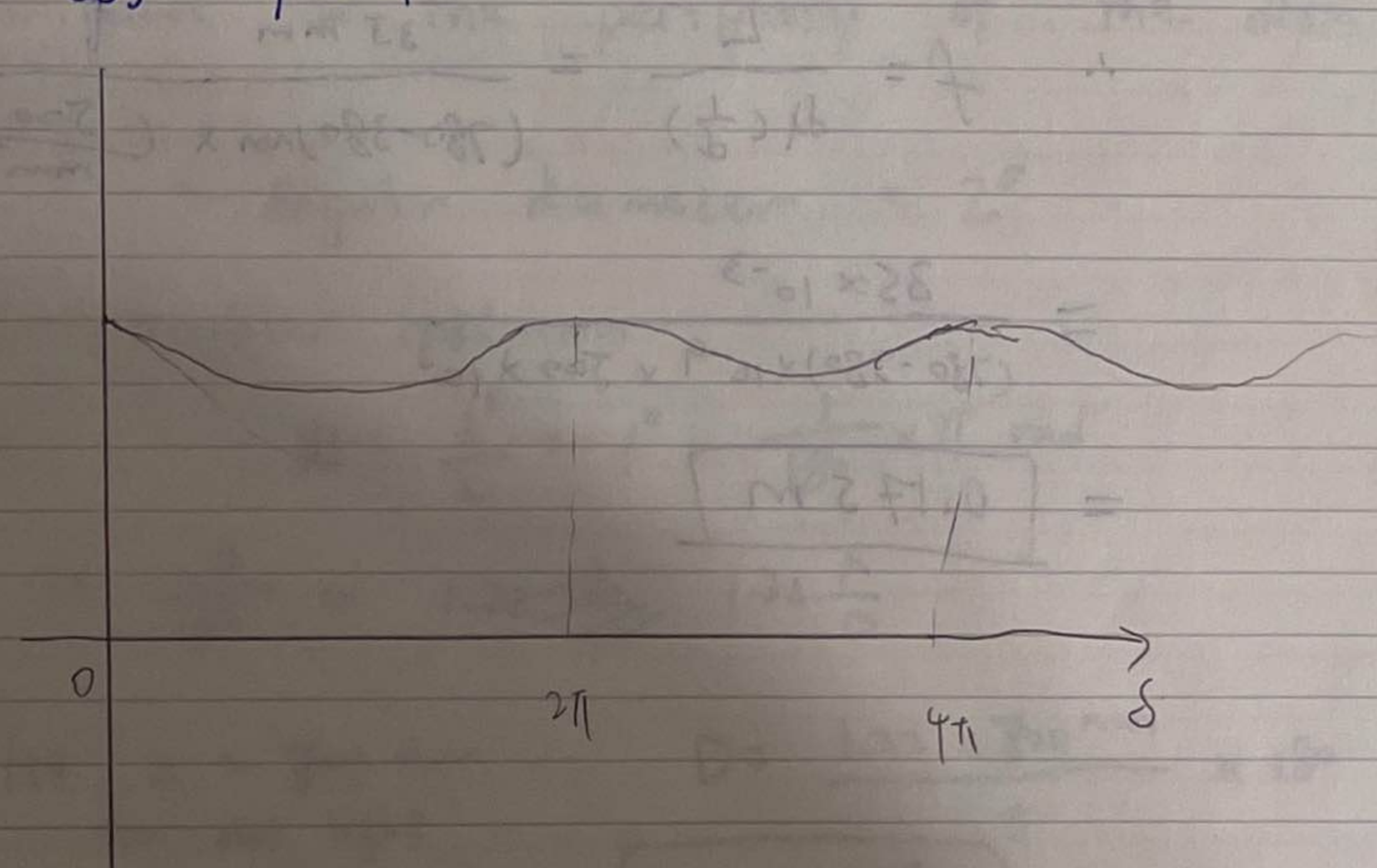
$$I(\delta) = I_0 \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2\left(\frac{\delta}{2}\right)}$$

where F is the Finesse

$\therefore$ (a) $F = 30$



(b) $F = 1$



ther

6. $E_x^2 + E_y^2 - 2E_x E_y \cos\phi = E_0^2 \sin^2\phi$

can be written as

$$(E_x, E_y)\begin{pmatrix} 1 & -\cos\phi \\ -\cos\phi & 1 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = E_0^2 \sin^2\phi$$

let $\vec{x} = (E_x, E_y)^T$, ~~we have~~, and

$Q = \begin{pmatrix} 1 & -\cos\phi \\ -\cos\phi & 1 \end{pmatrix}$, we have

$$\vec{x}^T Q \vec{x} = E_0^2 \sin^2\phi$$

Assume we can find matrix P such that

matrix $\tilde{Q} = P^T Q P$ is diagonal, then

$$Q = P\tilde{Q}P^T$$

$$\rightarrow \quad \vec{x}^T Q \vec{x} = (\vec{x}^T P)\tilde{Q}(P^T\vec{x}) = (P^T\vec{x})^T \tilde{Q}(P^T\vec{x}) = E_0^2 \sin^2\phi$$

So in the coordinate axis defined by $P^T\vec{x}$,

we see the major and minor axis of the ellipse coincide with the coordinate axes.

Now find P. , let eigenvalues $= \lambda$
eigenvectors $= v = (v_1, v_2)^T$

Eigenvalue equation:

$$\det\begin{pmatrix} (1-\lambda) & -\cos\phi \\ -\cos\phi & (1-\lambda) \end{pmatrix} = 0$$

$$\rightarrow \quad (1-\lambda)^2 - \cos^2\phi = 0$$

$$\therefore \quad \lambda^2 - 2\lambda + 1 - \cos^2\phi = 0$$

$$\therefore \quad \lambda^2 - 2\lambda + \sin^2\phi = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4\sin^2\phi}}{2} = 1 \pm \cos\phi$$

For $\lambda = 1 + \cos\phi$

$$\begin{pmatrix} -\cos\phi & -\cos\phi \\ -\cos\phi & -\cos\phi \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{-1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

For $\lambda = 1 - \cos\phi$

$$\begin{pmatrix} \cos\phi & -\cos\phi \\ -\cos\phi & \cos\phi \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad P^T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

This is a rotation matrix representing a rotation of $45°$

$\rightarrow$ Angle is $\boxed{45°}$

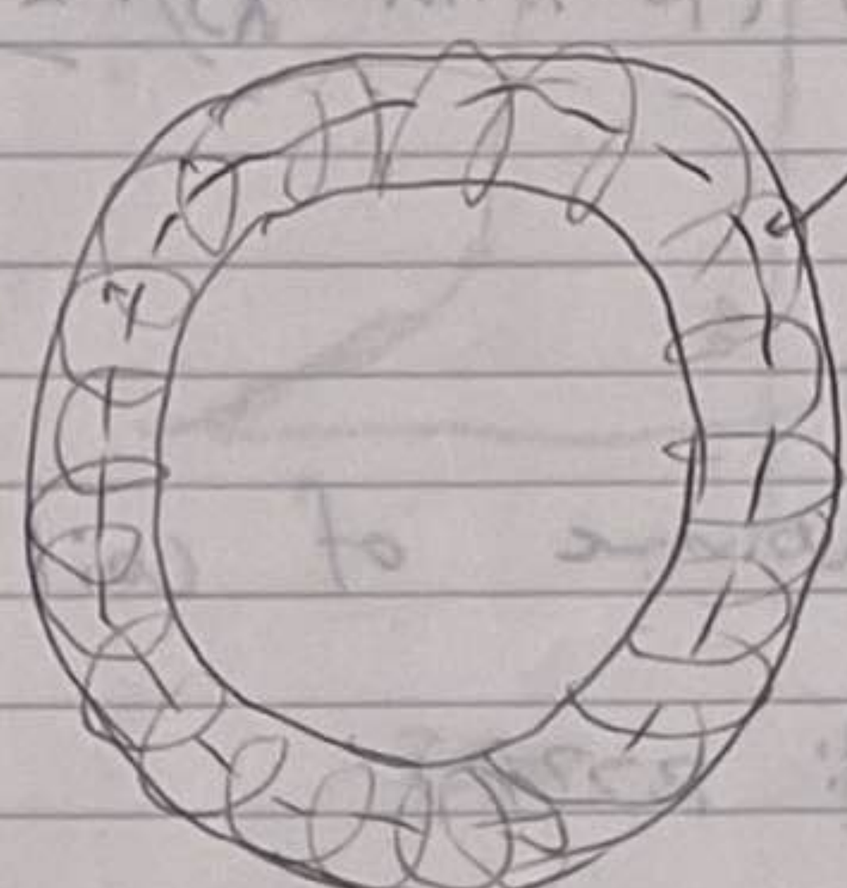
7. Magnetic permeability ~~is~~ μ is the ratio between magnetic field $\underline{B}$ and auxillary field $\underline{H}$ ~~in a linear~~

$$\underline{B} = \mu \underline{H}$$

Magnetic susceptibility χ is the ratio between the magnetisation $\underline{M}$ and auxillary field $\underline{H}$

$$\underline{M} = \chi \underline{H}$$

(a)



Amperien loop.

