Second Year Physics: Optics Problems

P. Ewart

Questions in shaded boxes are designed to cover the basic "bookwork" on the main topics in the course. Thorough understanding of these "simpler" questions should help in tackling the other questions which apply these principles and are at the level of Finals Part A.

1. Two long slits of negligible width, separated by a distance d are illuminated by monochromatic light of wavelength λ from a point source at a distance s on an axis perpendicular to the plane of the slits and mid-way between the slits (Figure 1(a)).

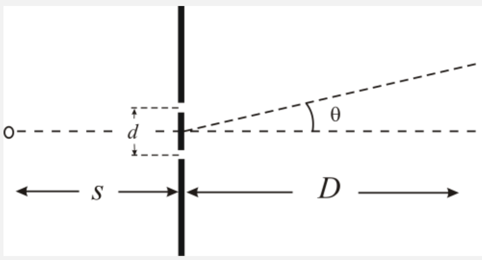


Figure 1(a)

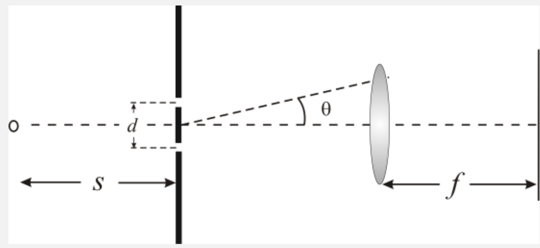


Figure 1(b)

By assuming an amplitude u_o of light emanating from each slit, calculate the *intensity* as a function of the angle θ to the axis at a distance D on the opposite side of the slits from the source when $D >> d$,

(a) using the analytical method, and

(b) by the method of phasors.

(c) What is the angular separation $\Delta\theta$ between the central maximum and the first maximum on either side?

A large aperture lens of focal length f, on the axis, is used to form an image of the fringes on a screen placed at a distance f from the lens. (Figure 1(b))

(d) Derive an expression for y, the separation on the screen of the first fringe maximum from the central fringe maximum.

(e) Make a sketch of the interference pattern seen on the screen and indicate quantitatively the positions of the principle maxima of the pattern.

(f) One of the slits is now covered with a thin attenuating film to reduce the amplitude to one half that emitted by the uncovered slit. Use either the analytical or phasor method to find the angular distribution of the intensity in this case. Sketch the form of the interference pattern seen on the screen in this case, on the same axes as for part (e), indicating quantitatively the intensity variation in the pattern.

(g) The film is now replaced by a 0.5 mm thick glass plate of refractive index 1.5 on the input side of the slit. Explain quantitatively any change that may occur in the fringe pattern.

(h) Explain what would happen to the pattern, when both slits are uncovered, if the source is moved a distance x perpendicularly from the axis in the plane of the diagram above.

(i) The original system, with unobstructed slits, is set up with a source emitting light of wavelength 500 nm at s = 20 mm from the slits which are separated by d = 0.5 mm. Explain what would be observed on the screen if a second, independent i.e. incoherent, source of equal intensity was placed beside the original source at a distance x off axis and x is gradually increased from zero perpendicularly to the axis. What will be the minimum value of x at which fringes can no longer be observed? Estimate the maximum size for a single source, of incoherent emitters, if fringes are to be visible.

2. (a) Explain what is meant by Fraunhofer diffraction. [Your answer should contain no reference to parallel light or observation at infinity.] Why is it of such great practical importance?

Plane waves of monochromatic light of wavelength λ fall on a slit of width *a:*

(b) Estimate the minimum distance from the slit of the plane of observation if the observed diffraction pattern is to fulfil the Fraunhofer condition.

(c) Derive an expression for the angular distribution of the intensity in the Fraunhofer pattern, and sketch it, indicating the relative distances from the centre to any mimina in intensity.

3. Calculate the size (i.e. *the separation of the first minima on either side of the central maximum*) of the Fraunhofer diffraction pattern created by collimated monochromatic light of wavelength 600 nm transmitted by an infinitely long slit of width 250 microns in the focal plane of a lens of focal length 500 mm.

4. The width of a slit, roughly 100 microns, is to be measured by studying its Fraunhofer diffraction pattern. Describe how you would carry out this measurement, including in your account reasoned order of magnitude estimates. Mention also all of the important experimental components, and how they would be used.

