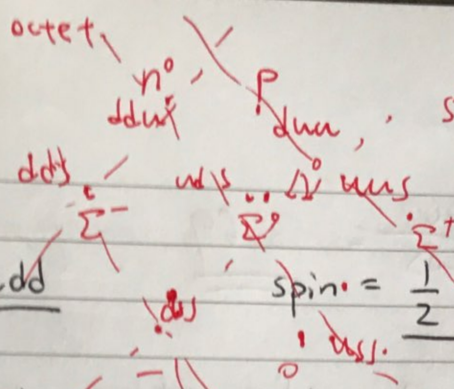
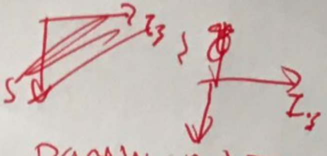
16B4 Q1





neutron $\underline{udd}$ spin $= \frac{1}{2}$

parity$(n) = (+1)(+1)(+1)(-1)^0 = 1$

it is in an octet

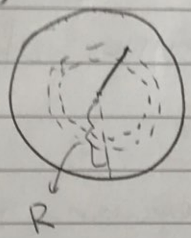
3

Λ^0 isospin singlet $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

$\Sigma^{\pm 0}$, isospin triplet $\uparrow\uparrow$, $\downarrow\downarrow$, $\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

– estimate mass :

(a) we want $\langle r^2 \rangle$, uniform density $\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi R^3}$



$$\langle r^2 \rangle = \frac{\int_0^R r^2 (4\pi r^2) dr}{\int_0^R 4\pi r^2 dr}$$

$$= \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{\frac{R^5}{5}}{\frac{R^3}{3}} = \frac{3}{5}R^2$$

$\therefore r_{rms} = \sqrt{\langle r^2 \rangle} = \sqrt{\frac{3}{5}}R$

assume $\Delta r = r_{rms} = \sqrt{\frac{3}{5}} R = 1.16\,\text{fm}$ ($R = 1.5\,\text{fm}$)

(b) $\Delta P_r \cdot \Delta r \geq \frac{\hbar}{2}$, $\therefore \Delta P_r \sim \frac{\hbar}{2\Delta r} = 4.53 \times 10^{-20}\,\text{kg m/s}$

$\Delta P_r = 84.86\,\frac{\text{MeV}}{c} = \langle P \rangle \approx P$

energy of 1 quark is

$E_1 = \sqrt{p^2 + m^2} = (84.86^2 + 5^2)^{\frac{1}{2}} = 85\,\text{MeV}$

(c)

total energy $E = 3 \times E_1 = 255\,\text{MeV}$

mass $m = 255\,\frac{\text{MeV}}{c^2}$

$\text{Ratio} = \frac{940}{255} = \underline{\underline{3.69}}$ ✓

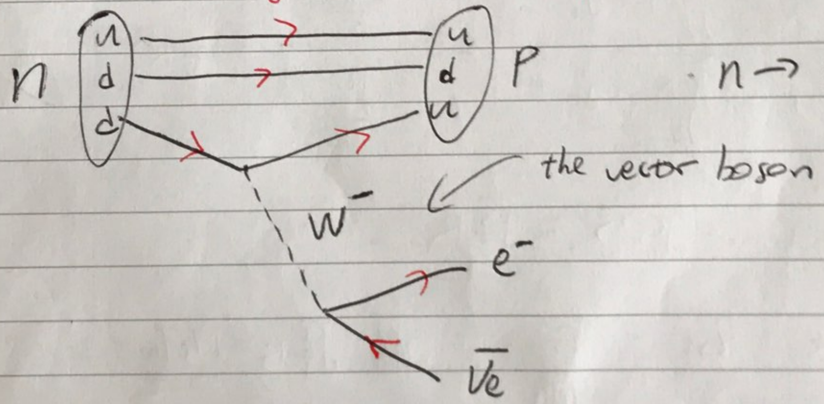
- Additional information needed:

  - The probability distribution (True wave-function) rather than uniform distribution ✓

  - The ~~masses~~ different masses of up and down quarks ✓

  - interaction energy between quarks. ✓

  - gluon momentum



$n \rightarrow p + e^- + \bar{\nu}_e$

Proton is stable because baryon number is, allways ~~in most cases~~, conserved and there is no lighter baryon number 1 particle ~~fro~~ for proton to decay to.

- 3 body decay

$n \rightarrow p + e^- + \bar{\nu}_e$

$\overset{n}{O} \quad \Rightarrow \quad \overset{p}{O} \quad \overset{e^-}{O} \quad \overset{\bar{\nu}_e}{O}$

Q4 momentum conservation $(c=1)$
metric $(1,-1,-1,-1)$.

$$P_n = P_p + P_e + P_\nu \qquad P_n = \begin{pmatrix} m_n \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad P_p = \begin{pmatrix} E_p \\ \underline{p_p} \end{pmatrix}$$

$$\therefore (P_n - P_e)^2 = (P_p + P_\nu)^2 \qquad P_e = \begin{pmatrix} E_e \\ \underline{p_e} \end{pmatrix} \quad P_\nu = \begin{pmatrix} p_\nu \\ \underline{p_\nu} \end{pmatrix}$$