Total free current enclosed

$$I_f = NI$$

Ampere's Law : $\oint \underline{H} \cdot d\underline{l} = I_f$

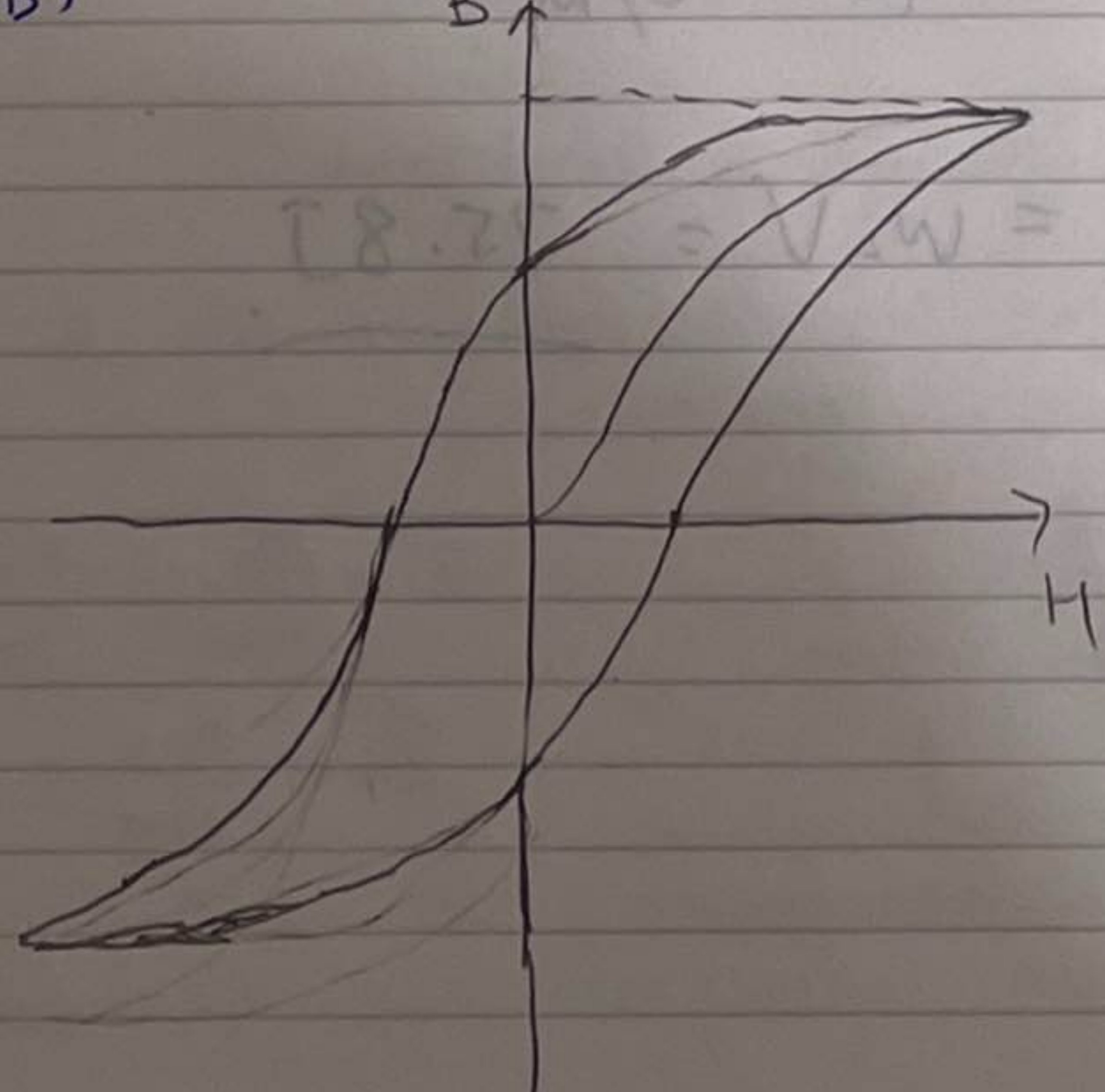
$$\therefore \quad H(2\pi R) = I_f = NI$$

$$\therefore \quad H = \frac{NI}{2\pi R}$$

$$H = \frac{B}{\mu_0} - m \rightarrow B = \mu_0(H + M) = \mu_0(1+\chi)H$$

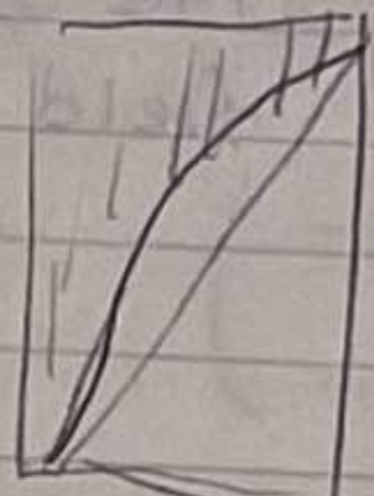
$$\rightarrow \quad B = \frac{\mu_0(1+\chi)NI}{2\pi R.}$$

(b)



Work done per unit volume to move along the ~~Hy~~ Hysterisis ~~too~~ curve is

$$w = \int H \, dB$$



To bring the system from origin to P along (i).

$\int H dB$ is approximately $\frac{1}{4}$ of ~~the~~ HB at point P

$\therefore \quad w_1 = \frac{1}{4} HB = \frac{1}{4}(1.5T)(10^4 A\cdot m^{-1} \times 5) = \cancel{37500\ J/m^3}$ $18750\ J/m^3$

energy needed to magnetise :

$W_1 = w_1 V$ ← volume of coil

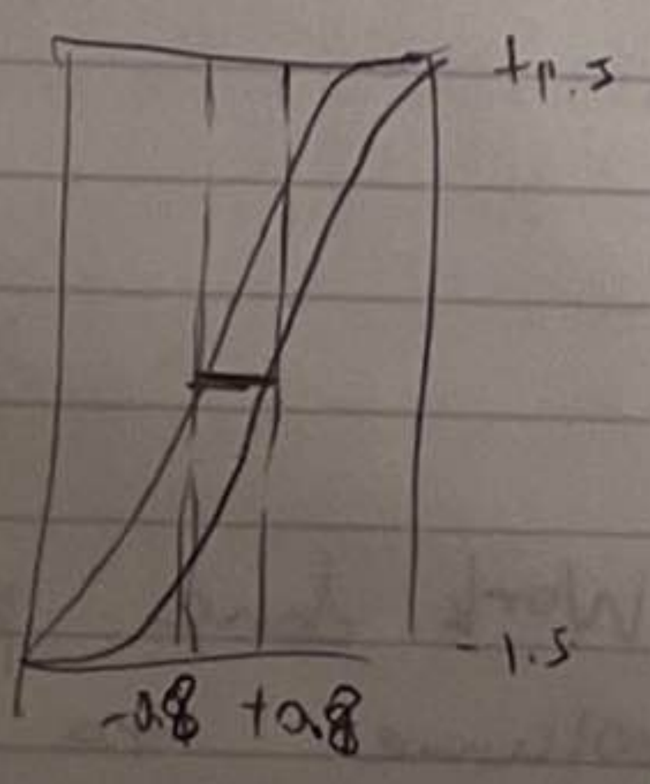
$V \approx \pi r^2 \cdot 2\pi R \quad (\because R \gg r)$

$= 2\pi^2 r^2 R = 1.58 \times 10^{-3} m^3$

$\therefore W_1 = (18750)(2\pi^2)(0.02)^2(0.2)$

$\approx \cancel{5.9J}$ $\underline{29.6J}$

To bring the system around the loop.

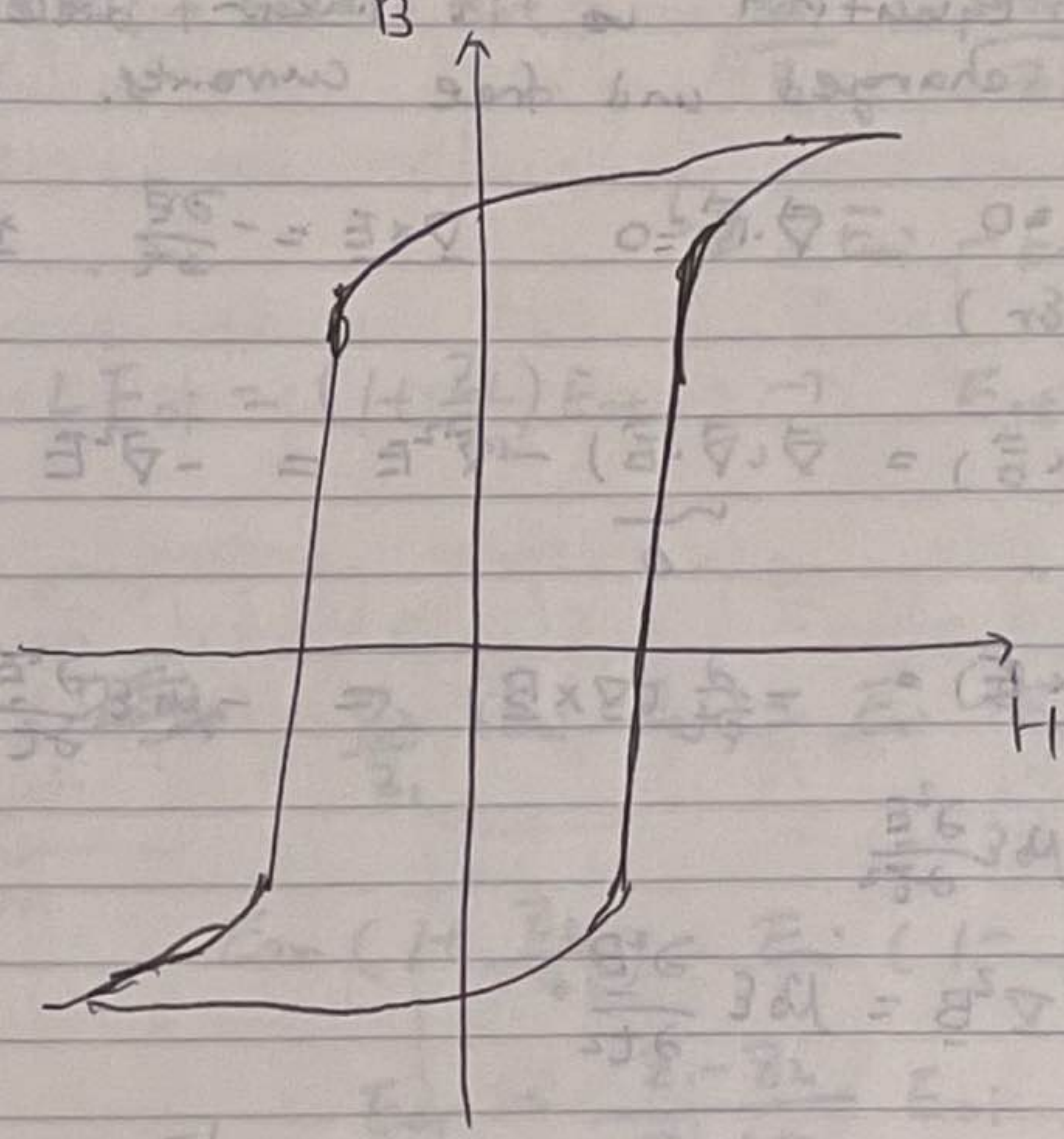


$w_2 \oint H \cdot dB \approx (1.5 \times 2T) \times (0.8 \times 2 \times 10^4 A\cdot m^{-1})$

$= 48000\ J/m^3$

$W_2 = w_2 V = \underline{75.8J}$.