5. The single most important result in physical optics is that a collimated beam of light, of width w, has an angular divergence $\sim\lambda/w$ or greater. Use this result to estimate approximately the size of the smallest patch of light one can get on the moon when a beam of diameter ~ 20 mm from a ruby laser is pointed at it in lunar ranging experiments. Find the new minimum size when the laser beam is expanded so as to fill the aperture of a Newtonian telescope having a mirror diameter of 3 m.

[*The wavelength of the light from a ruby laser may be taken to be 700 nm and the distance from the earth to the moon to be 390,000 km.*]

6. Viewed through a thin layer of cloud, the Moon sometimes appears to be covered by a translucent disk of light of angular diameter 2^{o}. The disk is white at its centre but reddish at its periphery. Explain this phenomenon and estimate the size of water droplets in the cloud.

7. The Fresnel-Kirchoff diffraction integral is,

$$u_p = -\frac{i}{\lambda}\int_S \frac{u_o dS}{r}\eta(\underline{n},\underline{r})\exp(ikr)$$

where u_p is the amplitude of the diffracted wave at position $\underline{r}$ from the centre of a diffracting aperture of area S, in which the amplitude is u_o per unit area and the other symbols have their usual meanings. Explain the physical significance of the factor $1/r$ in the integral and the quantity $\eta(\underline{n},\underline{r})$. Using this integral show that the angular distribution of the Fraunhofer diffraction pattern of a single long narrow slit, width a, may be represented by the Fourier transform of the amplitude distribution $u(x)$ in the aperture, making clear any assumptions and approximations required. x is the dimension transverse to the long axis of the slit.

8. The amplitude distribution in a slit of width a in the transverse, x, direction may be represented by the top-hat function:

$$v(x) = 1 \text{ for } |x| < \frac{a}{2} \text{ and } v(x) = 0 \text{ for } |x| > \frac{a}{2}$$

Find an expression for the Fraunhofer diffraction pattern of such a single slit.

9. State the Convolution Theorem.

Describe quantitatively how an aperture function $u(x)$ representing the amplitude transmission of two long slits of width, a, parallel to the y-axis and centred at positions $x = \pm d/2$, in the x-direction and where $d > a$, may be represented as a convolution of two simpler functions.

Use the Convolution Theorem to derive an expression for the Fraunhofer diffraction pattern of such a pair of slits illuminated by a plane monochromatic wave of wavelength λ.

10. An aperture lies in the plane $z = 0$ and has amplitude transmission function $T(y)$ independent of x. It is illuminated by coherent, monochromatic light of wavelength λ at normal incidence. Show that, in the Fraunhofer case, the diffracted intensity is proportional to $|A(\theta)|^2$, where

$$A(\theta) = \int_{-\infty}^{+\infty} T(y)\exp[-iky\sin\theta]dy$$

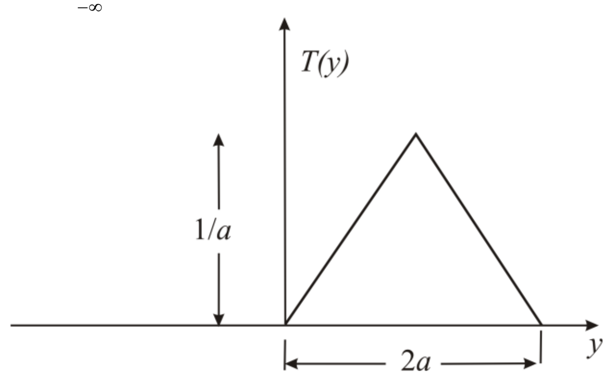


Figure 2 (a)

Figure 2 (a) shows a particular amplitude transmission function. Calculate and sketch the diffracted intensity as a function of sin θ.

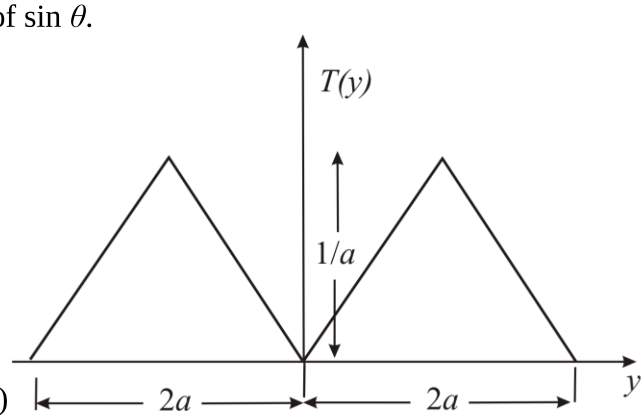


Figure 2 (b)

Figure 2 (b) shows another transmission function. Calculate and sketch the diffracted intensity as a function of sin θ. Comment on the location of the first zero of the diffracted intensity with regard to the spatial-frequency content of $T(y)$.

