To : Robin Nicholas

Statistical Physics 2 Ziyan Li

1.

$\cancel{dU = TdS - pdV}$ $dU = đQ + đW$

U does not change $\Rightarrow$ $dU = 0$ ①

~~Work~~ No work is done $\Rightarrow$ $đW = 0$ ②

Process is irreversible $\Rightarrow$ $đW > -pdV$ ③

①, ② $\Rightarrow$ $đQ = 0$

$dU = TdS - pdV$ and ① $\Rightarrow$ $TdS - pdV = 0$

$\Rightarrow$ ③ $\Rightarrow$ $TdS + đW > 0$

$đW = 0 \Rightarrow TdS > 0$

$T > 0 \Rightarrow dS > 0$

Everything is consistent.

$\therefore$ ~~There~~ $dU = TdS - pdV$ is

compatible with all the aforementioned conditions.

Tut Notes.

You cannot have a bunch of gas going spontaneously goes to a half of ~~to~~ its volume without doing any work.

(originally path no work, definition of reversible process is that you have to go ~~bat~~ back by doing the same amount of work).

2.
$$T_i = 90 + 273 = 363\,K$$

$$T_f = 18 + 273 = 291\,K$$

This is a reversible process

~~$đQ = đ$~~ $đQ_{rev} = đQ = C_V\,dT = cm\,dT$

$$\therefore\ dS = \frac{đQ_{rev}}{T} = cm\frac{dT}{T}$$

$$\Delta S = \int dS = cm\int_{T_i}^{T_f}\frac{dT}{T}$$

$$= cm\ln\frac{T_f}{T_i} = (4200\,JK^{-1}kg^{-1})(0.2\,kg)\ln\left[\frac{(291\,K)}{(363\,K)}\right]$$

$$= \boxed{-185.7\,JK^{-1}}$$

$$\Delta S = \Delta S_{system}$$

Now we find $\Delta S_{surrounding}$

~~The~~ If finally equilibrium is reached then

The surrounding has temperature $18°C = 291\,K$

and this value is a constant

$$\therefore\ \Delta S_{surrounding}\ \cancel{\Delta S} = \frac{\Delta Q}{T_f} = \frac{cm\Delta T}{T_f} = \frac{cm(T_i - T_f)}{T_f}$$

$$= cm\left(\frac{T_i}{T_f} - 1\right) = (4200)(0.2)\left(\frac{363}{291} - 1\right)$$

$$= \boxed{207.8\,JK^{-1}}$$

If Equilibrium is NOT reached then $T_f < 18°C$

which means $\Delta S_{surrounding} > \cancel{208.7}\ 207.8\ J/K$

$\therefore \Delta S_{surrounding} \geq 207.8\ JK^{-1}$

$\therefore \Delta S_{universe} = \Delta S_{system} + \Delta S_{surr}$

$\geq (-185.7 + 207.8)\ JK^{-1}$

$= 22.1\ JK^{-1} > 0$

$\therefore$ entropy of Universe increases.

18°C

3. (a) $m = 0.4\,kg$ $\quad C_V = 150\,JK^{-1}$

block = b $\quad$ lake = l

$T_i = 100°C = 373\,K$

$T_f = 10°C = 283\,K.$

$$\Delta S_b = \int dS = \int \frac{dQ_{rev}}{T} = \int \frac{dQ}{T}$$

reversible

$$= \int_{T_i}^{T_f} \frac{C_V\,dT}{T} = C_V \ln \frac{T_f}{T_i}$$

$$= (150) \ln \left(\frac{283}{373}\right) = -41.4\,J/K.$$

$$\Delta S_l = \frac{\Delta Q}{T} = \frac{C_V(T_i - T_f)}{T_f} = C_V\left(\frac{T_i}{T_f} - 1\right)$$

$$= 47.7\,J/K$$

$$\therefore \Delta S_{univ} = \boxed{6.3\,J/K}$$

(b) ~~Negligible heat is transfered to the block as the block los~~

As the block stops motion ~~heat~~ its kinetic energy is transferred into heat

No heat is transfered into the block

$\Delta S_b = 0$

$$\Delta S_l = \frac{\Delta Q}{T} = \frac{mgH}{T} = \frac{(0.4)(9.8)(100)}{283}$$

$$= \boxed{1.4\,J/K}$$

(c) $T_1 = 100°C = 373 K$

$T_2 = 10°C = 283 K$

$C_v(T_1 - T_f) + C_v(T_2 - T_f) = 0$ (No net heat flow into the system)

$$\therefore \quad T_f = \frac{T_1 + T_2}{2} = 328 K$$

$$\Delta S_1 = \int \frac{dQ_{rev}}{T} = \int \frac{dQ}{T}$$

$$= \int_{T_1}^{T_f} \frac{C_v \, dT}{T} = C_v \ln\left(\frac{T_f}{T_1}\right).$$

$$\Delta S_2 = \int_{T_2}^{T_f} \frac{C_v \, dT}{T} = C_v \ln\left(\frac{T_f}{T_2}\right).$$

$\Delta S_{surrounding} = 0$ because all heat lost by one is received by another.

$$\therefore \Delta S_{univ} = C_v \left[\ln\left(\frac{T_f}{T_1}\right) + \ln\left(\frac{T_f}{T_2}\right)\right].$$

$$= C_v \left[\ln\left[\frac{T_f^2}{T_1 T_2}\right]\right].$$

$$= 150 \ln\left(\frac{328^2}{373 \times 283}\right).$$

$$= \boxed{2.85 \ J/K}$$

(d) $C = 1\mu F$ $V = 100V$ $T = 273K$.

Capacitor = c surrounding = s

$\Delta S_c = 0$ $\because$ No heat transfered to the capacitor

Energy ~~stored~~ in Capacitor $\Delta U_c = \frac{1}{2}CV^2$

Energy supplied by the battery

$$W_B = \int V dq = \cancel{\int} VQ = V(CV) = CV^2$$

$\therefore$ Heat released from the capacitor to the surrounding is

$$\Delta Q = W_B - \Delta U_c = \frac{1}{2}CV^2.$$

$$\Delta S_s = \frac{\Delta Q}{T} = \frac{CV^2}{2T}.$$

$$\Delta S_{univ} = \Delta S_c + \Delta S_s$$

$$= \frac{CV^2}{2T} = \frac{(1\times10^{-6})(100)^2}{2\times273} =$$

$$= \boxed{1.83\times10^{-5}\ J/K}$$

(e)

After the capacitor is Totally discharged, all the energy stored in the capacitor is release into the surrounding as heat through the resistor

$\therefore \quad \Delta S_c = 0$

$$\Delta S_s = \frac{1}{2} C V^2 / T = \boxed{1.83 \times 10^{-5}\ J/K}$$

✓

(f)

$V_i = V \quad V_f = 2V$

Isothermal expansion $\quad dU = \text{đ}Q + \text{đ}W = 0$

$\therefore \text{đ}Q = -\text{đ}W = p\,dV$

$$\Delta S_g = \int dS = \int \frac{\text{đ}Q_{rev}}{T} = \int \frac{\text{đ}Q}{T}$$

$$= \int \frac{P}{T} dV = \underbrace{\int \frac{R}{V} dV}_{\text{1 mole of gas.}} = R\int_V^{2V} \frac{dV}{V} = \boxed{R\ln 2}$$