(c)



$$\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}$$

$$\therefore \quad \underline{B} = \mu_0 \underline{H} + \mu_0 \underline{M}$$

$\because$ at high $\underline{H}$, $\underline{M}$ is constant

$\therefore$ the slope is $\underline{\mu_0}$

8. Maxwell's equation in linear dielectric with no free charges and free currents.

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\cdot\vec{B}=0 \qquad \nabla\times\underline{E}=-\frac{\partial\underline{B}}{\partial t}, \quad \nabla\times\underline{B}=\mu_0\varepsilon\frac{\partial\underline{E}}{\partial t}$$

$(\varepsilon=\varepsilon_0\varepsilon_r)$

$$\rightarrow \quad \vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0})-\vec{\nabla}^2\vec{E}=-\vec{\nabla}^2\vec{E}$$

$$\nabla\times(\nabla\times\underline{E})=-\frac{\partial}{\partial t}(\nabla\times\underline{B})=-\mu_0\varepsilon\frac{\partial^2\underline{E}}{\partial t^2}$$

$$\rightarrow \quad \nabla^2\underline{E}=\mu_0\varepsilon\frac{\partial^2\underline{E}}{\partial t^2}$$

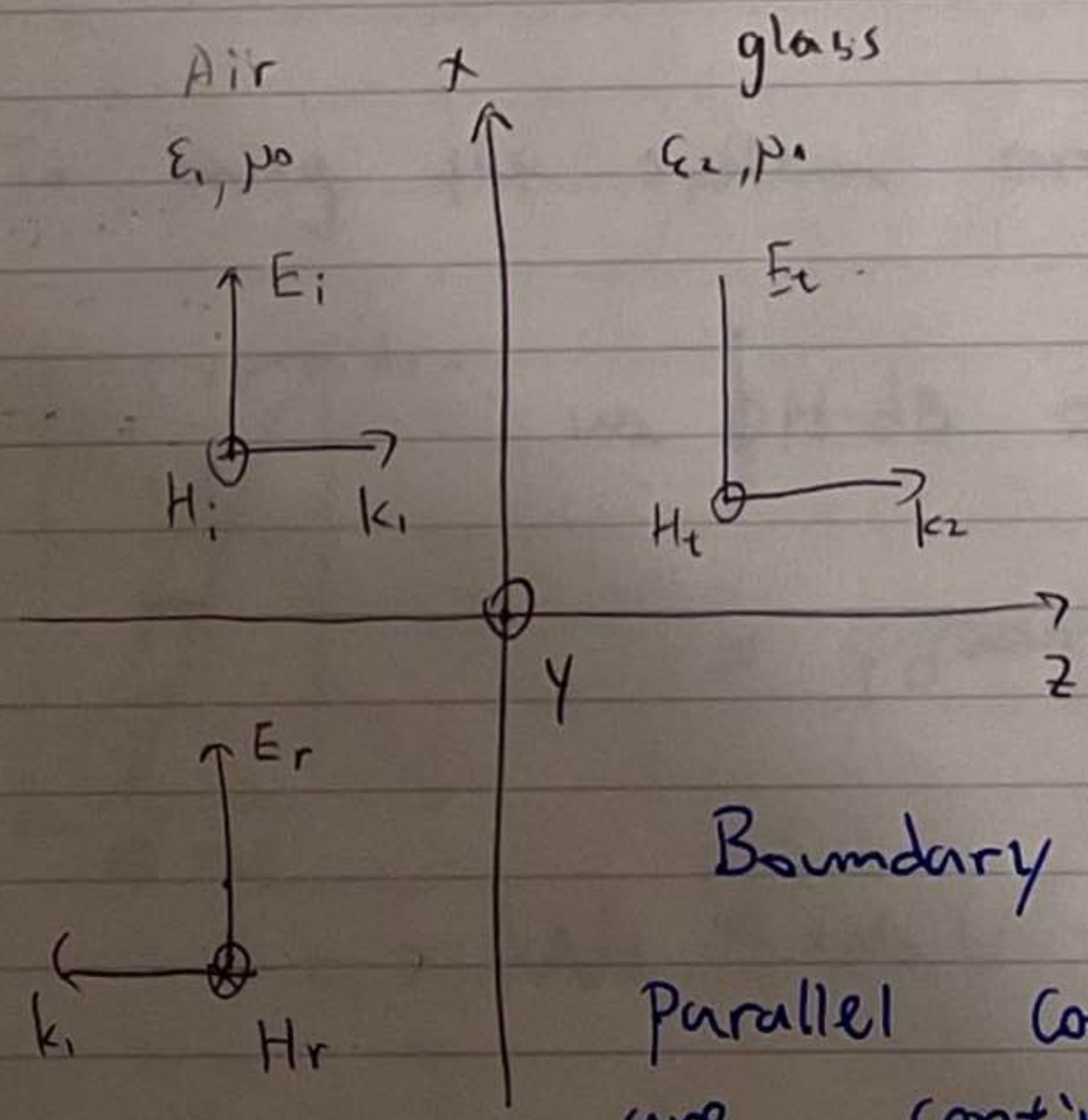
Similarly $\nabla^2\underline{B}=\mu_0\varepsilon\frac{\partial^2\underline{B}}{\partial t^2}$

Wave equations $\rightarrow$ phase velocity

$$\underline{V=\frac{1}{\sqrt{\mu_0\varepsilon}}=\frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}}$$

Definition of index of refraction $V=\frac{c}{n}$

$$\therefore \quad \frac{1}{\sqrt{\mu_0\varepsilon_0}}=\frac{1}{n}\frac{1}{\sqrt{\mu_0\varepsilon}} \qquad \therefore \quad n=\sqrt{\frac{\varepsilon}{\varepsilon_0}}=\underline{\sqrt{\varepsilon_r}}$$



$$\tilde{E}_i(z,t)=E_{0i}\,e^{i(k_1z-\omega t)}\hat{x}$$
$$\tilde{H}_i(z,t)=H_{0i}\,e^{i(k_1z-\omega t)}\hat{y}$$
$$\tilde{E}_r(z,t)=E_{0r}\,e^{i(-k_1z-\omega t)}\hat{x}$$
$$\tilde{H}_r(z,t)=-H_{0r}\,e^{i(-k_1z-\omega t)}\hat{y}$$

$$\tilde{E}_t(z,t)=E_{0t}\,e^{i(k_2x-\omega t)}\hat{x}$$
$$\tilde{H}_t(z,t)=H_{0t}\,e^{i(k_2x-\omega t)}\hat{y}$$

Boundary conditions:

Parallel components of $\underline{E}$ and $\underline{H}$ are continuous.

$$\rightarrow \quad E_{0i}+E_{0r}=E_{0t}.$$
$$H_{0i}-H_{0r}=H_{0t}$$

$$Z_1=\frac{E_{0i}}{H_{0i}}=\frac{E_{0r}}{H_{0r}}=\mu_1V_1=\mu_0V_1$$

$$Z_2=\frac{E_{0t}}{H_{0t}}=\mu_2V_2=\mu_0V_2$$

$E_{0i} + E_{0r} = E_{0t}$, $\quad \frac{E_{0i}}{Z_1} - \frac{E_{0r}}{Z_1} = \frac{E_{0t}}{Z_2} \rightarrow \frac{Z_2}{Z_1}(E_{0i} - E_{0r}) = E_{0t}$

$\hookrightarrow \; E_{0i} - E_{0r} = \frac{Z_1}{Z_2} E_{0t}$.

$2E_{0i} = \left(1 + \frac{Z_1}{Z_2}\right) E_{0t} \rightarrow E_{0t} = \frac{2Z_2}{Z_1 + Z_2} E_{0i}$

$\cancel{2E_{0t}} \quad \frac{Z_2}{Z_1}(E_{0i} - E_{0r}) = E_{0i} + E_{0r}$

$$E_{0r}\left(1 + \frac{Z_2}{Z_1}\right) = E_{0i}\left(1 - \frac{Z_2}{Z_1}\right)$$

$$\rightarrow E_{0r} = \frac{Z_1 - Z_2}{Z_1 + Z_2} E_{0i}$$

$$= \frac{v_1 - v_2}{v_1 + v_2} E_{0i}$$

$$= \frac{\frac{c}{n_1} - \frac{c}{n_2}}{\frac{c}{n_1} + \frac{c}{n_2}} E_{0i}$$

$$= \frac{n_2 - n_1}{n_2 + n_1} E_{0i} = \frac{n_{glass} - n_{air}}{n_{glass} + n_{air}} E_{0i} = r E_{0i}$$

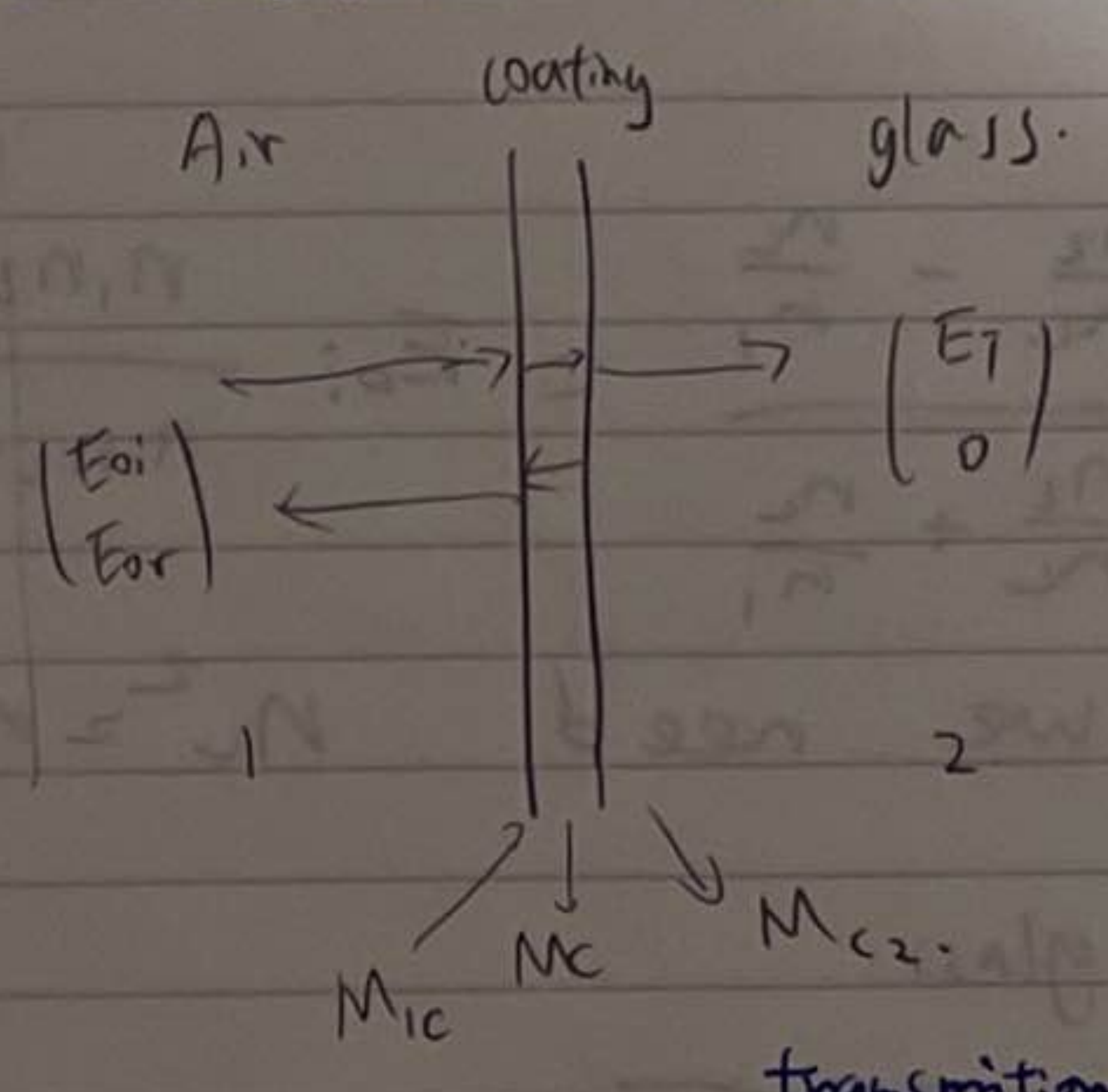
→ ~~ke~~ ~~reflection~~ reflectivity,

$$r = \frac{n_{glass} - n_{air}}{n_{glass} + n_{air}},$$

fraction of power reflected

$$R = r^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = 4\%$$

Use matrix method for the coating



$$M_{1c} = \begin{pmatrix} 1 + \frac{n_c}{n_1} & 1 - \frac{n_c}{n_1} \\ 1 - \frac{n_c}{n_1} & 1 + \frac{n_c}{n_1} \end{pmatrix}$$

$$M_c = \begin{pmatrix} e^{-ik_c d} & 0 \\ 0 & e^{+ik_c d} \end{pmatrix}$$

$$M_{c2} = \begin{pmatrix} 1 + \frac{n_2}{n_c} & 1 - \frac{n_2}{n_c} \\ 1 - \frac{n_2}{n_c} & 1 + \frac{n_2}{n_c} \end{pmatrix}$$