11. A monochromatic plane wave of wavelength λ is incident normally on a screen which transmits a fraction $\phi(x)$ of the amplitude incident upon it, where x is measured across the screen and $\phi(x)$ is given by the Gaussian function

$$\phi(x) = \exp\left[\frac{-x^2}{2d^2}\right]$$

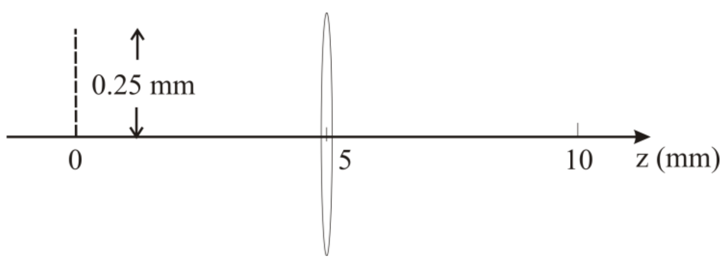
Where d is a constant. Sketch this function.

A lens of focal length f is placed behind the screen. Derive an expression for the intensity distribution in the focal plane of the lens. [*The point being that you get another Gaussian. This is because the amplitude distribution in the Fraunhofer pattern is the Fourier Transform, FT, of the transmission function of the screen, and the FT of a Gaussian is another Gaussian. Taking the modulus squared still keeps the Gaussian form. But of course you don't need to use FT's explicitly to do the question.*]

Explain briefly why the use of a screen of this type can be helpful in practice. Suppose the screen to be moved to a position beyond the lens, halfway between the lens and its focal plane. What would the intensity distribution then be in the focal plane? Would the pattern still be correctly described as Fraunhofer?

$$\left[\int_{-\infty}^{\infty} \exp(-\alpha x^2 + i\beta x)dx = \left(\frac{\pi}{\alpha}\right)^{1/2} \exp\left(\frac{-\beta^2}{4\alpha}\right)\right]$$

12.



A small transmission diffraction grating has width w = 0.25 mm and is composed of 16 narrow slits whose centres are spaced by 15 μm. It is situated in the z = 0 plane with its lines oriented along the y-direction, where the z-axis is defined to lie along the optical axis of a thin lens of focal length f = 2.5 mm and diameter 2w positioned at z = 2f. The grating is parallel to the lens but off-axis, extending from x = 0 to x = w. The grating is uniformly illuminated by a parallel beam of monochromatic light of wavelength 750nm propagating in the positive z-direction (the beam has width w and just fills the grating). Find the angle to the normal of the first-order diffracted beam as it leaves the grating, and draw a ray diagram showing rays leaving the edges of the grating in the zeroth and first-order diffraction directions and propagating through the lens to z = 10 mm.

Sketch on a large diagram the light intensity distribution in the plane at z = 7.5 mm, indicating the locations of the principal maxima and the separations of these maxima from the first adjacent minimum. Sketch on a large diagram the light intensity at the plane z = 10 mm.

Explain briefly how this example gives a useful insight into the resolving power of a microscope with coherent illumination.

13. A laser produces a beam of coherent light of wavelength λ with plane wave-fronts travelling along *z*. The laser beam is passed through a transparency whose amplitude transmission coefficient in the *x,y* plane is independent of *y* and varies with *x* as $A[1 + \cos(2\pi x/d)]$ where *A* is a constant and $d > \lambda$. Show that three beams emerge from the transparency, and find their angles to the *z*-axis.

An ordinary transmission diffraction grating illuminated normally by plane monochromatic light of wavelength 465 nm is examined under a microscope whose objective subtends an angle of 0.5 radians at the grating. What is the smallest grating spacing that can be seen in the image?

14. An opaque plane has two long, narrow and parallel slits of negligible width spaced by a distance *d*. The slits are illuminated by a plane wave of monochromatic light of wavelength λ.