~~(g) Adiabatic expan~~

But ~~the~~ since the process is reversible

$\therefore \Delta S_{univ} = \boxed{0} \quad (\Delta S_s = -R\ln 2)$ ✓

(g) Adiabatic process $\Rightarrow$ no heat exchange

$\Rightarrow \Delta S_{univ} = \boxed{0}$ ✓

(h) entropy is a state variable so ~~we~~ given the same initial and final temperature and volume as (f) we know that

$\Delta S_g = R\ln 2.$

If the container is thermally isolated

$\Delta S_s = 0$

$\therefore \Delta S_{univ} = \Delta S_g + \Delta S_s = \boxed{R\ln 2}$ ✓

4. (a) $C_V = 1000\ J/K \quad T_1 = 200K, \ T_2 = 100K.$

$$\underset{\text{block}}{\Delta S_b} = \int dS_b = \int \frac{dQ_{rev,b}}{T} = \int \frac{dQ_b}{T}$$

$$= \int \frac{C_V dT}{T} = C_V \int_{T_1}^{T_2} \frac{dT}{T} = C_V \ln\left(\frac{T_2}{T_1}\right)$$

$$\underset{\text{bath}}{\Delta S_s} = \frac{C_V(T_1 - T_2)}{T_2} = C_V\left(\frac{T_1}{T_2} - 1\right)$$

$$\therefore \Delta S = \Delta S_b + \Delta S_s = C_V\left[\frac{T_1}{T_2} - \left(\ln\frac{T_1}{T_2} + 1\right)\right]$$

$$= \boxed{306.8\ J/K}$$

(b) $T_1 = 200K, \ T_2 = 150K, \ T_3 = 100K.$

$T_1 \to T_2$

$$\Delta S_{12} = C_V\left[\frac{T_1}{T_2} - \left(\ln\frac{T_1}{T_2} + 1\right)\right]$$

$$= 45.7\ J/K.$$

$T_2 \to T_3$

$$\Delta S_{23} = C_V\left[\frac{T_2}{T_3} - \ln\left(\frac{T_2}{T_3}\right) - 1\right]$$

$$= 94.5\ J/K$$

$$\therefore \Delta S = \Delta S_{12} + \Delta S_{23} = \boxed{140.2\ J/K}$$

let T=100K Consider a series of bath with temperature

$T_2, T_3, \cdots, T_{n+1}$ with $\cancel{T_2 = T_1 + \Delta T, \; T_3 = T_1 + \ldots}$

$$T_2 = T_1 - \frac{T}{n}, \quad T_3 = T_1 - \frac{2T}{n}, \cdots, \quad T_{n+1} = T_1 - \frac{nT}{n} = T_1 - T$$

$$= 200K - 100K = 100K.$$

$$\Delta S = \sum_{k=1}^{n} C_V \left[\frac{T_k}{T_{k+1}} - \ln\left(\frac{T_k}{T_{k+1}}\right) - 1 \right]$$

$$= \sum_{k=1}^{n} C_V \left[\frac{T_{k+1} + \Delta T}{T_{k+1}} - \ln\left(\frac{T_{k+1} + \Delta T}{T_{k+1}} \right) - 1 \right].$$

$\Delta T = \frac{T}{n}$

$$= \sum_{k=1}^{n} C_V \left[\left(1 + \frac{\Delta T}{T_{k+1}}\right) - \ln\left(1 + \frac{\Delta T}{T_{k+1}}\right) - 1 \right].$$

$$\cancel{= C_V \sum_{k=1}^{n} \left[\frac{\Delta T}{T_{k+1}} - \ln\left(1 + \frac{\Delta T}{T_{k+1}}\right) \right]}$$

$$\cancel{\therefore \lim_{n\to\infty} \Delta S = C_V \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{\Delta T}{T_{k+1}} - \ln\left(1 + \frac{\Delta T}{T_{k+1}}\right) \right]}$$

$$\cancel{= C_V \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{\Delta T}{T_{k+1}} - \frac{\Delta T}{T_{k+1}} + \frac{\Delta T^2}{2T_{k+1}^2} - \frac{\Delta T^3}{3T_{k+1}^3} + \cdots \right]}$$

$$\cancel{= C_V \lim_{n\to\infty} \sum_{k=1}^{n} \sum_{m=2}^{\infty} \frac{(-1)^m}{m} \left(\frac{\Delta T}{T_{k+1}}\right)^m}$$

$$\cancel{< C_V \lim_{n\to\infty} \sum_{k=1}^{n} \sum_{m=2}^{\infty} \frac{1}{m} \left(\frac{\Delta T}{T_{k+1}}\right)^m} \quad \vec{0}$$

$\Delta T \to 0$ T_{k+1} is finite

because $\frac{\Delta T}{T_{k+1}} > 0$

$\frac{\Delta T}{T_{k+1}} \to 0$

$$\ln\left(1 + \frac{\Delta T}{T_{k+1}}\right) \approx \frac{\Delta T}{T_{k+1}} + O(\Delta T^2)$$

let $X_k = 1 + \frac{\Delta T}{T_{k+1}}$ then.

$$\Delta S = C_V \sum_{k=1}^{n} [X_k - \ln(X_k) - 1]$$

The plot of function ~~shows that~~.

$f(x) = x - \ln(x) - 1$ shows that.

the function is monotonically increasing in the interval $[1, \infty]$

we know all $X_k > 1$ $\left(\frac{\Delta T}{T_{k+1}} > 0\right)$ and that

~~and~~ $\cancel{X_k <}$ $\cancel{X_k = 1 + \frac{\Delta T}{T_{k+1}} < 1 + \frac{\Delta T}{T}}$

$X_k = 1 + \frac{\Delta T}{T_{k+1}} < 1 + \frac{\Delta T}{T}$ we call $X' = 1 + \frac{\Delta T}{T}$.

$T \leq T_{k+1}$

$$\therefore f(X_k) = X_k - \ln(X_k) - 1 \leq X' - \ln(X') - 1 = f(X')$$

$$\therefore \Delta S = C_V \sum_{k=1}^{n} [X_k - \ln(X_k) - 1].$$

$$< C_V \sum_{k=1}^{n} [X' - \ln(X') - 1]$$

$$= n[X' - \ln(X') - 1] \times C_V$$

$$= n\left[\cancel{1} + \frac{\Delta T}{T} - \ln\left(1 + \frac{\Delta T}{T}\right) - \cancel{1}\right] \times C_V$$

$$= n\left[\frac{\Delta T}{T} - \ln\left(1 + \frac{\Delta T}{T}\right)\right] \times C_V$$

$$= n\left[\frac{1}{n} - \ln\left(1 + \frac{1}{n}\right)\right] C_V$$

$\cancel{= C_V(1 + }$ $\cancel{= C_V(1 - \ln(1 + \frac{1}{n})}$

$\cancel{= C_V - C_V n}$

$$= C_v \left[1 - n \ln \left(1 + \frac{1}{n}\right) \right]$$

$$\therefore \lim_{n \to \infty} \Delta S \leq C_v \lim_{n \to \infty} \left[1 - n \ln \left(1 + \frac{1}{n}\right) \right]$$

$$= C_v \left[1 - \lim_{n \to \infty} n \ln \left(1 + \frac{1}{n}\right) \right]$$

$$\lim_{n \to \infty} n \ln \left(1 + \frac{1}{n}\right) = \lim_{n \to \infty} \ln \left(1 + \frac{1}{n}\right)^n$$

$$= \ln \left[\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \right] = \ln(e) = 1$$

$$\therefore \lim_{n \to \infty} \Delta S \leq C_v (1 - 1) = 0$$

~~$\therefore \Delta S$ is always ≥ 0~~

$\because \Delta S \geq 0$ by ~~defn~~ the second Law of Thermodynamics

$$\therefore \boxed{\lim_{n \to \infty} \Delta S = 0}$$

4 (b)