M_{1c}, M_{2c} are ~~propagation~~ transmition/reflection matrices

M_c is a propagation matrix

$k_c = \frac{2\pi}{\lambda}$ $\therefore k_c d = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$ $e^{\pm i k_c d} = e^{\pm i\frac{\pi}{2}}$
$= \pm i$

$\therefore M_c = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

$$\therefore \begin{pmatrix} E_{0i} \\ E_{0r} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{n_c}{n_1} & 1-\frac{n_c}{n_1} \\ 1-\frac{n_c}{n_1} & 1+\frac{n_c}{n_1} \end{pmatrix}}_{M_{1c}} \underbrace{\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}}_{M_c} \underbrace{\begin{pmatrix} 1+\frac{n_2}{n_c} & 1-\frac{n_2}{n_c} \\ 1-\frac{n_2}{n_c} & 1+\frac{n_2}{n_c} \end{pmatrix}}_{M_{c2}} \begin{pmatrix} E_T \\ 0 \end{pmatrix}$$

$$= M_{1c} M_c \begin{pmatrix} (1+\frac{n_2}{n_c})E_T \\ (1-\frac{n_2}{n_c})E_T \end{pmatrix}$$

$$= M_{1c} \begin{pmatrix} -i(1+\frac{n_2}{n_c})E_T \\ i(1-\frac{n_2}{n_c})E_T \end{pmatrix}$$

$$\rightarrow \frac{E_{0i}}{E_{0r}} = \frac{(1+\frac{n_c}{n_1})(-i - i\frac{n_2}{n_c}) + (1-\frac{n_c}{n_1})(i - i\frac{n_2}{n_c})}{(1-\frac{n_c}{n_1})(-i - i\frac{n_2}{n_c}) + (1+\frac{n_c}{n_1})(i - i\frac{n_2}{n_c})}$$

$$= \frac{\cancel{-i} - i\frac{n_c}{n_1} - \cancel{i\frac{n_2}{n_c}} - i\frac{n_2}{n_1} + \cancel{i} - i\frac{n_c}{n_1} - i\frac{n_2}{n_c} + \cancel{i\frac{n_2}{n_1}}}{\cancel{-i} + \frac{n_c}{n_1}i - i\frac{n_2}{n_c} + \cancel{i\frac{n_2}{n_1}} + \cancel{i} + \frac{n_c}{n_1}i - i\frac{n_2}{n_c} - \cancel{i\frac{n_2}{n_1}}}$$

$$= \frac{\frac{n_c}{n_1} + \frac{n_2}{n_c}}{-\frac{n_c}{n_1} + \frac{n_2}{n_c}}$$

$$\rightarrow E_{0r} = E_{0i}\frac{\frac{n_2}{n_c} - \frac{n_c}{n_1}}{\frac{n_2}{n_c} + \frac{n_c}{n_1}} = E_{0i}\frac{n_1 n_2 - n_c^2}{n_1 n_2 + n_c^2}$$

for $E_{0r} = 0$, we need $n_c^2 = n_1 n_2$

$\rightarrow n_{coat}^2 = n_{air} n_{glass}$

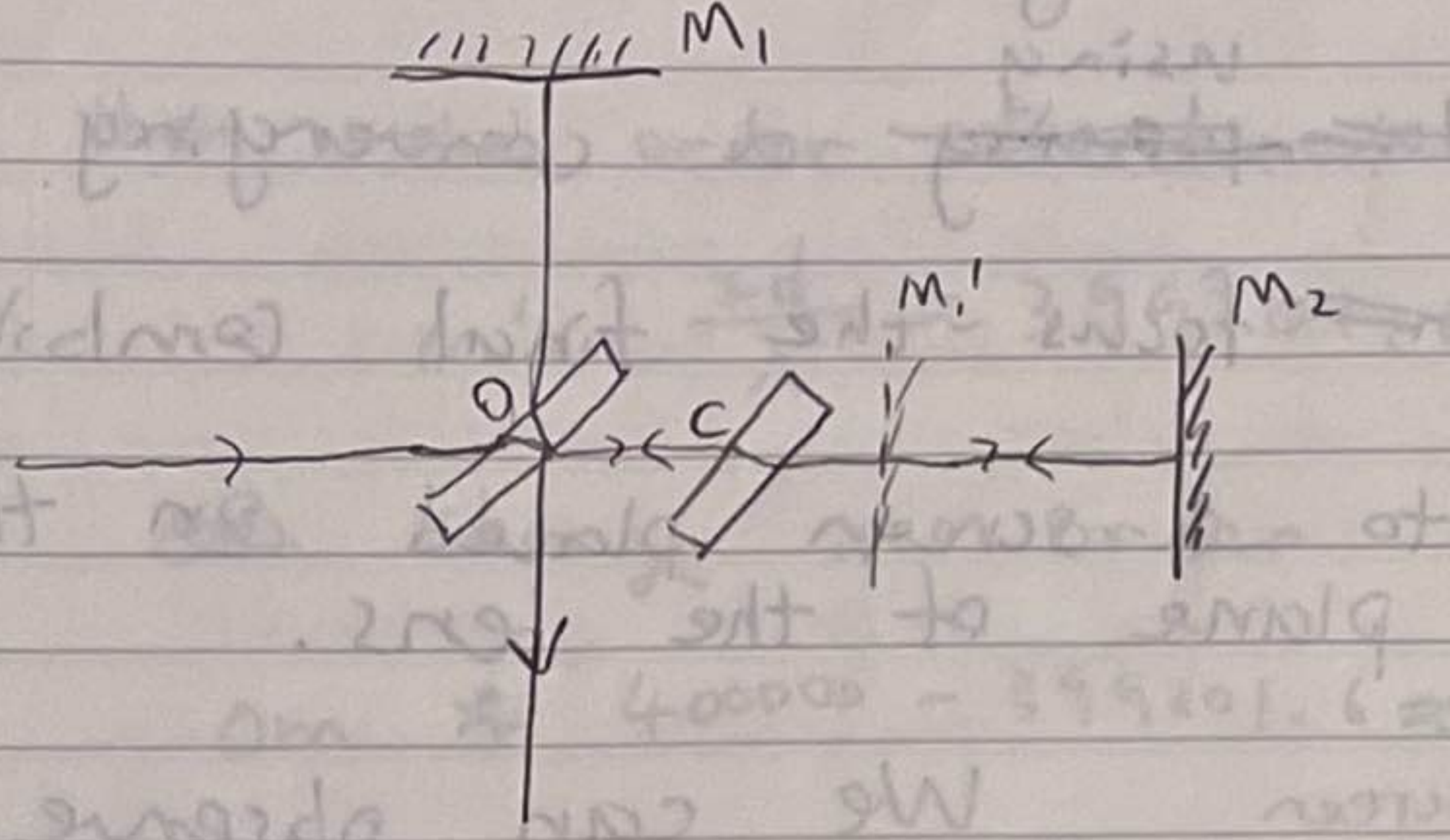
$\rightarrow$ $\underline{n_{coat} = \sqrt{n_{air} n_{glass}}}$

In practice, perfectly monochromatic light is impossible to make. ~~So~~ but in perfect anti-reflection coating the thickness of coating is exactly $\frac{\lambda}{4}$

Since there is a band width $\Delta\lambda$ we must have some reflection.

→ In porous materials the pores filled with air redmes the effective index of ~~refract~~ refraction

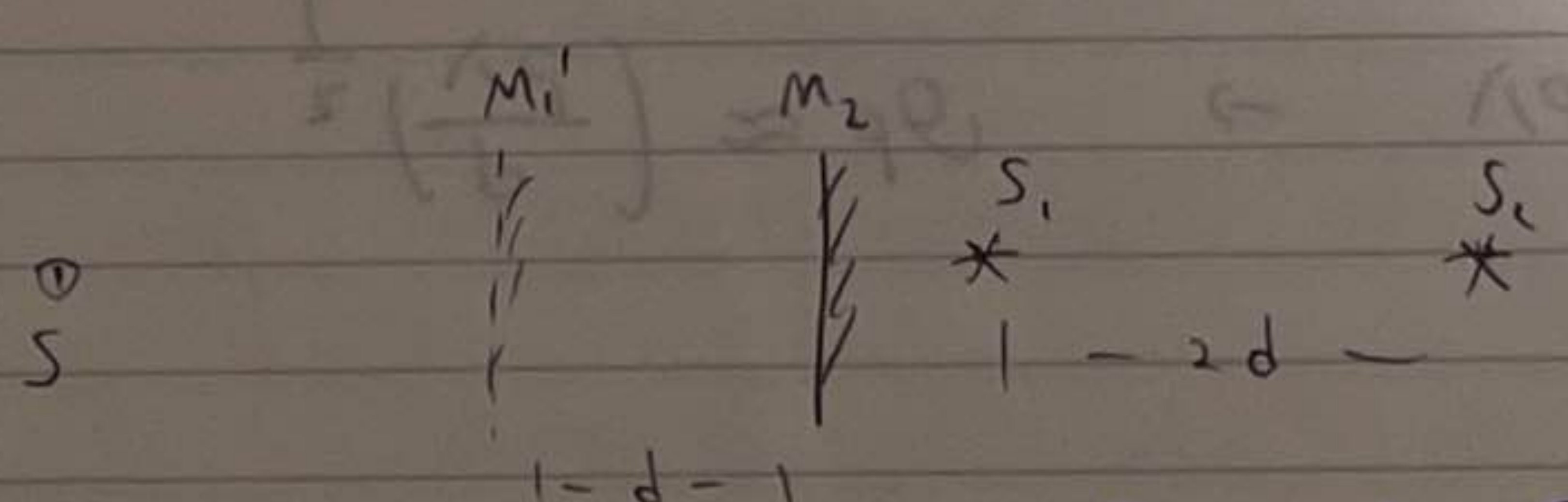
9.