(a) Make a quantitative sketch of the form of the *intensity* diffraction pattern observed on a screen at distance *D* from the plane of the slits where $D >> d$, i.e. label the angular separation of the fringe maxima or minima.
(b) On the same scale, sketch the pattern obtained if two extra slits are added, one on each side of the original pair of slits at spaced at the same distance, *d*.
(c) Explain what happens to the general features of these patterns as more slits are added (keeping the same values of *d* and *a* in each case).
(d) In the case of the four slit arrangement, the central two slits are covered by a film transmitting 25% of the incident intensity. By using phasors, or otherwise, estimate the change in intensity distribution and sketch the new intensity distribution on the screen.

15. A grating with *N* slits of spacing *d* is illuminated normally with monochromatic light of wavelength λ , and the p^{th} order principal maximum is observed at an angle θ_p.

(a) Show that the intensity at angle θ is given by

$$I(\delta) = \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \qquad \text{where } \delta = \frac{2\pi}{\lambda} d \sin\theta$$

Hence show that adjacent minima fall at angles $\theta_P \pm \delta\theta_P$ where

$$\delta\theta_p = \frac{\lambda}{Nd\cos\theta_p}$$

(b) Show that the dispersion $\frac{\partial\theta}{\partial\lambda}$ of the grating in the p^{th} order of interference is given by

$$\frac{d\theta}{d\lambda} = \frac{p}{d\cos\theta}.$$

(c) Hence show that the instrumental width of the grating (in terms of wavelength) is given by

$$\Delta\lambda_{INST} = \frac{\lambda}{Np}$$

(d) Deduce that the resolving power of the grating is *Np*.

Does the resolving power of a grating change if it is used in a medium of high refractive index?

16. A pair of slits, each of width a, and separated centre-to-centre by a distance d is illuminated normally by a plane monochromatic wave. Light transmitted by the slits is observed in the focal plane of a lens of focal length f. Draw sketch graphs showing the variation of the observed intensity in the focal plane of the lens for the cases (i) $d \gg a$ (ii) $d = 3a$ (iii) $d = 2a$. In each case your sketch graph should show quantitatively the scales being used on the co-ordinate axes.

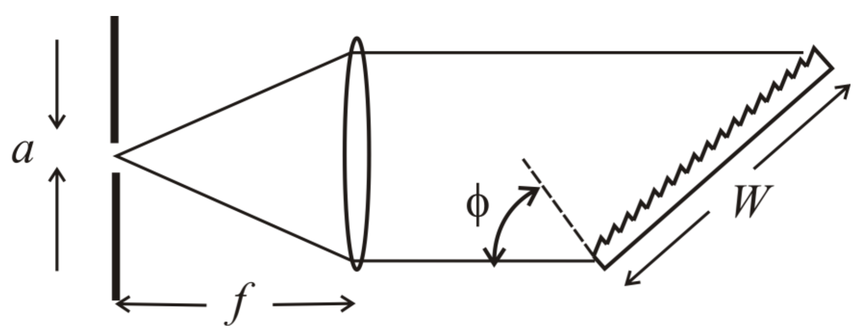
17. A diffraction grating has slits ruled on opaque material and has 20 slits per mm. The distance between the slit centres is twice the width of each slit. The grating is uniformly illuminated at normal incidence. The transmitted light passes through a lens of focal length 1m and then onto a screen that lies 1m from the lens. On the screen the first-order diffraction peak lies 10mm from the centre-line of the apparatus. Calculate the wavelength of the light and predict the location of the next visible diffraction peak.

18. A collimated beam from a white-light source is incident normally on a transmission grating with 500 lines per mm. The transmitted light then passes through a lens which is used to project the visible (380–780 nm) spectrum of the light source on to a strip of photographic film and to just cover its length of 35 mm. Calculate the focal length of the lens.

19. A plane wave of light of wavelength λ is incident at angle ϕ on a reflection grating of width W having δ lines per unit length. Derive an expression for the intensity of the reflected light as a function of the diffraction angle θ. Show that a maximum intensity is obtained at an angle θ given by

$$\theta = \sin^{-1}(\lambda\delta \pm \sin\phi)$$

Find an expression for the minimum wavelength difference $\Delta\lambda$ that may be resolved with this arrangement.