As for an infinite number of baths.

we can approximate such that $T_{bath} = T_{block}$

$$\delta S_{bath} = \frac{\delta Q}{T_{block}} = \frac{-C\,\delta T_{block}}{T_{block}} = \frac{-C(-100/N)}{T_{block}}$$

$$\therefore \Delta S_{bath} = \sum \delta S_{bath} = C\sum_{i=1}^{N} \frac{100}{N(200 - i\frac{100}{N})} = C\sum_{i=1}^{N} \frac{1}{2N-i}$$

$$= C\sum_{i=N}^{2N-1} \frac{1}{i} = C\int_{N}^{2N-1} \frac{1}{x}\,dx = C\ln\frac{2N-1}{N}$$

$$= C\ln 2 \quad \text{as} \quad N \to \infty$$

$$\Delta S_{block} = \cancel{C\ln\frac{200}{200-}} - \left[C\ln\frac{200}{T_1} + C\ln\frac{T_1}{T_2} + \dots + C\ln\frac{T_n}{100}\right]$$

$$= -C\ln\frac{200}{100} = -C\ln 2$$

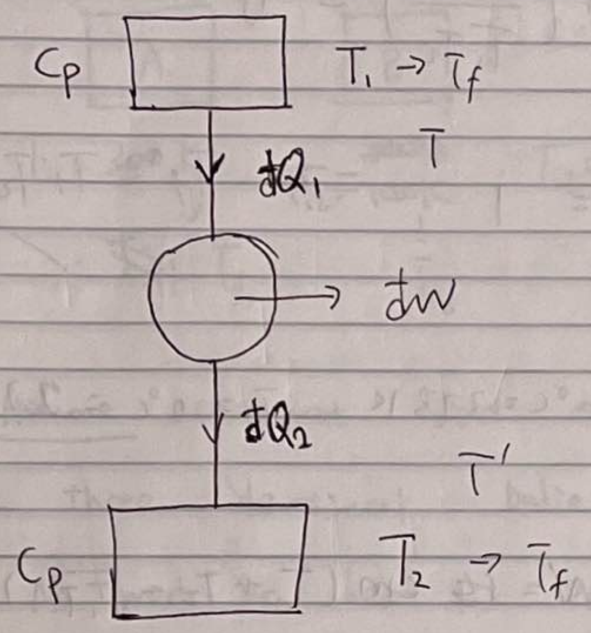
$$\therefore \Delta S = \Delta S_{bath} + \Delta S_{block} = C\ln 2 - C\ln 2 = \underline{0}$$

$$\Delta S_{bath} = C\int_{T_i}^{T_f} \frac{\Delta T}{T}$$

$$\Delta S_{block} = C\int_{T_i}^{T_f} \frac{-\Delta T}{T}$$

$$\Delta S = \Delta S_{bath} + \Delta S_{block} = 0$$

5



$$W = \Delta W = \int đW = \int đQ_1 - \int đQ_2$$

$$= C_p(T_1 - T_f) - C_p(T_f - T_2)$$

$$= \boxed{C_p\,(T_1 + T_2 - 2T_f)}$$

When the most efficient engine, i.e. the Carnot engine is used, since the whole process is reversible, the total change in entropy must be 0

$$\therefore \Delta S = 0$$

$$\therefore 0 = \Delta S = \int dS = \int dS_1 + \int dS_2 = \int \frac{đQ_1}{T} + \int \frac{đQ_2}{T'}$$

$$= \int_{T_1}^{T_f} C_p\, dT + \int_{T_2}^{T_f} C_p\, dT'$$

$$= C_p\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] = \ln\left(\frac{T_f^2}{T_1 T_2}\right) C_p$$

Hence $\ln\left(\frac{T_f^2}{T_1 T_2}\right) = 0$

$\Rightarrow \frac{T_f^2}{T_1 T_2} = 1 \Rightarrow T_f^2 = T_1 T_2$

QED.

For $T_1 = 100°C = 373K$, $T_2 = 0°C = 273K$

$C_p = cm$

We have $W = cm(T_1 + T_2 - \sqrt{T_1 T_2})$

$= (4200)(1)(373 + 273 - 2\sqrt{373 \times 273})$

$= 3.27 \times 10^4 J = \boxed{32.7\ kJ}$

6

C	C	C
A	B	C
$T_{i_1} = 300\,K$	$T_{i_2} = \cancel{300\,K}\ 100\,K$	$T_{i_3} = 300\,K$
$\downarrow$	$\downarrow$	$\downarrow$
T_{f_1}	T_{f_2}	T_{f_3}

<u>Assume</u> constant heat capacity $C_V = C$ for the three identical bodies

No work or heat is supplied from outside

~~By First law of Thermodynamics~~

$$\therefore \Delta W = 0, \quad \Delta Q = 0$$

$$\Delta Q = 0 \implies \Delta Q_1 + \Delta Q_2 + \Delta Q_3 = 0$$

$$\implies C(T_{f_1} - T_{i_1}) + C(T_{f_2} - T_{i_2}) + C(T_{f_3} - T_{i_3}) = 0$$

$$\implies T_{f_1} + T_{f_2} + T_{f_3} = T_{i_1} + T_{i_2} + T_{i_3}$$

No heat is transferred to the surroundings

$$\implies \Delta S_{surr} = 0$$

2nd Law of Thermodynamics $\implies$

$$0 \le \Delta S_{univ} = \Delta S_{surr} + \Delta S_{sys} = 0 + \Delta S_{sys}$$

$$\implies \Delta S_{sys} \geq 0$$

(OR, $\because$ system is isolated $(\Delta W = 0, \Delta Q = 0)$ $\therefore \Delta S_{sys} \geq 0$)

$$\therefore 0 \le \int \frac{dQ_1}{T_1} + \int \frac{dQ_2}{T_2} + \int \frac{dQ_3}{T_3}$$

$$= C\left[\int_{T_{i_1}}^{T_{f_1}} \frac{dT_1}{T_1} + \int_{T_{i_2}}^{T_{f_2}} \frac{dT_2}{T_2} + \int_{T_{i_3}}^{T_{f_3}} \frac{dT_3}{T_3}\right]$$

$$= C \ln\left(\frac{T_{f_1} T_{f_2} T_{f_3}}{T_{i_1} T_{i_2} T_{i_3}}\right)$$

$\therefore\ T_{f_1} T_{f_2} T_{f_3} \geq T_{i_1} T_{i_2} T_{i_3}$ let $T_{i_1} T_{i_2} T_{i_3} = B$

and let $T_{f_1} T_{f_2} T_{f_3} = T_{i_1} T_{i_2} T_{i_3} + \delta = B + \delta = B' \ (\delta \geq 0)$

and let $T_{f_1} + T_{f_2} + T_{f_3} = T_{i_1} + T_{i_2} + T_{i_3} = A$

$(A = 700K,\ B = 9000000\,K^3,\ \delta \geq 0)$.