$\underline{P_p} \cdot \underline{P_\nu} = |P_p||P_\nu|\cos\theta$

$$\therefore \quad P_n^2 + P_e^2 - 2P_n \cdot P_e = P_p^2 + P_\nu^2 + 2P_p \cdot P_\nu$$

$$\therefore \quad m_n^2 + m_e^2 - \cancel{2} 2 m_n E_e = m_p^2 + 0 + 2P_p \cdot P_\nu$$

max $\Leftrightarrow$ neutrino energy ?

$$\therefore \quad E_e = \frac{(m_n^2 + m_e^2 - m_p^2) - 2P_p \cdot P_\nu}{2m_n}$$

$$E_e = \frac{1}{2m_n}\left(m_n^2 + m_e^2 - m_p^2 - 2E_\nu(E_p - p_p \cos\theta)\right)$$

range of E_e depends on ~~$P_p \cdot P_\nu = E_p p_\nu - p_p p_\nu$~~ ~~$= (E_p - p_p) p_\nu$~~

~~$P_p \cdot P_\nu = E_p p_\nu - p_p p_\nu$~~ $P_p \cdot P_\nu$

- maximum energy

Now 2 ways to do this

① assume $m_\nu = 0$ $\quad P_\nu = \begin{pmatrix} p_\nu \\ \underline{p_\nu} \end{pmatrix} \quad P_p = \begin{pmatrix} E_p \\ \underline{p_p} \end{pmatrix}$

$P_p \cdot P_\nu$ in the rest frame of proton $= \begin{pmatrix} m_p \\ \underline{0} \end{pmatrix} \cdot \begin{pmatrix} p_\nu' \\ \underline{p_\nu'} \end{pmatrix}$

$$= m_p p_\nu' \geq 0$$

$\therefore P_p \cdot P_\nu \geq 0 \qquad P_p \cdot P_\nu = 0$ when $p_\nu = 0$

$$\therefore E_{e,max} = \frac{m_n^2 + m_e^2 - m_p^2}{2m_n}$$

② neutrino has small but non-zero mass

$$P_p \cdot P_\nu = \cancel{m_p m_\nu \gamma} \; m_p m_\nu \gamma_u^2$$

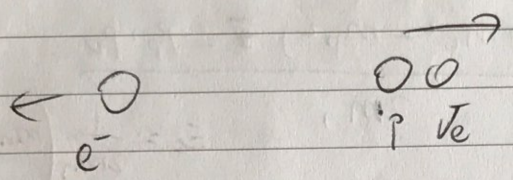
where γ_u is $\dfrac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$ $\quad u$ = relative velocity between neutrino and proton

$\therefore P_p \cdot P_\nu$ is minimum when $\gamma_u = 1$

$$P_p \cdot P_\nu = m_p m_\nu \simeq 0$$

$$\therefore E_{e,max} = \frac{m_n^2 + m_e^2 - m_p^2}{2m_n} = \cancel{1.3\,eV} \; \underline{\underline{1.3\,MeV}}$$

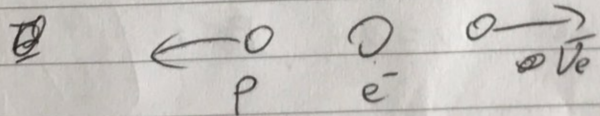
when neutrino ~~k~~ and proton travel togerther with no relative velocity



$$\underline{P_{e^-} = P_p + P_{\bar{\nu}_e}}$$

- minimum energy

This is when electron is at rest



momenta of p and $\bar{\nu}_e$ balance

$$\underline{P_p = P_{\bar{\nu}_e}}$$

$$E_{e,min} = m_e = \underline{\underline{0.511\ MeV}}$$

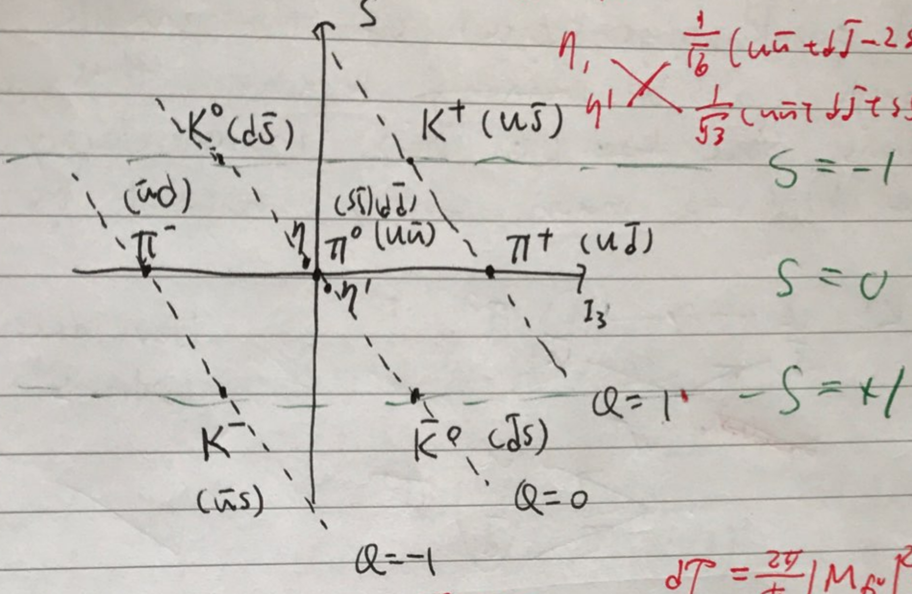
- decay width. $\tau = \frac{\hbar}{\Gamma}$