Such a reflection grating is used in an optical arrangement shown in the diagram and is similar to that used to select the operating wavelength of a laser. Light from a source is incident on the slit of width a, is collimated by the lens of focal length f and falls on the reflection grating at angle ϕ. The grating has 2400 lines mm^{-1} and a total width W of 50 mm.

(a) At what value of ϕ will a maximum intensity of light of wavelength 600nm from the source return through the slit?
(b) What factors influence the choice of slit width a to obtain the narrowest possible range of wavelengths returning to the light source?
(c) Estimate the optimum value of a when the lens has a focal length of 200 mm.

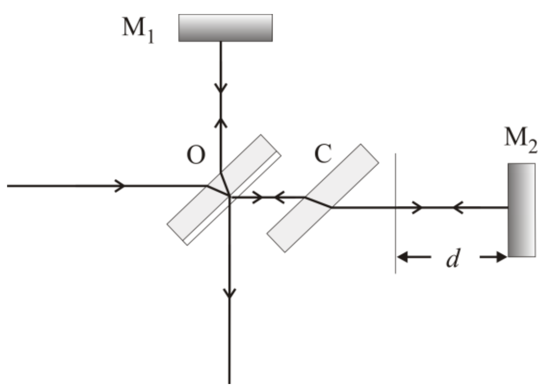
20. Give a brief account of the principal features of a Michelson interferometer used with visual observation and illuminated by an extended monochromatic source. Explain what configurations of the mirrors will give (i) circular fringes (ii) straight, equally spaced fringes. State where the fringes are localized and why. Why in case (ii) does the mirror spacing have to be small?

21. A Michelson interferometer is set up to give circular fringes when illuminated by light of wavelength λ.
(a)What will be the order of interference, p_o, for the fringe on the axis when the difference in the lengths of the arms of the interferometer is t?
(b) What condition must be fulfilled to give a bright circular fringe at angle α to the axis when the interferometer is illuminated by an extended source of wavelength λ.
(c) Explain why the order of interference p of a fringe at angle α to the axis is less than p_o.
(d) Derive an expression for the radius, r_p, of the p^{th}-order fringe formed in the focal plane of a lens of focal length f. Hence show that the difference in radii of the p^{th}–order and $(p+1)^{th}$–order fringes is given by

$$r_p^2 - r_{p+1}^2 = \frac{f^2\lambda}{t}$$

(e) The path length of the arm containing the moving mirror is initially larger than the path in the arm containing the fixed mirror. The path difference t is slowly decreased towards the zero path difference condition. Describe what happens to the fringes as the path difference decreases and then increases as the moving mirror moves past the position of zero path difference.

22.



The figure shows the elements of a Michelson interferometer: O, a half-silvered glass plate; C, a glass plate; M_1, a fixed mirror; and M_2, an adjustable mirror which can be moved towards or away from O. Also shown are paths of rays in a collimated beam from a coherent, monochromatic light source of wavelength λ = 500 nm. Explain the functions of elements O and C. Explain why interference fringes are formed and describe how they might be observed.
Mirror M_2 is initially set at a distance d= 0·1m from the image (in the half-silvered mirror) of M_1 and accurately parallel to it. Its position is then finely adjusted so as to produce a central dark fringe. Make an estimate of the order of this fringe.
Show that the angular radius of the pth order dark ring around the central dark fringe is given by

$$\theta_p \cong \left(\frac{p\lambda}{d}\right)^{1/2}$$

The monochromatic source is now replaced by a source with a range of wavelengths $\Delta\lambda$ = 1 nm. Discuss whether or not the fringe pattern still exists.

23. A Michelson interferometer is set up to give circular fringes and illuminated with light in the spectral range 800 -900 nm from an atomic caesium ^{133}Cs discharge source. The intensity *I(x)* at the centre of the interference pattern is recorded as a function of the distance *x* of the moving mirror from that corresponding to zero path difference. It is found to have the form

$$I(x) = 3I_0 + 3I_0 \cos K_1 x \cos K_2 x - I_0 \sin K_1 x \sin K_2 x$$

for $0 \le x \le 5$ mm, where $K_1 = 1.44 \times 10^7$ m^{-1} , $K_2 = 3.48 \times 10^5$ m^{-1} and I_o is a constant. Show that this can be written as the sum of the patterns due to two monochromatic spectral components. Hence determine, for the two caesium lines in this wavelength range:

(a) their mean wavenumber, $\bar{\nu}$,
(b) their wavenumber separation, $\Delta\bar{\nu}$
(c) their relative intensities.