* To arrive at this ~~start~~ stage alternatively we can use the Clausius Inequality $\oint$, define A, B, C to be the baths and the heat ~~eig~~ engines between them is the system, then define heap goes into the system to be positive and heat leaves out of the system to be negative

then $\oint \frac{dQ'}{T'} \leq 0 \Rightarrow \int \frac{dQ_1'}{T_1'} + \int \frac{dQ_2'}{T_2'} + \int \frac{dQ_3'}{T_3'} \leq 0$

$dQ_i' = -dQ_i$

$$\Rightarrow C\left[\int_{T_{f_1}}^{T_{i_1}} \frac{dT'}{T'} + \int_{T_{f_2}}^{T_{i_2}} \frac{dT'}{T'} + \int_{T_{f_3}}^{T_{i_3}} \frac{dT'}{T'}\right] \leq 0$$

$$\Rightarrow \ln\left(\frac{T_{i_1} T_{i_2} T_{i_3}}{T_{f_1} T_{f_2} T_{f_3}}\right) \leq 0$$

$$\therefore\ T_{f_1} T_{f_2} T_{f_3} \geq T_{i_1} T_{i_2} T_{i_3}$$

If we want to maximize T_{f3}

let $T_{f3} = z$, $T_{f1} = x$, $T_{f2} = y$, then.

The function we are trying to maximize is

$$T_{f3} = f(x,y,z) = z$$

two constraints are

$$g(x,y,z) = x+y+z-A = 0$$

$$h(x,y,z) = xyz - B' = 0$$

Use Lagrange multiplier :

$\boxed{\text{At extreme values}}$

$$\vec{\nabla} f = \lambda_1 \vec{\nabla} g + \lambda_2 \vec{\nabla} h$$

$\therefore \quad (0,0,1) = \lambda_1 (1,1,1) + \lambda_2 (yz, xz, xy)$

$$\therefore \begin{cases} \lambda_1 + yz\lambda_2 = 0 & ① \\ \lambda_1 + xz\lambda_2 = 0 & ② \\ 1 + xy\lambda_2 = 1 & ③ \\ x+y+z = A & ④ \\ xyz = B' & ⑤ \end{cases}$$

① − ② ⟹ $(x-y)z\lambda_2 = 0$

For $\lambda_2 \neq 0$, $z \neq 0$ we have $x=y$

$\therefore \quad x = y = -\dfrac{\lambda_1}{\lambda_2 z}$, substitute into ⑤ gives

$$\frac{\lambda_1^2}{\lambda_2^2 z^2} z = B' \Rightarrow \frac{\lambda_1^3}{\lambda_2^2 z} = B'$$

$$\therefore \quad B' z = \frac{\lambda_1^2}{\lambda_2^2} = \text{constant}$$

$\therefore$ Z is indirectly proportional to B',

$\therefore$ the smaller the B', the larger the ~~max~~ maximum value of Z is.

$\because B' = B + \delta \geq B \quad \therefore$ we set $\underline{B' = B}$

(This correspond to the situation where ~~of~~ $\Delta S_{sys} = 0$, same argument applies to both x and y if we wish to maximize them instead of z)

$\therefore$ ④ $\Rightarrow 2x + z = A \Rightarrow z = (700 - 2x)$

sub into ⑤ $\Rightarrow \quad x^2(700 - 2x) = 9000000$

$$\therefore \boxed{x^3 - 350x^2 + 4500000 = 0 \quad ⑥}$$

If we do nothing to the system ~~also~~ Then $x = T_{i1}$, $y = T_{i2}$, $z = T_{i3}$ should satisfy the equation.

$\therefore$ try $x = 300$, ~~x^3~~ we have

~~$x^3 - 350x^2 + 4500000 = 1000000$~~

~~$300^3 - 350$~~ $\quad 300^2 (700 - 2 \times 300) = 9000000$

$\therefore$ ① has a factor $(x - 300)$

$\therefore$ ⑥ $\Rightarrow \quad (x-300)(x^2 - 50x - 15000) = 0$

$\Rightarrow \quad (x-300)(x+100)(x-150) = 0$

~~X=~~ $\therefore \quad x = 300, -100, \text{ or } 150.$

$x = 300$ correspond to no charge $\therefore$ eliminated.

$x = -100 < 0 \quad \therefore$ eliminated

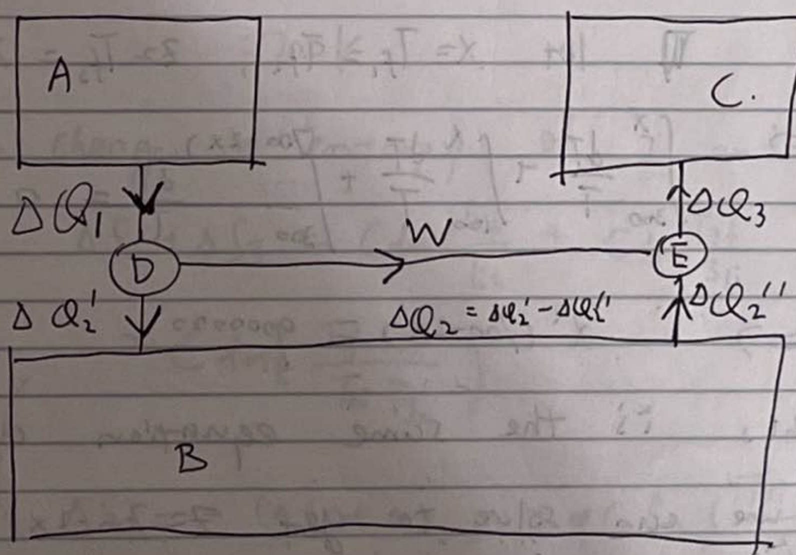
$\therefore$ we have ~~$x = 100$~~ $x = 150$.

$z = 700 - 2 \times 150 = 400\,K$.

Similar arguments can be ~~shown~~ used to show that the theoretical maximum for T_{f_1} and T_{f_2} is also 400 K.
(Just change $f(x,y,z) = z$ to $f = x$ or $f = y$).

Therefore we conclude that theoretically the highest temperature to which any one of ~~the~~ A, B, C in the ~~isolated~~ thermally isolated system can be raised is 400 K

Now we show that this temperature is achievable by heat engines:



Consider Carnot engines D and E

D runs forwards and E runs backwards.

make all the ΔQ s positive.

D extract heat ΔQ_1 and does work W

then thermve heat $\Delta Q_2'$ to B.

E extract heat $\Delta Q_2''$, with the ~~help~~ help

of W it puts heat ΔQ_3 into C.

$\therefore$ ~~ΔQ~~ ~~$W = \Delta Q_1 - \Delta Q_2''$~~ $W = \Delta Q_1 - \Delta Q_2'$

$\Delta Q_3 = W + \Delta Q_2'' = \Delta Q_1 - (\Delta Q_2' - \Delta Q_2'') = \Delta Q_1 - \Delta Q_2$

$\rightarrow$ No more work can be given when temperature of A and B are equal so this is the end of the process

$\rightarrow$ Both engines are carnot engines also ensures

that $\Delta S_{sys} = 0$ ⑦

$\therefore$ let $x = T_{f_1} = T_{f_2}$, $z = T_{f_3} = 700 - 2x$

$$⑦ \Rightarrow \int_{300}^{x} \frac{dT}{T} + \int_{100}^{x} \frac{dT}{T} + \int_{300}^{(700-2x)} \frac{dT}{T} = 0$$

$$\Rightarrow x^2(700-2x) = 9000000$$

this is the same equation as before

so we can solve to get $z = 700 - 2x = 400K$.