→ O is used as a beam splitter to ~~splitt~~ split the incoming light beam into two beams that hit M_1 and M_2 respectively. It also introduces a phase shift of π between the two beams as the two sides of the Beam Splitter have different change in refractive index (one from small to large, the other from large to small)

→ C is used to make the two beams travel the same distance within ~~the~~ glass. So the phase difference is only due to the separation between mirrors.

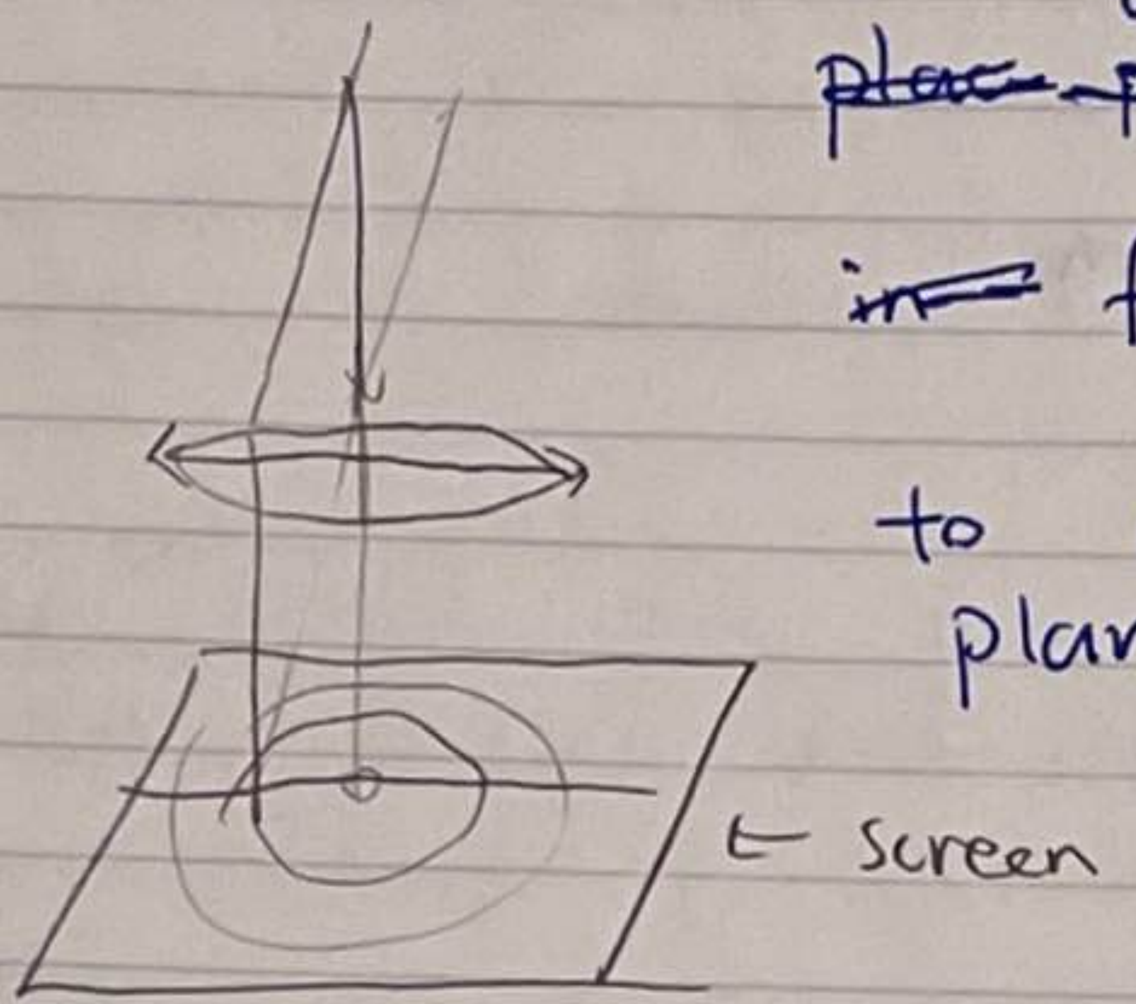
→



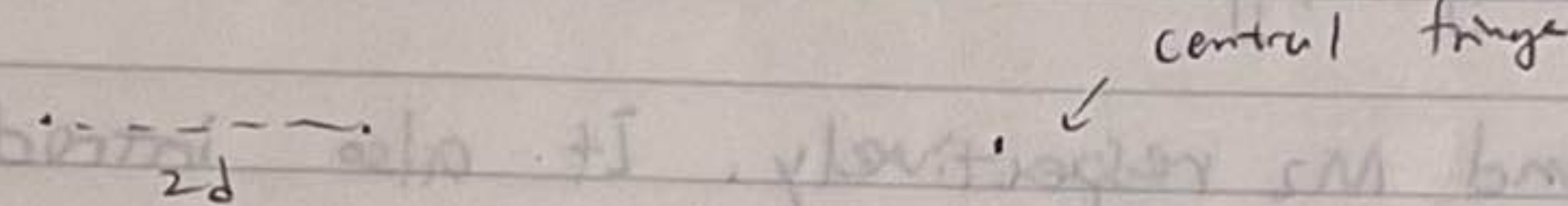
The image of mirror M_1, M_1', and the mirror M_2 ~~&~~ togather form two images of the light source S, namely S_1 and S_2.

S_1 and S_2 ~~acts~~ act like two light sources and the light "produced" by them interfere.

The interference fringes can be observed by ~~placing~~ using a converging ~~les~~ lens to ~~in~~ focus the final combined rays to a screen placed on the focal plane of the lens.



We can observe the fringes on the screen.



Because of the π phase shift:

central fringe is dark if $2d = m\lambda$

→ the order m is then ~~$m = \frac{\lambda}{2d}$~~

~~$m = \frac{(500nm)}{2\times 0.1}$~~ $\quad m = \frac{2d}{\lambda} = \frac{2\times 0.1m}{(500nm)} = \underline{4\times 10^5}$

the pth order dark ring is the $(m-p)$th order of interference

$$\rightarrow \quad 2d = m\lambda$$
$$2d\cos\theta_p = (m-p)\lambda$$
$$\rightarrow \quad 2d(1-\cos\theta_p) = p\lambda$$
$$\text{small } \theta_p \rightarrow \quad 2d\left(1-\left(1-\frac{\theta_p^2}{2}\right)\right) \approx p\lambda$$
$$\rightarrow \quad d\theta_p^2 = p\lambda \quad \rightarrow \quad \theta_p \approx \left(\frac{p\lambda}{d}\right)^{\frac{1}{2}}$$

$\because \Delta\lambda = 1\,nm \quad \therefore$ Consider $\lambda' = \lambda + \Delta\lambda = 501\,nm$

then the order of central fringe is

$$m' = \frac{2d}{\lambda'} \approx 399201.6$$

change in order

$$\Delta m \approx 400000 - 399201.6 \approx \underline{798.4 \text{ orders}}$$

→ wavelengths between 500 nm and ~~...~~ 501 nm will cause many orders to overlap, and we <u>cannot</u> see the fringes.

10. Fraunhofer condition:

Fraunhofer diffraction is the diffraction pattern in which phase ~~difference~~ difference of light at an observation point is a linear function of position for all points in the diffracting aperture

Wavefront deviates from a plane-wave by a very small amount.

direct Fourier Transform: ~~$\tilde{G}(\beta) = \int_{-\infty}^{\infty}$~~

$$A(\beta) = \int_{-\infty}^{\infty} T(y) e^{-i\beta y}\, dy$$

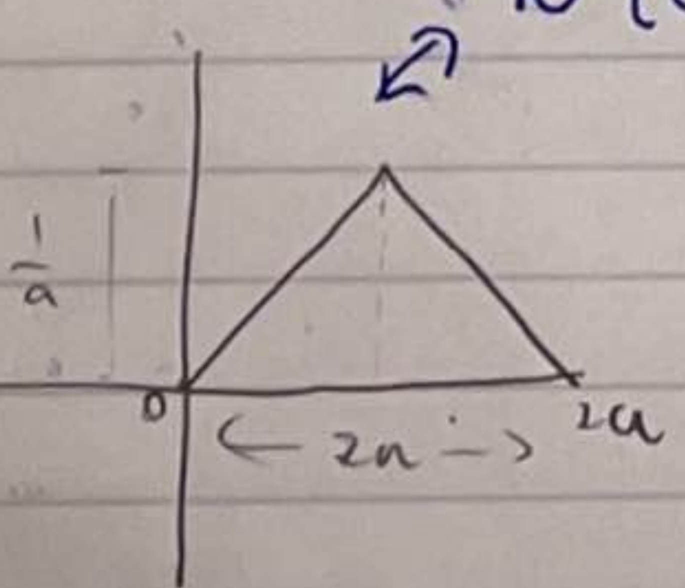
inverse:

$$T(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\beta) e^{i\beta y}\, d\beta$$

~~$\beta = ky\sin\theta$~~ $\left(\beta = k\sin\theta = \frac{2\pi}{\lambda}\sin\theta\right)$

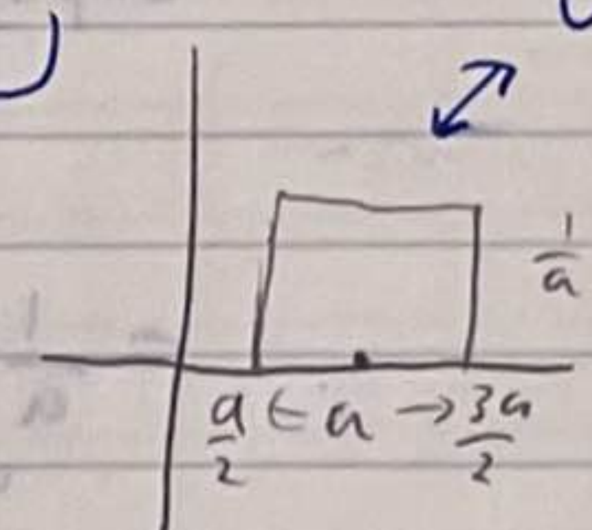
~~In Fraunhofer case, phase is linear~~ ~~$\rightarrow A(\beta) = \int_{-\infty}^{\infty} T(y)\, e^{-i\frac{2\pi}{\lambda}y\sin\theta}\, dy$~~

$T(y)$

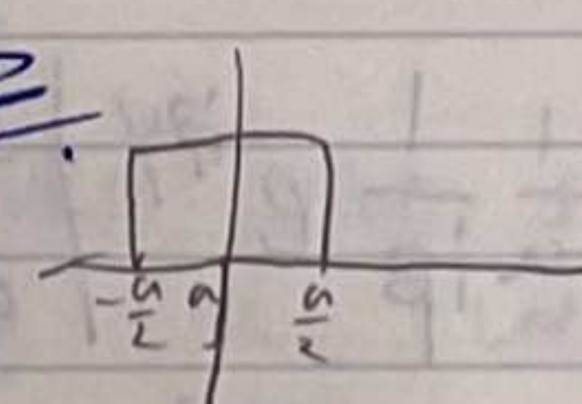


is the $\underbrace{\text{convolution of}}$ function

$u(y)$



~~Convolute~~ with ~~itself~~.