When the interference pattern is recorded over a larger range of path difference it is found that the periodic terms in the pattern are in fact multiplied by a function

$$f(x) \propto \exp(-K_3^2 x^2)$$

where $K_3 = 5.02$ m^{-1}. Make a rough (but reasonably realistic) sketch of the interference pattern as a function of x over a range large enough to show the effect of this term.

[*Optional extra*.This factor arises from the Doppler broadening of spectral lines. This broadening is due to the different Doppler shift of the frequency, *f*, radiated by atoms moving with different speeds within a Maxwell-Boltzmann distribution. The Doppler shift in frequency is $\Delta f = f(v/c)$ where *v* is the speed of the atom. Taking an average speed to be approximately $\sqrt{3kT/M}$, where *k* is Boltzmann's constant, *M* is the atomic mass and *T* is the temperature, the width of the frequency spread may be used to estimate the temperature. Use the values in the question to estimate the temperature of the gas of Cs atoms.]

24. (a) Show that monochromatic light of wavelength λ from an extended source incident on two parallel reflecting surfaces, separated by a distance *d*, will form fringes of equal inclination at angle θ and order *p* where

$$p\lambda = 2d\cos\theta$$

(b) . Prove that the transmission function of a Fabry-Perot interferometer for monochromatic light of wavelength λ incident at an angle θ to the normal is

$$I(\phi) = I_0\left[1 + \left(\frac{4F^2}{\pi^2}\right)\sin^2\left(\frac{\delta}{2}\right)\right]^{-1}$$

where *I(0)* is the incident intensity, $\delta = (2\pi/\lambda)2nt\cos\theta$, where *t* is the plate separation, and *n* is the refractive index of the medium between the reflecting surfaces.

(c) Obtain an expression for *F* in terms of the intensity reflectivity of the plates, *R*, and explain how it determines the sharpness of the fringes.

25. Find the instrument width and resolving power of the Fabry-Perot etalon described in question 24, as follows.

(a) Show that the values of δ at which the intensity has dropped to 50% of maximum near the p^{th} order of interference are

$$\delta = 2\pi\left(p \pm \frac{1}{2F}\right)$$

(b) Show that two spectral lines for which the p^{th} maximum of one falls on the $(p+1)^{th}$ maximum of the other, the Free Spectral Range, are separated by $\delta\bar{\nu}_{FSR}$ given by

$$\delta\bar{\nu}_{FSR} = \left(\frac{1}{2nt}\right)$$

(c) From (a) and (b), show that the instrumental width, $\Delta\bar{\nu}_{INST}$, (FWHM, full width at half maximum intensity) of the etalon is given by

$$\Delta\bar{\nu}_{INST} = \frac{1}{F} \times \frac{1}{2nt}$$

(d) Hence show that the resolving power is Fp.

26. (a) Describe, with the aid of a labelled diagram including all essential components, how a Fabry-Perot etalon can be used to measure the separation of components in one of the spectral lines emitted by a discharge lamp.
(b) The discharge lamp is known to emit a spectral line consisting of two closely spaced components, with a wavenumber separation of $\Delta\bar{\nu}_S$. Explain how you would decide upon the value of the etalon spacing, *d*, in order to study this spectrum and suggest a suitable value for *d*.
(c) Assuming firstly that the spectral linewidth $\Delta\bar{\nu}_C$ of each component is negligible find an expression for the minimum value of plate reflectivity *R* required to resolve the two components. How would the appearance of the fringes be affected if $\Delta\bar{\nu}_C = \Delta\bar{\nu}_S$?

27. A certain spectral line is known to consist of two equally intense components with a wavenumber separation $\Delta\bar{\nu}_S$ less than 20 m^{-1}. The Fabry-Perot fringes produced by this line are photographed, using a plate separation of 25 mm. The diameters of the smallest rings are found to be, in mm:

$$1.82,\ 3.30,\ 4.84,\ 5.57,\ 6.60,\ 7.15.$$

Explain why this experiment does not allow $\Delta\bar{\nu}_S$ to be determined uniquely, but gives two possible values. What are they? Suggest a further experiment which could be carried out to resolve the ambiguity.

28. Explain what are meant by the *instrumental function* and the *instrumental width* associated with a spectroscopic device. Why is the instrumental width such an important property?