$\therefore$ 400K is achievable by the combinations of heat engines.

$\therefore$ The highest temperature to which any one of these bodies can be raised by the operation of heat engines is $\boxed{400K}$

6. $\because$ Bodies 1 and 2 are of the same temperature

$\therefore$ we can increase the temperature of one of them by using the temperature difference between bodies 2 & 3 to drive a heat engine that does work on body ~~a~~ 1 ~~until~~ until bodies 2 & 3 reach thermal equilibrium T_{2f}

As all 3 bodies are identical, conservation of energy gives

$$\cancel{T_{1f}}\ T_{1i} + T_{2i} + T_{3i} = 2T_{2f} + T_{1f} = 300K + 300K + 100K$$

$$= 700K$$

The change in entropy of 2 and 3:

$$\Delta S_2 + \Delta S_3 = C\ln\frac{T_{2f}}{T_{i2}} + C\ln\frac{T_{3f}}{3_{3i}}$$

$$= C\ln\left(\frac{T_{2f}T_{3f}}{T_{2i}T_{3i}}\right)$$

reversible gives the most efficiency

$$\Delta S_1 = -(\Delta S_2 + \Delta S_3) = C\ln\left(\frac{T_{2i}T_{3i}}{T_{2f}^2}\right)$$

Also $\Delta S_1 = C\ln\frac{T_{1f}}{T_{1i}}$

$$\therefore \frac{T_{2i}T_{3i}}{T_{2f}^2} = \frac{T_{1f}}{T_{1i}}$$

$\therefore T_{1i} = T_{2i}$

$$\therefore T_{2f}^2 = \frac{T_{1i}\,T_{2i}\,T_{3i}}{T_{1f}} = \frac{300 \times 300 \times 100}{T_{1f}} = \frac{9 \times 10^6}{T_{1f}}$$

$$\therefore 2\sqrt{\frac{9 \times 10^6}{T_{1f}}} + T_{1f} = 700$$

$$6 \times 10^3 \sqrt{\frac{1}{T_{1f}}} + T_{1f} = 700$$

$$\Rightarrow \boxed{T_{1f} = 400\,K}$$

7. (a)

$$\boxed{\begin{aligned} U &= U \\ H &= U + pV \\ F &= U - TS \\ G &= U - TS + pV \end{aligned}}$$

$$\boxed{dU = TdS - pdV} \quad ①$$

$$dH = dU + pdV + Vdp = TdS - \cancel{pdV} + \cancel{pdV} + Vdp$$

$$= \boxed{TdS + Vdp} \quad ②$$

$$dF = T\cancel{dS} - pdV - \cancel{TdS} - SdT$$

$$= \boxed{-pdV - SdT} \quad ③$$

$$dG = \cancel{d}\, T\cancel{dS} - p\cancel{dV} - T\cancel{dS} - SdT + p\cancel{dV} + Vdp.$$

$$= \boxed{-SdT + Vdp} \quad ④$$

(b)

① $\Rightarrow$ $T = \left(\frac{\partial U}{\partial S}\right)_V$, $-P = \left(\frac{\partial U}{\partial V}\right)_S$

$$\frac{\partial^2 U}{\partial S \partial V} = \frac{\partial^2 U}{\partial V \partial S} \Rightarrow \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V}$$

② $\Rightarrow$ $T = \left(\frac{\partial H}{\partial S}\right)_P$, $V = \left(\frac{\partial H}{\partial P}\right)_S$

$$\frac{\partial^2 H}{\partial S \partial P} = \frac{\partial^2 H}{\partial P \partial S} \Rightarrow \boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$$

③ $\Rightarrow$ $-P = \left(\frac{\partial F}{\partial V}\right)_T$, $-S = \left(\frac{\partial F}{\partial T}\right)_V$

$$\frac{\partial^2 F}{\partial V \partial T} = \frac{\partial^2 F}{\partial T \partial V} \Rightarrow \boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$$

(4) $\Rightarrow$ $-S = \left(\frac{\partial G}{\partial T}\right)_p$, $V = \left(\frac{\partial G}{\partial p}\right)_T$

$$\frac{\partial^2 G}{\partial T \partial p} = \frac{\partial^2 G}{\partial p \partial T} \Rightarrow \boxed{-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p}$$

8 (i) $\left(\frac{\partial T}{\partial V}\right)_U \left(\frac{\partial U}{\partial T}\right)_V = -\left(\frac{\partial U}{\partial V}\right)_T$

$\cancel{\left(\frac{\partial U}{\partial V}\right)_T}$ $dU = T\,dS - p\,dV$

$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$

Maxwell relation $\Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$

$\therefore \left(\frac{\partial T}{\partial V}\right)_U \left(\frac{\partial U}{\partial T}\right)_V = -\left(\frac{\partial U}{\partial V}\right)_T = -\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]$

$\because C_V = \left(\frac{\partial U}{\partial T}\right)_V$

$\left(\frac{\partial T}{\partial V}\right)_U C_V = -\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]$

$\therefore \left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]$

(ii) $\cancel{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T}$

cyclic

$\cancel{= -\left(\frac{\partial T}{\partial S}\right)_V \left(-\left(\frac{\partial U}{\partial S}\right)_V\right) \left(\frac{\partial S}{\partial V}\right)_T}$

$\cancel{= -\left(\frac{\partial T}{\partial U}\right)_V \left(\frac{\partial U}{\partial S}\right)_V}$

(ii) $$\left(\frac{\partial T}{\partial V}\right)_S \underbrace{=}_{\text{cyclic}} -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

$$= -\left(\frac{\partial T}{\partial U}\right)_V \left(\frac{\partial U}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

$$= -\frac{1}{(\partial U/\partial T)_V} \left(\frac{\partial U}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V .$$

$$dU = T\,dS - p\,dV \Rightarrow \left(\frac{\partial U}{\partial S}\right)_V = T.$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$\therefore \left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V} T \left(\frac{\partial P}{\partial T}\right)_V$$

8 (ii) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$

$\because C_V = \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$\therefore \frac{1}{C_V} = \frac{1}{T}\left(\frac{\partial T}{\partial S}\right)_V$

$\therefore \left(\frac{\partial T}{\partial S}\right)_V = \frac{T}{C_V}$

$\therefore \left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V}\left(\frac{\partial S}{\partial V}\right)_T = -\frac{T}{C_V}\left(\frac{\partial P}{\partial T}\right)_V$

maxwell relation

(iii) $dH = TdS + Vdp$

$$\therefore \left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial p}\right)_T + V$$

$$-\left(\frac{\partial H}{\partial p}\right)_T = -T\left[-\left(\frac{\partial V}{\partial T}\right)_p\right] - V$$

maxwell relation

$$\therefore -\left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial V}{\partial T}\right)_p - V$$

$$\left(\frac{\partial T}{\partial p}\right)_H \left(\frac{\partial H}{\partial T}\right)_p = \cancel{T\frac{\partial V}{\partial}}\ T\left(\frac{\partial V}{\partial T}\right)_p - V$$