$$\therefore \Gamma = \frac{\hbar}{\tau} = \frac{1.05 \times 10^{-34}\ J \cdot s}{15 \times 60\ s} = \underline{\underline{7.3 \times 10^{-19}\ eV}}$$

16B4Q2

$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

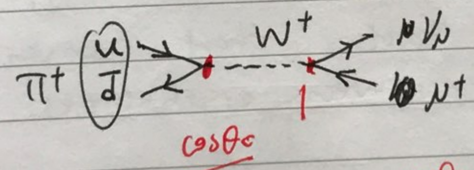
η, η': $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

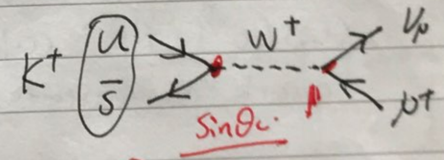


$d\Gamma = \frac{2\pi}{\hbar}|M_{fi}|^2 \frac{dN}{dE}$

$dN = \frac{\cancel{4\pi} p^2 dp}{(2\pi\hbar)^3} \sim Q^2$

$\pi^+ \to \mu^+ \nu_\mu$ $\qquad$ $K^+ \to \mu^+ + \nu_\mu$



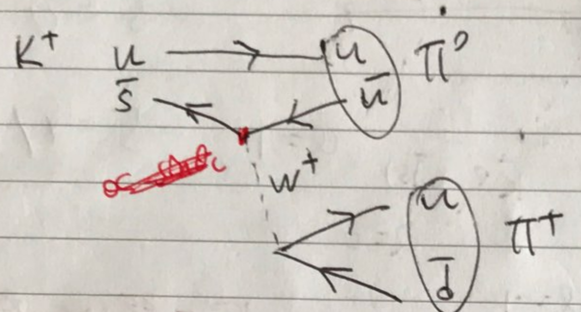


$M \sim \frac{\cos\theta_c \cdot 1}{M_W}$ $\qquad$ → ~~Cabbi~~ Cabbibo suppressed

- suppressions
  - the phase space factor suppresses the pion decay relative to kion decay, because pion has smaller mass than kion. There is more freedom for the μ^+ and ~~ν~~ ν_μ to ~~de pn~~ be produced for kion decay than ~~for~~ pion decay. (freedom of energy & momentum) ✓

    Kaon not Kion
  - the Cabbibo factor suppresses the Kion decay relative to pion decay, Because pion$^+$ is made of $u, \bar{d}$, the lightest quark/antiquarks ∴ pion can't decay into hadrons. Kion, on the other ~~hand~~ hand, can decay into ~~hadrons~~ the

~~weak~~ quark eigenstates of the weak interaction through cabbibo ~~mixing~~ mixing, thus decay into hadrons. Since ~~kion~~ kion has more decay channels, the kion to muon decay channel is suppressed.



kion decay to hadrons.

- A flavour change ⇒ weak interaction.

The production of π^+ $(u\bar{d})$ is suppressed by $\cos\theta_c$, the cabbibo factor that converts strong quark eigenstates to weak eigenstates.

— $\pi^+\pi^-$ scattering $\quad \sigma(E) = (2J+1)\dfrac{B_{in}B_{out}\Gamma_{tot}^2}{(E^2-m^2)^2+E^2\Gamma_{tot}^2}$

$$\cancel{\sigma_{\rho^0} = (2J_{\rho}+1)\frac{B^2(\rho^0\to\pi^+\pi^-)\Gamma_{\rho}^2}{E^2\Gamma_{\rho}^2+(E^2-m_{\rho^0}^2)}}$$

$$\cancel{\sigma_{K^0} = (2J_{K^0}+1)\frac{B^2(K^0\to\pi^+\pi^-)\Gamma_K^2}{E^2\Gamma_K^2}}$$

~~We can write them is in this way because~~

$\pi^+\pi^-$ scattering (assume elastic scattering)