$\rightarrow u'(y)$

Convolution Theorem:

Fourier Transform of products of 2 functions is

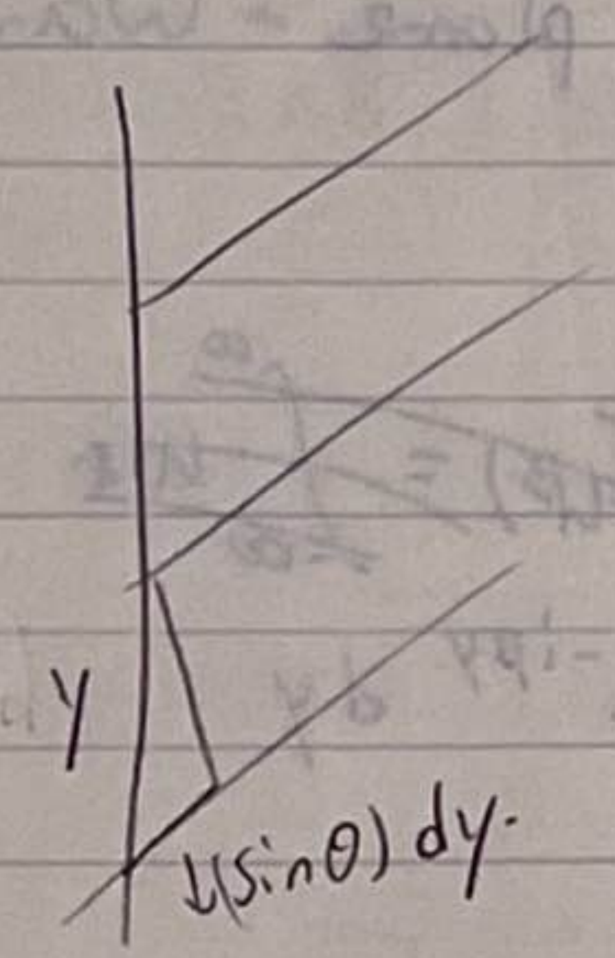
$\propto$ the product of the Fourier Transform of two functions.

$\rightarrow$ ~~$A(\beta) \propto B(\beta)$~~ $A(\beta) \propto (B(\beta))^2$ where

$B(\beta)$ is the F.T. of $u(y)$

$$\cancel{B(\beta) = \int_{-\infty}^{\infty} u(y)\, e^{-i\beta y}\, dy}$$

In the Fraunhofer case:



phase is linear:

$\rightarrow$ phase at y is $e^{-ik\sin\theta\, y}$

Sum all the phase and weight with the amplitude function

$$A(\beta) = \int_{-\infty}^{\infty} T(y)\, e^{-ik\sin\theta\, y}\, dy$$

$$\rightarrow A(\theta) = \int_{-\infty}^{\infty} T(y) \exp\left(-i\left[\frac{2\pi}{\lambda} y \sin\theta\right]\right) dy$$

Similarly

$$B(\beta) = \int_{-\infty}^{\infty} u(y)\, e^{-i\beta y}\, dy$$

$$= \frac{1}{a}\int_{a/2}^{3a/2} e^{i\beta y}\, dy$$

$$= \frac{1}{a}\frac{1}{i\beta} e^{i\beta y}\Big|_{a/2}^{3a/2}$$

$$= \frac{1}{a}\frac{1}{i\beta}\left(e^{\frac{i3a\beta}{2}} - e^{\frac{ia\beta}{2}}\right)$$

$$= \frac{1}{ia\beta} e^{ia\beta}\underbrace{\left(e^{\frac{ia\beta}{2}} - e^{-\frac{ia\beta}{2}}\right)}_{2i\sin\left(\frac{a\beta}{2}\right)}$$

$$= e^{ia\beta}\frac{2i\sin\left(\frac{\beta a}{2}\right)}{i\beta a} = e^{ia\beta}\frac{\sin\left(\frac{\beta a}{2}\right)}{\left(\frac{\beta a}{2}\right)}$$

$$\rightarrow B'(\beta) = \int_{-\infty}^{\infty} u'(y)\, e^{-i\beta y} = \frac{\sin\left(\frac{\beta a}{2}\right)}{\frac{\beta a}{2}}$$

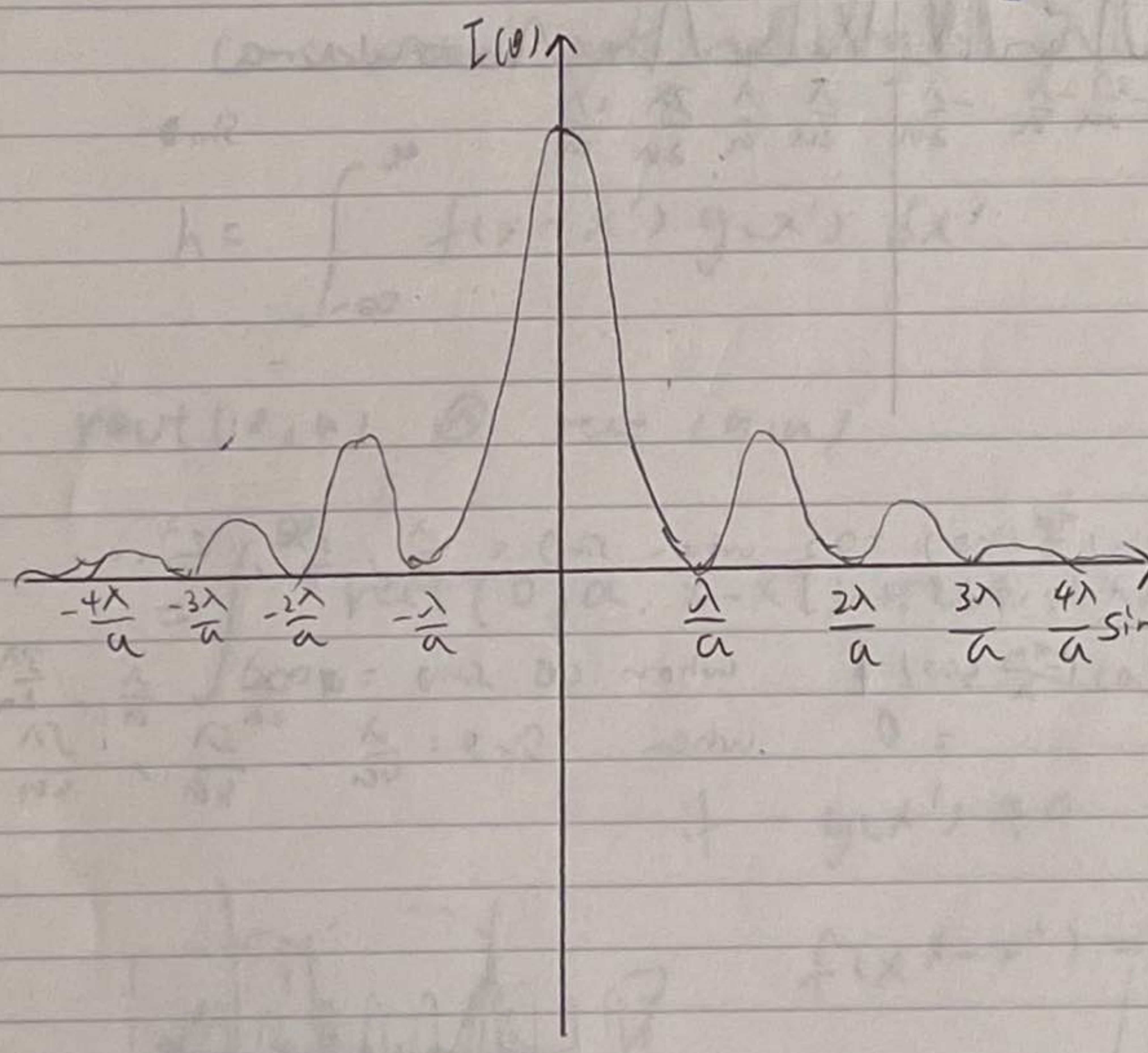
$$\rightarrow \quad A(\beta) = (u(\beta))^{\prime} = e^{i a\beta} \frac{\sin^2(\frac{\beta a}{2})}{(\frac{\beta a}{2})^2}$$

$$\times (u'(\beta))$$

Intensity pattern is

$$I(\beta) \propto |A(\beta)|^2 \qquad \because \beta = \frac{2\pi}{\lambda}\sin\theta$$

$$\rightarrow I(\theta) = I_0 \frac{\sin^4(\frac{\pi a}{\lambda}\sin\theta)}{(\frac{\pi a}{\lambda}\sin\theta)^4}$$



(b) The left triangle gives the same amplitude function but replacing a with $-a$