$dU = đQ + đW = đQ - pdV \quad \therefore đQ = dU + pdV$

$\cancel{C_p =}$

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

$$= \left(\frac{\partial U}{\partial T}\right)_p + \left[\frac{\partial (pV)}{\partial T}\right]_p = \left[\frac{\partial (U+pV)}{\partial T}\right]_p$$

$$= \left(\frac{\partial H}{\partial T}\right)_p$$

$$\therefore \left(\frac{\partial T}{\partial p}\right)_H C_p = T\left(\frac{\partial V}{\partial T}\right)_p - V$$

$$\therefore \left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$$

(b) $PV = nRT$ for an ideal gas

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{1}{C_V}\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]$$

$$= -\frac{1}{C_V}\left[\underbrace{T\left[\frac{nR}{V}\right] - \frac{nRT}{V}}_{0}\right] = 0$$

$$\cancel{\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V}}$$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

$$= \frac{1}{C_P}\left[\underbrace{T\left(\frac{nR}{P}\right) - \frac{nRT}{P}}_{0}\right] = 0$$

$$\cancel{\left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V}P} \quad \left(\frac{\partial T}{\partial V}\right)_S = -\frac{1}{C_V}T\left(\frac{\partial P}{\partial T}\right)_V$$

$$= \frac{-1}{C_V}T\frac{nR}{V}$$

Along constant S (adiabatic)

$$\frac{dT}{dV} = -\frac{1}{C_V}T\frac{nR}{V} \Rightarrow \int_{T_1}^{T_2}\frac{dT}{T} = -\frac{nR}{C_V}\int_{V_1}^{V_2}\frac{dV}{V}$$

$\left(\begin{matrix} nR = C_p - C_V \\ \gamma = \frac{C_P}{C_V}\end{matrix}\right)$

$$\Rightarrow \ln\frac{T_2}{T_1} = -\frac{C_p - C_V}{C_V}\ln\frac{V_2}{V_1}.$$

$$\ln\frac{T_2}{T_1} = (1-\gamma)\ln\left(\frac{V_2}{V_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_2^{1-\gamma}}{V_1^{1-\gamma}}\right).$$

$$\therefore T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad \because T_1 \propto P_1V_1,\ T_2 \propto P_2V_2$$

$$\therefore P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow PV^{\gamma} = \text{const}$$

9. $C_V = \left(\frac{\partial U}{\partial T}\right)_V$, $C_P = \left(\frac{\partial U}{\partial T}\right)_V + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\left(\frac{\partial V}{\partial T}\right)_P$

$$\therefore C_P - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\left(\frac{\partial V}{\partial T}\right)_P$$

$$V\beta_b = V\left(\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right) = \left(\frac{\partial V}{\partial T}\right)_P$$

$$\frac{C_P - C_V}{V\beta_b} = \frac{\left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\cancel{\left(\frac{\partial V}{\partial T}\right)_P}}{\cancel{\left(\frac{\partial V}{\partial T}\right)_P}} = \left(\frac{\partial U}{\partial V}\right)_T + P$$

$$\therefore \frac{C_P - C_V}{V\beta_b} - P = \left(\frac{\partial U}{\partial V}\right)_T \cancel{+P} \cancel{-P} = \left(\frac{\partial U}{\partial V}\right)_T$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = \frac{C_P - C_V}{V\beta_b} - P$$

✓

10. (a)

tension f is proportional to the temperature T if the length L is held constant.

$\therefore f(L,T) = K(L)\,T$ where $K(L)$ is an arbitrary function of L

$dU = TdS + f\,dL$

write $F = U - TS$

then $dF = dU - TdS - SdT = f\,dL - S\,dT$

$\therefore f = \left(\frac{\partial F}{\partial L}\right)_T \quad , \quad -S = \left(\frac{\partial F}{\partial T}\right)_L$

second order derivatives commute:

$$\left(\frac{\partial f}{\partial T}\right)_L = -\left(\frac{\partial S}{\partial L}\right)_T = K(L)$$

$$\left(\frac{\partial U}{\partial L}\right)_T = T\left(\frac{\partial S}{\partial L}\right)_T + f$$

$$= T(-\cancel{K(L)}) + T\cancel{K(L)}$$

$$= 0$$

$$dU = \left(\frac{\partial U}{\partial L}\right)_T dL + \left(\frac{\partial U}{\partial T}\right)_L dT = \underbrace{\left(\frac{\partial U}{\partial T}\right)_L}_{C_L} dT$$

$\therefore U(T, L) = U(T)$

U is ~~only~~ a function only of temperature.

only change in T can result in change in U

(c) $\left(\frac{\partial L}{\partial T}\right)_f \left(\frac{\partial f}{\partial L}\right)_T \left(\frac{\partial T}{\partial f}\right)_L = -1$ ⓪

① ② ③

② ⇒ $\because \left(\frac{\partial L}{\partial f}\right)_T > 0$ and $\left(\frac{\partial L}{\partial f}\right)_T \left(\frac{\partial f}{\partial L}\right)_T = 1$

$\therefore \left(\frac{\partial f}{\partial L}\right)_T > 0$

③ ⇒ $f = K(L)\,T$

$\therefore \left(\frac{\partial f}{\partial T}\right)_L = K(L)$

$\because f > 0,\ T > 0 \quad \therefore K(L) > 0$

$\therefore \left(\frac{\partial f}{\partial T}\right)_L > 0$

$\because \left(\frac{\partial f}{\partial T}\right)_L \left(\frac{\partial T}{\partial f}\right)_L = 1 \quad \therefore \left(\frac{\partial T}{\partial f}\right)_L > 0$

then ⓪ gives $\boxed{\left(\frac{\partial L}{\partial T}\right)_f < 0}$

$\therefore$ The band will contract if warmed while kept under constant tension

(d) $dU = TdS + fdL$ adiabatic ⇒ $dS = 0$

$\therefore dU = fdL$ Also $\because$ ~~$dU = C_L dT$~~ U is a function only of T

$\therefore$ We can define $dU = C_L dT$ where C_L is the heat capacity and $C_L > 0$

$\therefore C_L dT = fdL \Rightarrow \frac{dT}{dL} = \frac{f}{C_L} > 0$

$\therefore$ Adiabatic stretching results in increase in Temperature

11. $dU = TdS + \gamma dA$

define $F = U - TS$

$$dF = dU - TdS - SdT = \cancel{TdS} + \gamma dA - \cancel{TdS} - SdT$$

$$= -SdT + \gamma dA$$

$$-S = \left(\frac{\partial F}{\partial T}\right)_A \qquad \gamma = \left(\frac{\partial F}{\partial A}\right)_T$$

$$\frac{\partial^2 F}{\partial A \partial T} = \frac{\partial^2 F}{\partial T \partial A} \Rightarrow -\left(\frac{\partial S}{\partial A}\right)_T = \left(\frac{\partial \gamma}{\partial T}\right)_A$$

$$\Rightarrow \left(\frac{\partial S}{\partial A}\right)_T = -\left(\frac{\partial \gamma}{\partial T}\right)_A = -\frac{(70-75)\times 10^{-3}}{35^\circ C - 5^\circ C} = \boxed{1.67 \times 10^{-3}\ JK^{-1}m^{-2}}$$