$\cancel{\pi^+}$ $\pi^+ + \pi^- \to \rho^0 \to \pi^+ + \pi^-$

$\pi^+ + \pi^- \to K^0 \to \pi^+ + \pi^-$

$B_{in} = B_{out} = Br(\rho^0\to\pi^+\pi^-)$ for ρ^0 production

$Br(K^0\to\pi^+\pi^-)$ for K^0 production

∴ each pion beam has energy equal to half of the kaon mass

∴ total cm energy $E = 2 \times \frac{1}{2} m_K = M_K = 2 \times 249\,\text{MeV} = 498\,\text{MeV}$

$$\therefore \sigma_\rho \propto 2J_\rho + 1 \frac{B_\rho^2 \Gamma_\rho^2}{(m_{\cancel{K}}^2 - m_\rho^2)^2 + M_K^2 \Gamma_\rho^2}$$

$$\approx 3 \times \frac{(1)^2 (147)^2}{(769^2 - 498^2)^2 + 498^2 (147)^2} = 5.26 \times 10^{-7} \frac{m^2}{\hbar^2 c^2}$$

$$\sigma_K = 2J_K + 1 \frac{B_K^2 \cancel{\Gamma_K^2}}{\underbrace{(m_K^2 - m_K^2)^2}_{0} + m_K^2 \cancel{\Gamma_K^2}}$$

$$= \underbrace{2J_K + 1}_{1} \frac{B_K^2}{m_K^2} = \frac{(0.69)^2}{(498)^2} = 1.92 \times 10^{-6} \frac{m^2}{\hbar^2 c^2}$$

$B_{out} = 100\%$ definitely

$Br(K^0 \to \pi^+\pi^-) = 69\% = B_{in} \cdot B_{out}$

$$\therefore \text{ratio } R = \frac{\sigma_\rho}{\sigma_K} = 0.27$$

(b) in this case $E = 2 \times 250\,\text{MeV} = 500\,\text{MeV}$

$$\therefore \sigma_\rho = 3 \times \frac{(1)^2 (147)^2}{((769^2 - 500^2))^2 + 500^2 (147)^2} = 5.32 \times 10^{-7} \frac{m^2}{\hbar^2 c^2}$$

$$\sigma_K = 1 \times \frac{\cancel{K^2} \times (7.4 \times 10^{-12})^2 \times (0.69)^2}{(500^2 - 498^2)^2 + 500^2 (7.4 \times 10^{-12})^2}$$

$$= 1.37 \times 10^{-29} \frac{m^2}{\hbar^2 c^2}$$ ~~2.8 · 10^23~~

$$\therefore R = \frac{5.32 \times 10^{-7}}{1.37 \times 10^{-29}} = 3.88 \times 10^{22} = \frac{\sigma_\rho}{\sigma_K} = 8.13 \times 10^{22}$$

$7.6 \cdot 10^{22}$ (rounding?)

- It is impractical to study resonant K^0 production ~~because~~ in $\pi^+\pi^-$ scattering because we need extremely accurate pion beam energy to ~~produce~~ have reasonable probability to produce resonant Kaon. Such an accuracy is difficult to achieve.

(c) pure force $\frac{dm_0}{dt} = 0$

$$\underline{f} = \frac{d}{dt}(\gamma m_0 \underline{u}) = \gamma m_0 \underline{a} + m_0 \frac{d\gamma}{dt} \underline{u}$$

4 velocity $U = \begin{pmatrix} \gamma c \\ \gamma \underline{u} \end{pmatrix}$, 4 force $F = \begin{pmatrix} \frac{\gamma}{c}\frac{dE}{dt} \\ \gamma \underline{f} \end{pmatrix} = \frac{dP}{d\tau}$

$$\therefore U \cdot F = \gamma^2 \left(-\frac{dE}{dt} + u \cdot f\right)$$

$$= \begin{pmatrix} c \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{d}{d\tau}(m_0 c^2) \\ 0 \end{pmatrix} = -c^2 \frac{dm_0}{d\tau} = 0$$

rest frame

if pure force

$$\Rightarrow \underline{f} \cdot \underline{u} = \frac{dE}{dt}$$

$$\therefore \frac{d\gamma}{dt} = \frac{1}{m_0 c^2} \frac{dE}{dt} = \frac{1}{m_0 c^2} \underline{f} \cdot \underline{u}$$

$$\therefore \underline{f} = \gamma m_0 \underline{a} + \frac{\underline{f} \cdot \underline{u}}{c^2} \underline{u}$$

magnetic field - Lorentz force $\underline{f} = q \underline{u} \times \underline{B}$ $\therefore \underline{f} \cdot \underline{u} = 0$

$$\therefore \underline{f} = \gamma m_0 \underline{a} = \gamma m_0 \frac{d\underline{u}}{dt}$$

$$\frac{d}{dt}(u^2) = 2\underline{u} \cdot \frac{d\underline{u}}{dt} = \frac{2\underline{f}}{\gamma m_0} \cdot \underline{u} = 0$$

$\therefore$ velocity u is constant

$\Rightarrow \gamma$ is also constant

$\because \gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$

the motion is circular , $a = \frac{v^2}{r}$

$$\gamma m_0 \frac{v^{\cancel{2}}}{r} = q\cancel{v}B$$

$$\therefore \frac{p}{r} = qB \implies \underline{p = qBr}$$

~~ref~~ mass ~~m~~ of $\pi^{\pm}$ is $m_\pi = 139.57 \frac{MeV}{c^2}$

$$\therefore p = \sqrt{E^2 - m_\pi^2} = \sqrt{(249)^2 - (139.57)^2} = 206.21 \frac{MeV}{c}$$

$$\therefore r = \frac{p}{qB} = \frac{206.21\ MeV/c}{1eV \cdot s \cdot m^{-2}}$$

$$= \frac{206.21 \times 10^6\ m^2/s}{3 \times 10^8\ m/s} = \underline{\underline{0.69 m}}$$

16B4Q3

(a) proton proton fusion

$$p + p \rightarrow D + e^{+} + \nu_e$$

$Q = 2m_p - m_D - m_e$ (assume neutrinos are massless)

$= 2(938.27) - (1875.61) - (0.511) = \underline{\underline{0.42 \text{ MeV}}}$ ✓

↑ where from?