29. Calculate the minimum width a diffraction grating with 500 lines mm^{-1} must have, working at normal incidence, if it is to resolve two equally intense spectral lines with wavelengths spaced 0.04 nm apart at 600 nm. The collimating lens of the spectrograph has a focal length of 500 mm; estimate the maximum slit width one could use without significantly affecting the resolution of the lines. How far would it be necessary to scan a Michelson Fourier Transform Spectrometer to resolve these lines? Would a Fabry-Perot interferometer be a good choice of instrument to measure their separation?

30. Light of wavelength, λ, falls at normal incidence from air onto a glass plate of refractive index n_G covered by a film of dielectric material with refractive index n_d and thickness ℓ. Show that the ratio of the reflected, E', to incident amplitude, r, is given by

$$\frac{E_o'}{E_o} = r = \frac{\cos k_1\ell - i(n_G/n_d)\sin k_1\ell + in_d \sin k_1\ell - n_G \cos k_1\ell}{\cos k_1\ell - i(n_G/n_d)\sin k_1\ell - in_d \sin k_1\ell + n_G \cos k_1\ell}$$

where $k_1 = 2\pi n_d/\lambda$

31. A layer of a non-absorbing homogeneous dielectric of refractive index n_2, is deposited on the surface of glass with refractive index n_1. In the case where the thickness of the layer, ℓ is given by $\ell = \lambda/4n_2$, a quarter wave layer, show that the condition for all the energy to be transmitted by the layer is

$$n_2^{\,2} = n_1.$$

32. (a) Explain what is meant by *polarized light* and describe the different types of polarization that are possible. Explain how a uniaxial birefringent material of length l, with ordinary and extra-ordinary refractive indices , n_o and n_e respectively can be used to change the relative phase of two orthogonal components of the electric field in a light wave of wavelength λ. Describe briefly the principle of operation of a polarizing device made from a birefringent material.

(b) Demonstrate analytically that unpolarized light can be converted to circularly polarized light by passing the radiation through a linear polarizer followed by a quarter-wave plate at a suitable orientation.

(c) A beam of initially unpolarized light passes through this system, and is then reflected back normally through it from a plane mirror. Describe the state of polarization of the beam in each part of its path through the system.

(d) Show that any elliptically polarized light can be converted to plane polarized light using a suitably oriented quarter-wave plate. Is the converse also true? Show also that plane polarized light remains plane polarized after passing through a half-wave plate, but with the direction of the electric vector altered. How would you use a half-wave plate to change this direction by 40^{o}?

33. A beam of light is elliptically polarized with the major axis vertical. The ratio of the major and minor axes of the ellipse is $a : b$. Explain how you would use a quarter-wave plate to obtain linearly polarized light, and determine the angle of the plane of polarization to the vertical.

A beam of light consisting of a mixture of elliptically polarized and unpolarized light is passed through a linearly polarizing filter. The maximum of the transmitted light intensity is observed when the transmission axis of the filter is vertical, and is twice the minimum intensity. In a second experiment, the beam is passed through a quarterwave plate with the fast axis vertical followed by the polarizing filter. The maximum is now observed when the transmission axis is at 33.21^{o} to the vertical. Calculate the ratio of the intensities of the polarized to unpolarized components of the light.

How could the handedness of the elliptically polarized component be determined?

34. A beam of, initially unpolarized, light is incident normally on a piece of polarizing sheet (polaroid) **A**, the transmitted light then falling on a second similar sheet **B**. **A** and **B** are set with their axes at right angles. A third polaroid sheet **C** is placed between the first two with its axis at an angle ϕ to that of sheet **A**.

(a) What fraction of the incident energy passes through the system?

(b) In a Young's double slit experiment, both slits are covered by polarizing sheets, one with its axis horizontal, the other with its axis vertical. Explain why no fringes are seen in the plane of observation.

(c) Given a third polarizing sheet and a half-wave mica plate, explain where you would place them, and in what orientation, so that fringes would appear. Could the pattern be produced using only the half-wave plate?

Suggested Problem Sets for Tutorials

1. (Question 1 is optional revision of first year material.) Questions: 2, 6, 7, 8, 9, 10

2. Questions 14, 15, 16, 18, 19

3. Questions 20, 21, 23, 24, 25, 27

4. Questions 30, 31, 32, 33, 34