$$dU = TdS + \gamma dA$$

$$\left(\frac{\partial U}{\partial A}\right)_T = T\left(\frac{\partial S}{\partial A}\right)_T + \gamma = \gamma - T\left(\frac{\partial \gamma}{\partial T}\right)_A$$

$$= (278)(1.67 \times 10^{-3}) + (75 \times 10^{-3})$$

$$= \boxed{121.4 \times 10^{-3}\ Jm^{-2}}$$

radius of small drop

$$r = \frac{0.1}{2}\mu m = 0.05 \mu m$$

weight of small drop

$$m = \frac{4}{3}\pi r^3 \rho = \left(\frac{4}{3}\pi\right)(0.05 \times 10^{-6})^3 (1.0 \times 10^3)$$

$$= 5.236 \times 10^{-19}\ kg$$

weight of big drop

$$M = 1.0 \times 10^{-3}\ kg$$

$\therefore$ ~~Nm~~ Number of drops $N = \frac{M}{m} = 1.91 \times 10^{15}$

Surface area of small drop

$$a = 4\pi r^2 = 3.14 \times 10^{-14}\ m^2$$

Surface area of big drop

$$A = 4\pi R^2 = 4\pi \left[\frac{3M}{4\pi\rho}\right]^{2/3}$$

$$= 4.836 \times 10^{-4}\ m^2.$$

$\therefore$ Change in total area is

$$\Delta A = \cancel{Na} \; A - Na = -59.97\ m^2$$

$$\cancel{\Delta Q_{surface} = \left(\frac{\partial Q}{\partial A}\right)_T \Delta A = -T\left(\frac{\partial S}{\partial A}\right)_T \Delta A}$$

~~No heat exchange with surroundings~~

$$\cancel{\Rightarrow \Delta Q_{surface} + \Delta Q_{bulk} = 0}$$

$$\cancel{\therefore \Delta Q_{bulk} = -T\left(\frac{\partial S}{\partial A}\right)_T \Delta A = T\left(\frac{\partial \gamma}{\partial T}\right)_A \Delta A}$$

$$\cancel{= (35+273)(0.167 \times 10^{-3})(59.97)}$$

$$\cancel{= 3.085\ J.}$$

$$\cancel{\Delta T = \frac{\Delta Q_{bulk}}{C_p m} = \frac{3.085}{(4200)(10^{-3})} =}$$

Lost in surface energy

$$\Delta E = \left(\frac{\partial U}{\partial A}\right)_T \Delta A = (121.3 \times 10^{-3})(-59.97)$$

$$= -7.274\ J$$

This goes into the bulk of water drop as heat $\Rightarrow$ $\Delta Q = -\Delta E = 7.274\ J.$

$$\Delta T = \frac{\Delta Q}{m c_p} = \frac{7.274}{(4200)(10^{-3})} = \boxed{1.73\,K}$$

change in entropy:

$$\Delta S = \Delta S_{drop} + \Delta S_{surface}$$

$$= \frac{\Delta Q}{T} + \left(\frac{\partial S}{\partial A}\right)_T \Delta A$$

$$= \frac{7.274}{308} + (0.167 \times 10^{-3})(-59.97)$$

$$= \boxed{13.6 \times 10^{-3}\,J K^{-1}}$$

OR $$\Delta S = \int_{308}^{308+1.78} \frac{C_p m \, dT}{T} + \left(\frac{\partial S}{\partial A}\right)_T dA$$

$$= \boxed{13.5 \times 10^{-3} \, JK^{-1}}$$

mass doesn't change, (volume doesn't change approximately.)

12. $c_B = \frac{C_B'}{V}$, $c_M = \frac{C_m'}{V}$

$$\cancel{S} \quad C_B' = T\left(\frac{\partial S}{\partial T}\right)_B \qquad C_m' = T\left(\frac{\partial S}{\partial T}\right)_m$$

$$S = S(T, B) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_B dT + \left(\frac{\partial S}{\partial B}\right)_T dB$$

$$\left(\frac{\partial S}{\partial T}\right)_m = \left(\frac{\partial S}{\partial T}\right)_B + \left(\frac{\partial S}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m$$

$$\therefore \frac{C_B'}{T} - \frac{C_m'}{T} = -\left(\frac{\partial S}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m$$

Maxwell relation : $\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B = -\left(\frac{\partial m}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m$

$$\therefore \cancel{\frac{\partial B}{\partial T}} \quad \frac{C_B'}{T} - \frac{C_m'}{T} = \left(\frac{\partial m}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m^2$$

$$dU = TdS - m\,dB$$

$$\uparrow \; P \qquad \uparrow \; dV$$

Maxwell relation $\left[\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V\right] \Rightarrow \left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B = -\left(\frac{\partial m}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m$

$$\frac{C_B' - C_m'}{T} = \left(\frac{\partial m}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_m^2$$

$$\because M = \frac{\chi B}{\mu_0} = \frac{CB}{\mu_0 T} \Rightarrow m = MV = \frac{CBV}{\mu_0 T}$$

$$\Rightarrow B = \frac{m\mu_0}{CV} T$$

$$\therefore \cancel{\frac{\partial B}{\partial}} \left(\frac{\partial m}{\partial B}\right)_T = \frac{CV}{\mu_0 T}, \quad \left(\frac{\partial B}{\partial T}\right)_m = \frac{m\mu_0}{CV} = \frac{B}{T}$$

$$\frac{C_B' - C_m'}{T} = \left(\frac{cV}{\mu_0 T}\right)\left(\frac{B^2}{T^2}\right) = \frac{cB^2 V}{\mu_0 T^3}$$

$$\therefore \frac{C_B - C_m}{T} = \frac{cB^2}{\mu_0 T^3} \Rightarrow C_B - C_m = \frac{cB^2}{\mu_0 T^2}$$

$$\because C_m = \frac{a}{T^2}$$

$$\therefore C_B = \frac{a}{T^2} + \frac{cB^2}{\mu_0 T^2}$$

$$\therefore \boxed{C_B = \frac{a}{T^2}\left(1 + \frac{B^2 c}{\mu_0 a}\right)}$$

If a sample is initially at temperature T_1 in an applied field of flux density B_1

$$\because \left(\frac{\partial T}{\partial B}\right)_S = -\frac{TVB}{\mu_0 C_B'}\left(\frac{\partial \chi}{\partial T}\right)_B \quad \because C_B = \frac{C_B'}{V}, \ \chi = \frac{c}{T}$$

$$\therefore \left(\frac{\partial T}{\partial B}\right)_S = -\frac{TB}{\mu_0 C_B} \cdot c \cdot -\frac{1}{T^2} = \frac{Bc}{\mu_0 C_B T}$$

Adiabatic process $\Rightarrow$ ~~Sis~~ S is constant

$$\therefore \frac{dT}{dB} = \frac{Bc}{\mu_0 C_B T} = \frac{Bc}{\mu_0 T\left(\frac{a}{T^2}\right)\left(1+\frac{B^2 c}{\mu_0 a}\right)}$$

$$\int_{T_1}^{T_2} \frac{1}{T}\, dT = \int_{B_1}^{0} \frac{c}{\mu_0 a} \cdot \frac{B\, dB}{1 + \frac{B^2 c}{\mu_0 a}}$$

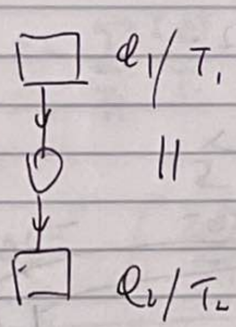
$$\Rightarrow \ln\frac{T_2}{T_1} = -\frac{1}{2}\ln\left(1 + \frac{B_1^2 c}{\mu_0 a}\right) + \frac{1}{2}\underbrace{\ln 1}_{0}$$

$$\therefore \frac{T_2}{T_1} = \left(1 + \frac{B_1^2 c}{\mu_0 a}\right)^{-\frac{1}{2}}$$

$$\therefore \boxed{T_2 = \frac{T_1}{\left(1 + \frac{B_1^2 c}{\mu_0 a}\right)^{1/2}}}$$

2l. reversible?