(b)

For each 4 ~~he~~ hydrogen nuclei being burned, the radiated energy is

$$4E_p = Q + E_{ee^+} - E_\nu$$

$$= 24.68 + 2\times 1.02 - 2\times 0.26$$

$$= 26.20 \text{ MeV}$$ ✓

⇒ radiated energy per proton burned is

$E_p = 6.55 \text{ MeV} = 6.55\times 10^{6}$ eV

total energy burned

$$E_{tot} = (3.86\times 10^{26} \text{ J/s})\times(4.6\times 10^{9}\times 365\times 24\times 60\times 60 \text{ s})$$

$$= 5.6\times 10^{43} \text{ J}$$

$$= 3.5\times 10^{62} \text{ eV}$$

~~#~~ number of hydrogen nuclei is burned

$$N_b = \frac{E_{tot}}{E_p} = \frac{3.5\times 10^{62}}{6.55\times 10^{6}} = \underline{5.34\times 10^{55}}$$

~~Time take~~

~~the~~ time to burn all hydrogen nuclei is

$$T_{tot} = \frac{9\times10^{56}}{5.34\times10^{55}}(4.6 \text{ billion years}) = \underline{77.5 \text{ billion years}}$$

~~a~~ Time left to burn is

~~more~~

$$T_{longer} = 77.5 - 4.6 = \underline{\underline{72.9 \text{ billion years.}}}$$

(c) SEMF:

$$M(Z,A) = \underset{①}{Z(m_p + m_e)} + \underset{②}{(A-Z)m_n} - \underset{③}{a_v A} + \underset{④}{a_s A^{2/3}}$$

$$+ \underset{⑤}{a_c \frac{Z^2}{A^{1/3}}} + \underset{⑥}{a_a \frac{(Z-\frac{A}{2})^2}{A}} \underset{⑦}{\pm a_p A^{-1/2}}$$

① the mass of constituent protons and electron. there are Z of each.

② There are A−Z number of neutrons, each with mass m_n

③ – ⑦ are factors of binding energy

③ From the roughly constant binding energy per nucleon data we can say that the strong nuclear force that attracts the nucleons togather only act between nearest neighbours.

Each nucleon ~~of~~ binds the same number of nearest neighbours, independently of the size of the nucleus, so total binding energy should be proportional to the number of nucleons. This energy is attractive, so proportionality constant is negative. We have the term (this is the volume term)

$$-a_V A \qquad (r^3 \propto A)$$

④ In modelling the ~~neighbours~~ nearest neighbour force we ~~ov~~ ought to make a correction for that the surface has less neighbours. So binding energy (attractive energy) is reduced by a factor ~~of~~ proportional to the surface area $\propto A^{2/3}$
$(r^2 \propto A^{2/3})$

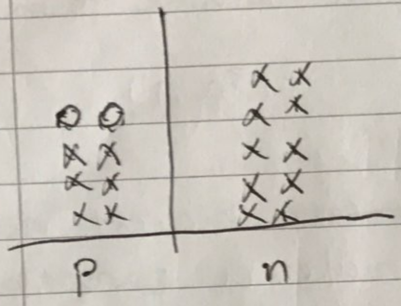
$$\therefore \quad +a_S A^{2/3}$$

⑤ This is the Coulomb term, and is repulsive (so a positive term in energy). ~~There are~~ the number of interaction pairs is proportional to $Z(Z-1) \simeq Z^2$, and Coulomb energy is $\propto \frac{1}{r} \propto \frac{1}{A^{1/3}}$ $\quad (r \propto A^{1/3})$

$$\therefore \text{ The term is } +a_C \frac{Z^2}{A^{1/3}}$$

⑥ This is the asymmetry term. Protons & neutrons are both fermions. No two protons and no two neutrons can be in the same state, but a proton and a neutron can be in the same energy state because they are not identical. If, for example, there are more neutrons than protons $(N > Z,\ N + Z = A)$ ~~than~~

$\Rightarrow Z < \frac{A}{2})$, then ~~there~~ the additional neutrons must ~~p~~ be placed in higher energy levels than additional protons



$\therefore$ If $Z \neq \frac{A}{2}$ there will be a term that increases the total energy, thus a positive term

$$+a_a \frac{(Z-\frac{A}{2})^2}{A}$$

⑦ This is the ~~parity term~~ pairing term.

nucleus with even number of protons or even number of neutrons tend to be more stable than those odd nuclei. The pairing term is 0 for ~~a~~ odd-A nuclei. If both Z, N are even the nucleus is more tightly bond; If both Z, N are odd the nucleus is less tightly bond