$\rightarrow$

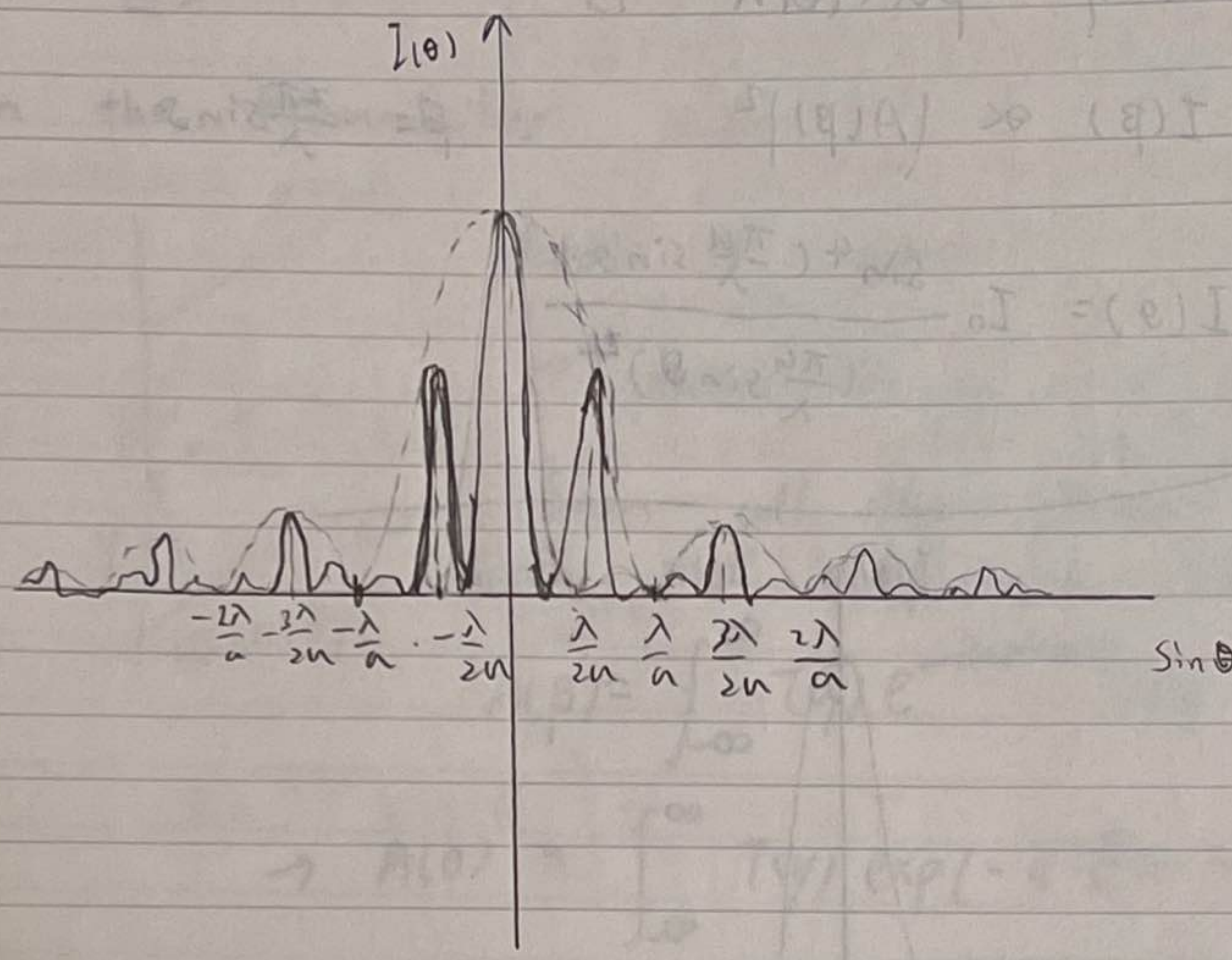
$$A'(\beta) = A(\beta, a) + A(\beta, -a)$$

$$= \frac{\sin^2(\frac{\beta a}{2})}{(\frac{\beta a}{2})} (e^{i\beta a} + e^{-i\beta a})$$

$$= 2 \frac{\sin^2(\frac{\beta a}{2})}{(\frac{\beta a}{2})} \cos(\beta a)$$

~~$\rightarrow A'(\theta) =$~~

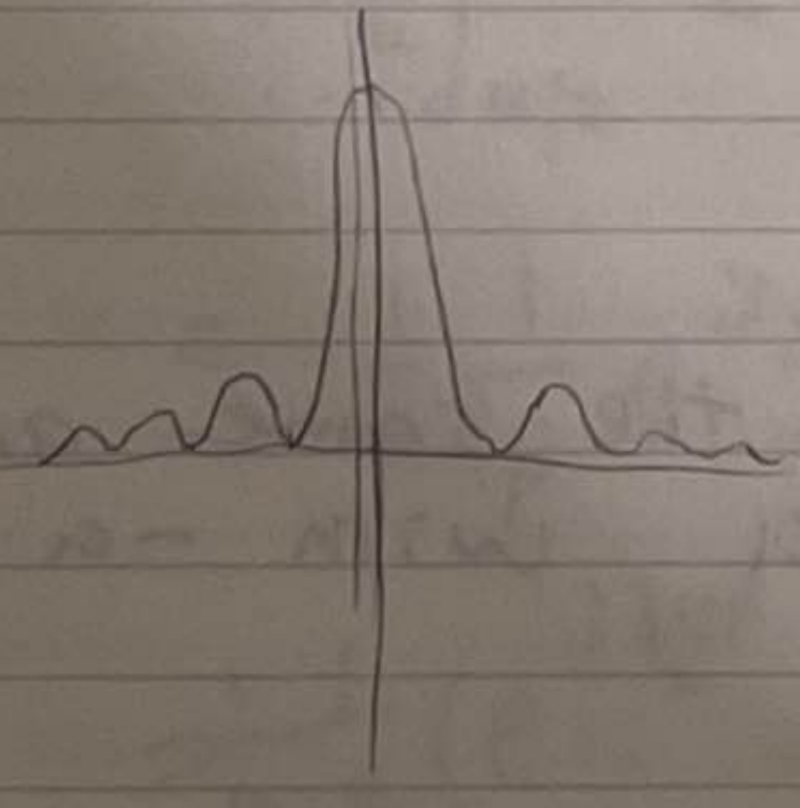
$$\rightarrow I'(\theta) = I_0 \cos^2\left(\frac{2\pi a}{\lambda}\sin\theta\right) \frac{\sin^4\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

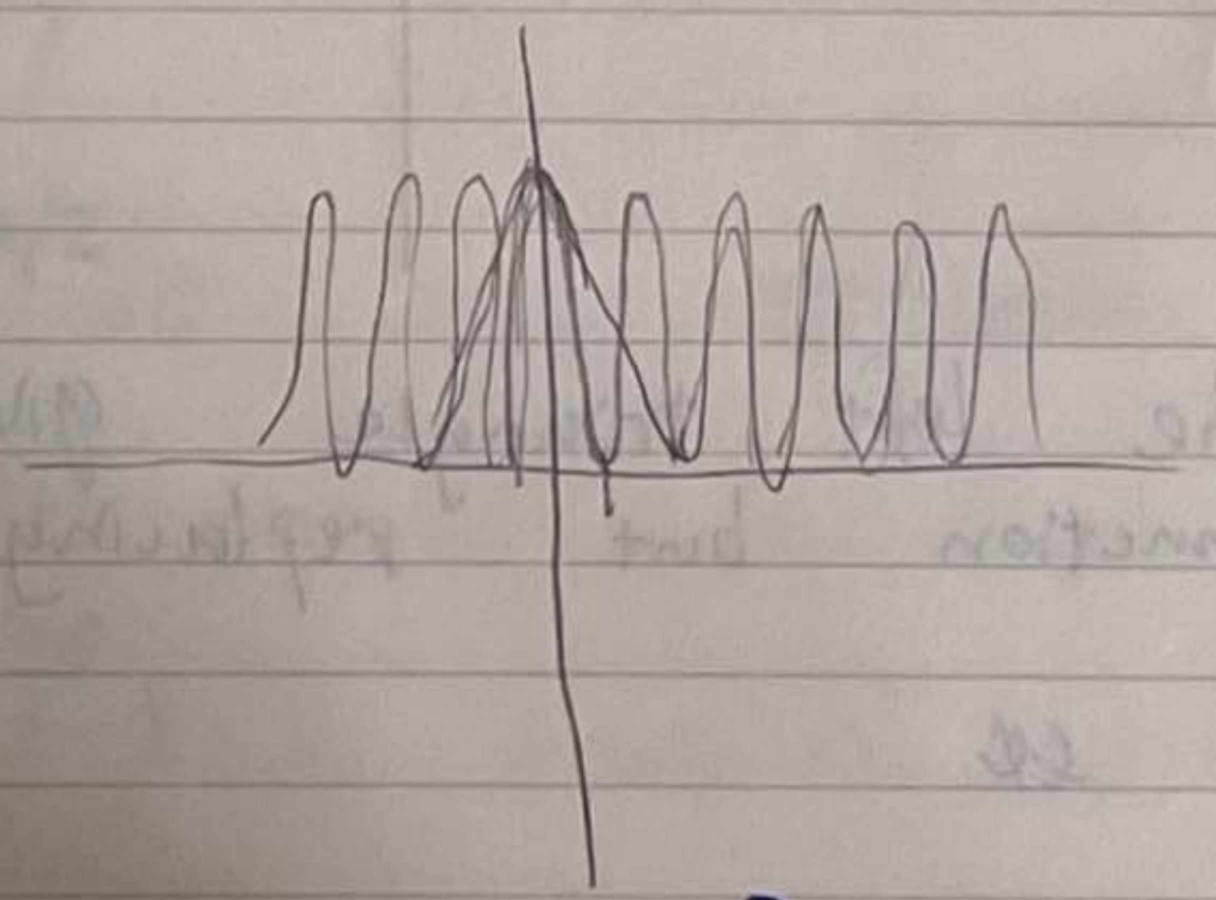


$\left.\begin{array}{l}\sin\left(\frac{\pi a}{\lambda}\sin\theta\right) = 0 \\ \left(\frac{\pi a}{\lambda}\sin\theta\right) \neq 0\end{array}\right\}$ when $\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{2\lambda}{a} \ldots$

$\cos\left(\frac{2\pi a}{\lambda}\sin\theta\right) = 1$ when ~~cos~~ $\sin\theta =$ ~~a~~ $\frac{\lambda}{2a}, \frac{\lambda}{a}, \frac{3\lambda}{2a}$

$= 0$ when $\sin\theta = \frac{\lambda}{4a}, \frac{3\lambda}{4a}, \frac{5\lambda}{4a}$



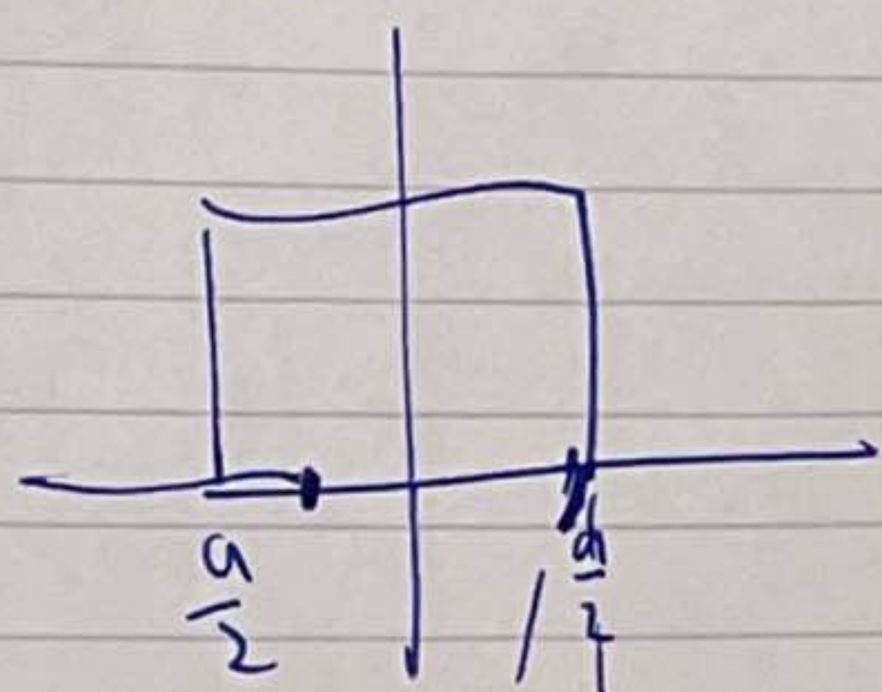


First zeros : $\sin\theta = \pm\frac{\lambda}{2a}$.