3(a)



reversible.

infinitely many bath.

$$dS = \frac{dQ}{T} = \frac{CdT}{T}$$

cooling with Carnot engine

3(b)

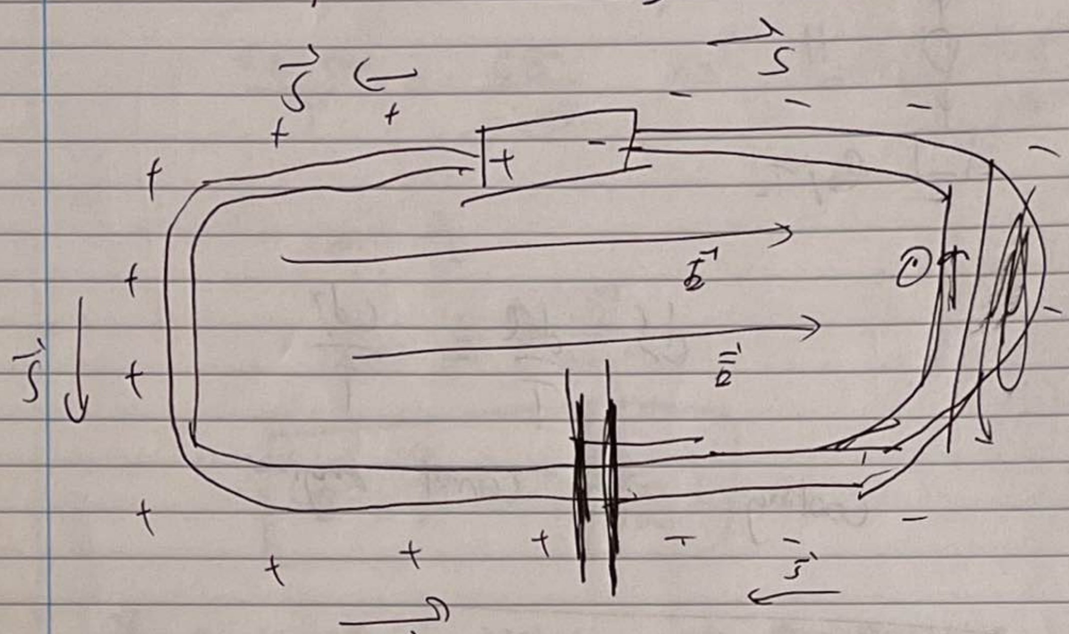
wave has ~~kinetic~~ potential energy

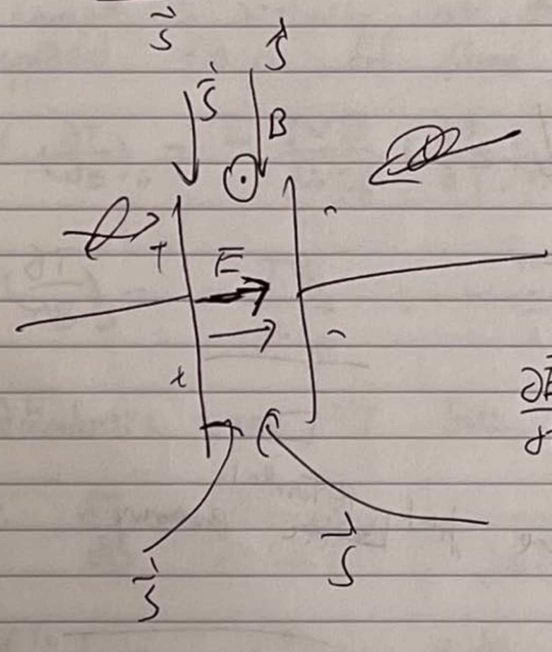
false seems same

3(c) more and more bath, more and more Q more and more quasi-static, more and more reversible.

(3) (d)

dissipate $CV^2 - \frac{1}{2}CV^2$.





$\frac{\partial \vec{E}}{\partial t}$ has same effect as current in generating $\vec{B}$ field.

poynting vector points in from the sides

Block $\Delta S = -\sum_{T_i} \frac{C\delta T}{T_i} = -\int \frac{CdT}{T}$

Bath $\Delta S = \int \frac{CdT}{T}$

---

6. to ~~use the~~ ~~get~~ the ~~most effic~~ highest.

Temperature we use most effisiont

engine => $\Delta S_{system} = 0$.

13.

P, V, T — extensive

$(f, L, T$ — intensive

$$dU = TdS + fdL$$

$$\left(\frac{\partial f}{\partial S}\right)_L = \left(\frac{\partial T}{\partial L}\right)_S$$

$$dU = fdL + TdS$$

$$dU = \left(\frac{\partial U}{\partial L}\right)_S dL + \left(\frac{\partial U}{\partial S}\right)_L dS$$

$$\left(\frac{\partial f}{\partial S}\right)_L = \left(\frac{\partial T}{\partial L}\right)_S.$$

$$dF = -SdT + fdL.$$

$$\left(\frac{\partial f}{\partial T}\right)_L = -\left(\frac{\partial S}{\partial L}\right)_T.$$

$$U = U(f, L, T, S).$$

$$L = L(f, S).$$

$$f = (L, S).$$

$$S = (L, T).$$

$$\left(\frac{\partial T}{\partial L}\right)_S > 0$$

$$dU = f\,dL$$

$$\left(\frac{\partial U}{\partial L}\right)_S = \cancel{f}\ \frac{\partial}{\partial L}(f\,dL) = f\left(\frac{\partial L}{\partial L}\right)_S = f > 0$$

$$\left(\frac{dU}{dL}\right)_S = c_L\left(\frac{\partial T}{\partial L}\right)_S = \cancel{a}$$

$$dU = f\,dL$$

$$dU = c_L\,dT. \qquad c_L\,dT = f\,dL.$$

w 11.

$$dU = TdS + \gamma dA$$

$$dU = \left(\frac{\partial U}{\partial A}\right)_T dA + \left(\frac{\partial U}{\partial T}\right)_A dT$$

T

C_A.

$$dU = 0$$

$$\left(\frac{\partial U}{\partial A}\right)_T dA + C_A dT = 0$$

$$\boxed{\left(\frac{\partial U}{\partial A}\right)_T \Delta A + C_A \Delta T = 0}$$

$\cancel{C_A}$ $dA = 0 \Rightarrow dU = dQ$

$$\left(\frac{\partial U}{\partial T}\right)_A = \left(\frac{\partial Q}{\partial T}\right)_A = C_A$$

$dU = TdS + \cancel{B}Bdm$

$C_V = \frac{a}{T^2}$.