(d)

$$n + {}^{235}_{92}U \rightarrow {}^{92}_{37}Rb + {}^{140}_{55}Cs + 4n$$

use SEMF:

$$m_U = 92(m_p+m_e) + (235-92)m_n - (15.56)(235) + (17.23)(235)^{2/3} + (0.697)\frac{92^2}{(235)^{1/3}} + (93.14)\frac{(92-\frac{235}{2})^2}{235}$$

$$+(\cancel{12})(235)^{-1/2} \quad \cancel{218745\ MeV}$$

odd ~~od~~ even nucleus

$$= \boxed{218947\ MeV/c^2} \quad \cancel{218744\ MeV}$$

$$m_{Rb} = 37(m_p + m_e) + (92-37)m_n - (15.56)(92) + (17.23)(92)^{2/3} + (0.697)(37^2)/(92)^{1/3}$$

$$+ (93.14)\frac{(37 - \frac{92}{2})^2}{92} + (12)(92)^{-\frac{1}{2}}$$

odd odd

$$= \boxed{85628 \text{ MeV}/c^2}$$

$$M_{Cs} = 55(m_p + m_e) + (140-55)m_n - (15.56)(140) + 17.23(140)^{2/3} + (0.697)(55^2)/(140)^{1/3}$$

$$+ (93.14)\frac{(55 - \frac{140}{2})^2}{140} + 12(140)^{-\frac{1}{2}}$$

odd-odd

$$= \boxed{130344 \text{ MeV}/c^2}$$

d.

$$Q = m_n + m_0 - 4m_n - m_{Rb} - m_{Cs}$$

$$= m_0 - m_{Rb} - m_{Cs} - 3m_n$$

$$= 218947 + \cancel{939.6} - 85628 - 130344 - 3 \times 939.6$$

$$= \underline{\underline{156.2 \text{ MeV}}}$$

Each year energy burned is

$$E = (100\,M\,J/s)(365 \times 24 \times 60 \times 60\,S) / (1.6 \times 10^{-19}\,J/eV)$$

$$= 1.97 \times 10^{28}\ MeV$$

number of uranium atoms is

$$N = \frac{E}{Q} = \frac{1.97 \times 10^{28}}{156.2}$$

$$= \underline{\underline{1.26 \times 10^{26}}}$$ ✓

2

how many ~~grams~~ .

(e)

$$\frac{E_f}{E_i} = \frac{M_c^2 + m_n^2}{(M_c + m_n)^2} = 0.8569$$

$$\therefore \frac{0.025\,eV}{2 \times 10^6\,eV} = (0.8569)^N$$

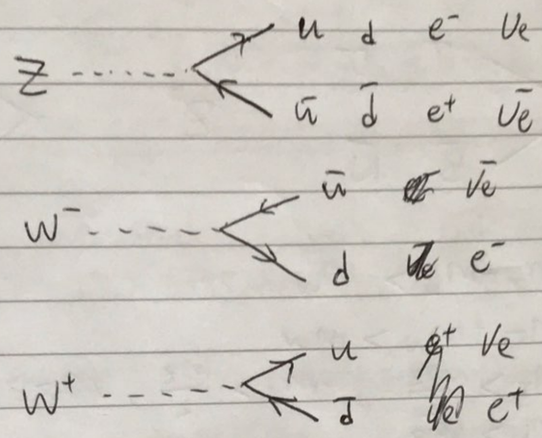
number of collisions

$$N = \frac{\ln\left(\frac{0.025}{2 \times 10^6}\right)}{\ln(0.8569)} = \underline{\underline{118}}$$ ✓

16B4Q4

(a) first generation leptons e^-, ν_e

quarks ~~quark~~ u, d



– branching ratios

$Br(W \to l\bar{l}) = \frac{1}{1+3} = \frac{1}{4}$

$Br(W \to q\bar{q}) = \frac{3}{1+3} = \frac{3}{4}$

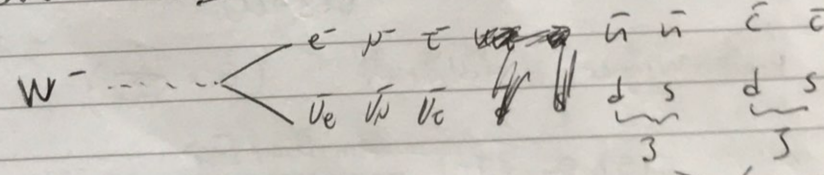
$Br(W^- \to l\bar{l}) = \frac{3}{9} = \frac{1}{3}$

$Br(W^- \to q\bar{q}) = \frac{2}{3}$

(b)

– branching ratio, use all generations

~~$Br(W \to l\bar{l})$ $Br(W \to q\bar{q})$~~



$Br(e^-\bar{\nu}_e) = \frac{\Gamma_{e^-\bar{\nu}_e}}{\Gamma} = \frac{1}{1+1+1+3+3} = \frac{1}{9}$

the measured ratio is $Br(e^-\bar{\nu}_e) = \frac{232 \text{ MeV}}{2085 \text{ MeV}} \simeq \frac{1}{9}$

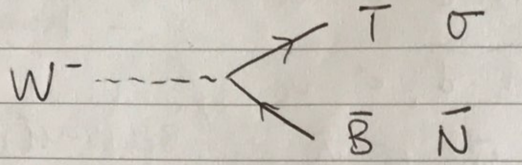
9 !!