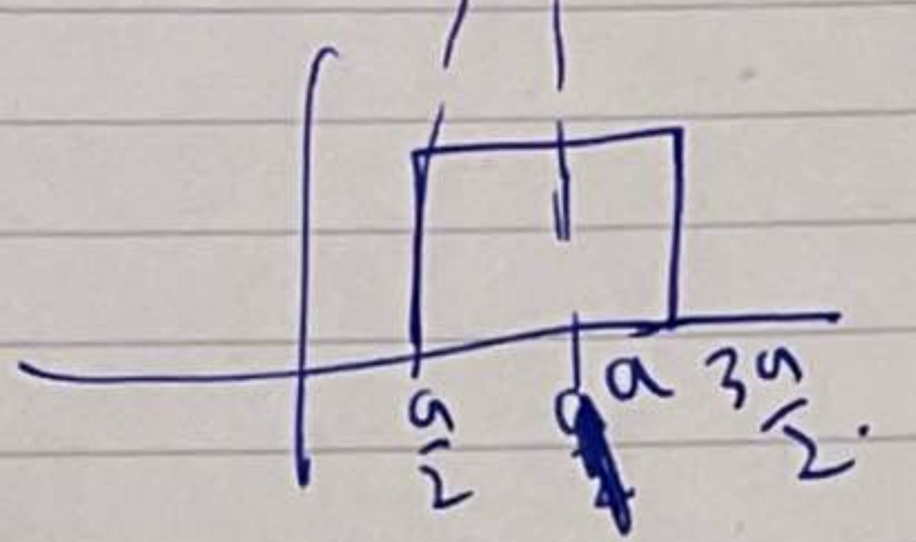
spatial frequency $= \frac{1}{2a}$

$\therefore$ First zero location $= \pm\lambda \times$ (spatial frequency)

Appendix A. Q10.



$rect(0, a; x) = f(x)$



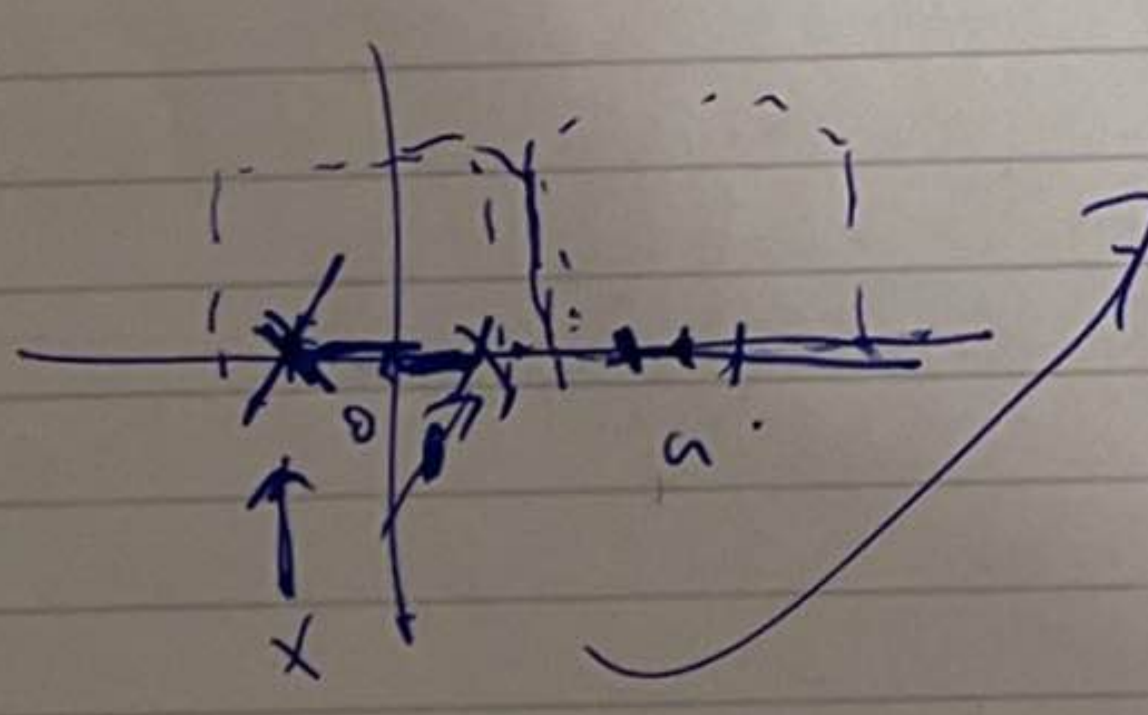
$rect(a, a; x) = g(x)$

convolution of f, g. $h = f \otimes g$

$$h = \int_{-\infty}^{\infty} f(x-x')\, g(x')\, dx'$$

$rect(0,a) \otimes rect(a,a)$

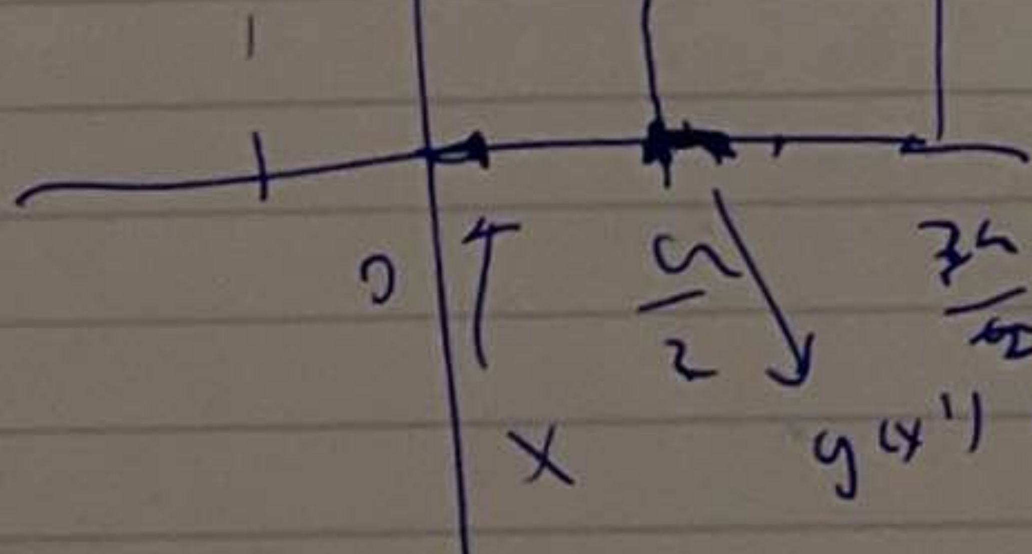
$$= \int_{-\infty}^{\infty} rect(0,a,x-x')\, rect(a,a,x')\, dx'$$

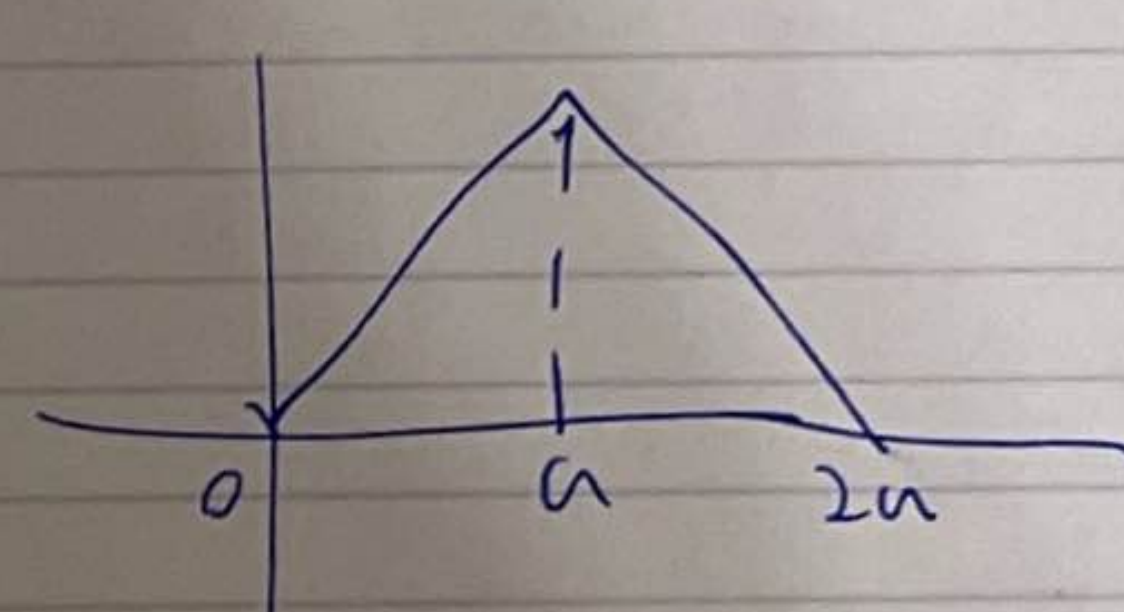


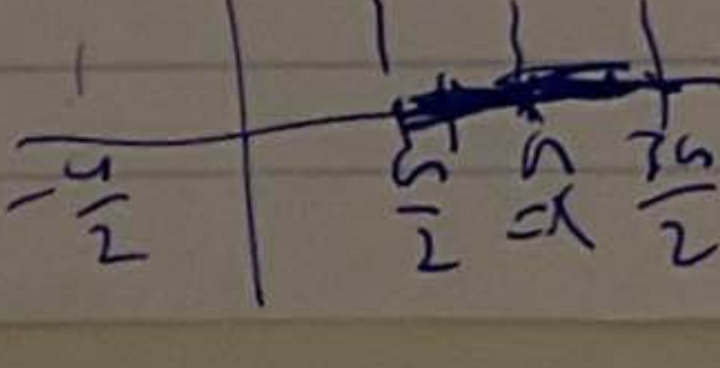
if $g(x') \neq 0$.

$f(x-x') = 0$.

$g(x') = 0$







→ maximum allowed range if $x = a$. for $g(x')$