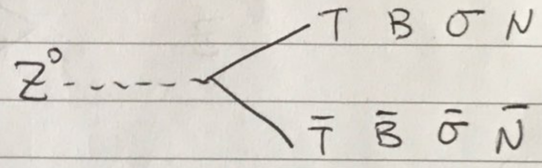
This consistency shows that ~~3~~ distinguishable fermion pairs are contributing to the W boson width.

9

(c) The 4th generation of quarks and leptons

since we do not observe the decays : ~~they have to be massive~~





we need
$$\begin{cases} m_T + m_B > m_W \\ m_\sigma + m_N > m_W \\ m_T > \frac{m_Z}{2}, \quad m_B > \frac{m_Z}{2}, \quad m_\sigma > \frac{m_Z}{2}, \\ m_N > \frac{m_Z}{2} \end{cases}$$

(d) we measured $R = \frac{\sigma(W \to l\nu)}{\sigma(Z \to ll)}$

Breit-Wigner distribution $\left(B_i := \frac{\Gamma_i}{\Gamma_{tot}} \right)$ ~~$\therefore \Gamma_i = B_i \Gamma_{tot}$~~

$$\sigma(E) \propto 2J+1 \frac{B_{in} B_{out} \Gamma_{tot}^2}{(E^2-m^2)^2 + m^2 \Gamma_{tot}^2} = 2J+1 \frac{\Gamma_{in} \Gamma_{out}}{(E^2-m^2)^2 + m^2 \Gamma_{tot}^2}$$

$$\sigma(W \to l\nu) = 2J+1 \frac{\Gamma_{in}^W \Gamma_{out}^W}{m^2 \Gamma_{tot}^2}$$ (because E is close to mass of W (resonance) $\Rightarrow E^2 - m^2 \to 0$)

$$= 2J+1 \frac{\Gamma_{in}^W \Gamma_{out}^W}{m_W^2 \Gamma_W^2}$$

Similarly $\sigma(Z \to ll) = 2J+1 \frac{\Gamma_{in}^Z \Gamma_{out}^Z}{M_Z^2 \Gamma_Z^2}$

Same $J=1$ as W

$$\Gamma_W^{4} = \frac{m_Z \Gamma_{\nu\ell}^{Z}}{m_W} \sqrt{\frac{\Gamma_i^W \Gamma_o^W}{\Gamma_i^Z \Gamma_o^Z} R^{-1}}$$

i. $$\frac{\Gamma_{in}^W \Gamma_{out}^W \, m_Z^2 \Gamma_Z^2}{\Gamma_{in}^Z \Gamma_{out}^Z \, m_W^2 \Gamma_W^2} = R$$

$$\Rightarrow \quad \Gamma_W = \left[\frac{\Gamma_{in}^W \Gamma_{out}^W m_Z^2 \Gamma_Z^2}{\Gamma_{in}^Z \Gamma_{out}^Z m_W^2 R} \right]^{\frac{1}{2}}$$

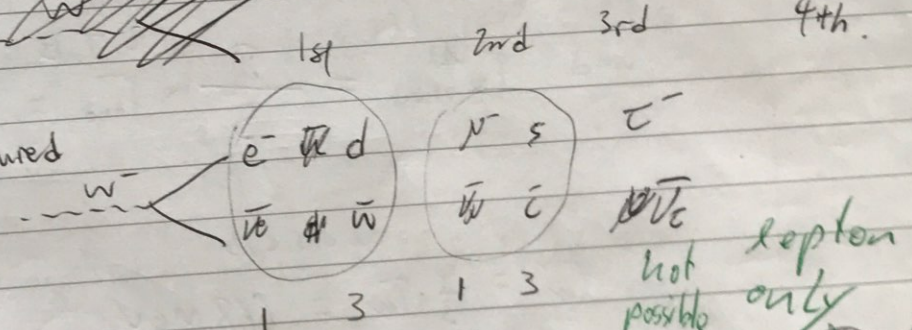
(e) measured values, which only allow 3 generations, are $\Gamma_W = 2085$ MeV and $\Gamma_Z = 2495$ MeV

~~for W~~ in 4-generation model all decays are kinematically possible.

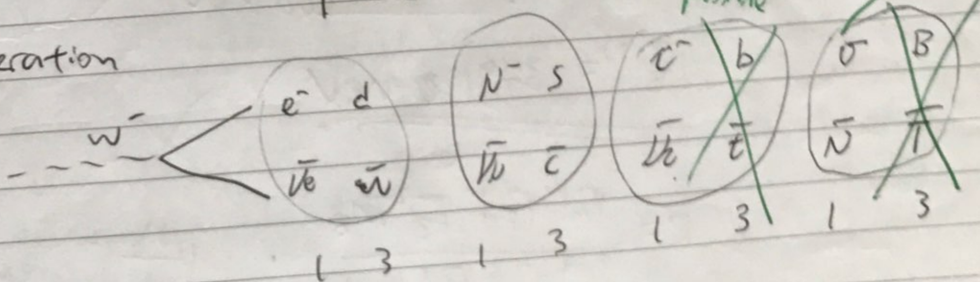
- ~~measured~~

- For W:

- measured



4-generation



$$\therefore \Gamma_W^{(4)} = (2085) \times \frac{(1+3)\times 4}{(1+3)\times 2 + 1} = \underline{\underline{3707 \text{ MeV}}}$$

2317

$\times \frac{10}{9}$

$$\Gamma_Z^4 = \Gamma_Z^{SM} + 501 \text{ MeV} + 84 \text{ MeV}$$

$$= (2495 + 501 + 84) \text{ MeV}$$

— For Z :

the measured values

~~$\Gamma_Z = \Gamma_{Z\to\nu\nu} + \Gamma_{Z\to ee} + \Gamma$~~

$$\Gamma_Z = \Gamma_{Z\to\nu\nu} + 3\times\Gamma_{Z\to ee} + \Gamma_{Z\to had}$$

$$\therefore \Gamma_{Z had} = \Gamma_Z - \Gamma_{Z\to\nu\nu} - 3\Gamma_{Z\to ee} = 1742 \text{ MeV}$$

Now add one more generation ⇒ 4-generation

no ——— only leptons

New $\Gamma_Z^{(4)}$ should include $\Gamma_{Z\to ll} = 4\Gamma_{Z\to ee} = 336$ MeV

and $\Gamma^{(4)}_{Z\to had} = \frac{8}{5} \times \Gamma_{had} = 2787$ MeV

$8 \to$ + t, T, B

$5 \to$ u, d, s, c, b

$$\Gamma_W^{(4)} = \frac{4}{3}\times\Gamma_W = 668 \text{ MeV}$$

$$\therefore \Gamma_Z^{(4)} = 3791 \text{ MeV}$$

$$\because R = \frac{\Gamma_{in}^{W}\,\Gamma_{out}^{W}\,\Gamma_Z^2\, m_Z^2}{\Gamma_{in}^{Z}\,\Gamma_{out}^{Z}\,\Gamma_W^2\, m_W^2}$$

$\cdot \left(\frac{m_Z}{m_W}\right)^2$ remains the same

~~∵ proton-proton~~ ∵ $p\bar{p}$ collision, ∴ colliding quarks remain the same

$\therefore \frac{\Gamma_{in}^W}{\Gamma_{in}^Z}$ remains the same

fractional effect of R is dependent on $\frac{\Gamma_{out}^W\,\Gamma_Z^2}{\Gamma_{out}^Z\,\Gamma_W^2}$

$$R = \left(\frac{\Gamma_Z^4}{\Gamma_Z^{SM}}\right)^2 \left(\frac{\Gamma_W^{SM}}{\Gamma_W^4}\right)^2$$

Γ_Z^2 ~~goes~~ changes by a fraction $\left(\frac{3791}{2495}\right)^2$ $= 1.23.$

Γ_W^2 changes by a fraction $\left(\frac{3707}{2085}\right)^2$

$\Gamma^W_{only} = \Gamma_{W\to l\nu}$ changes by a factor $\frac{4}{3}$ no

$\Gamma^Z_{only} = \Gamma_{Z\to ll}$ changes by a factor $\frac{4}{3}$ no

$\Gamma^{W,Z}_{only}$ don't change ∵ we observed it in a specific channel still observe same channel

∴ total fractional change

$$= \left(\frac{3791}{2495}\right)^2 \times \left(\frac{2085}{3707}\right)^2 = \underline{\underline{0.73}}$$ 1.23

$(1-0.73)\times 100\% = 27\%$

(f) R changes by $R^m = R_{SM}(1 \pm 1.9\%)$

∴ # of σ $= \frac{27}{1.9} = \underline{\underline{14}}$: 12

$(1.23-1)/1.9 = 12\%$

Γ_Z changes by $3791 - 2495 = 1296$ MeV

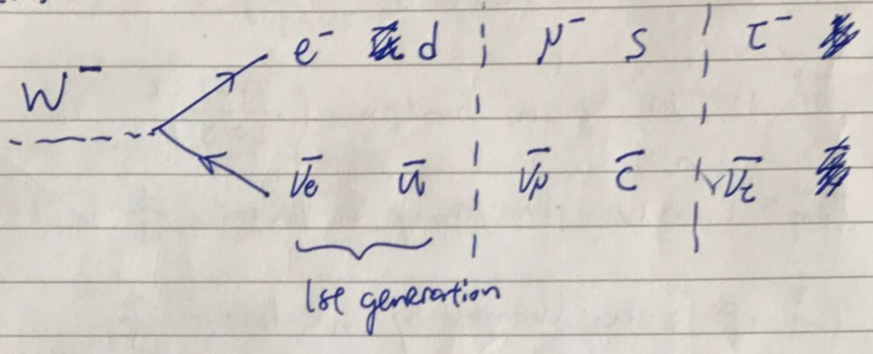
$$\# \text{ of } \sigma = \frac{1296}{2.3} = \underline{\underline{563}}$$ 254 (✓)

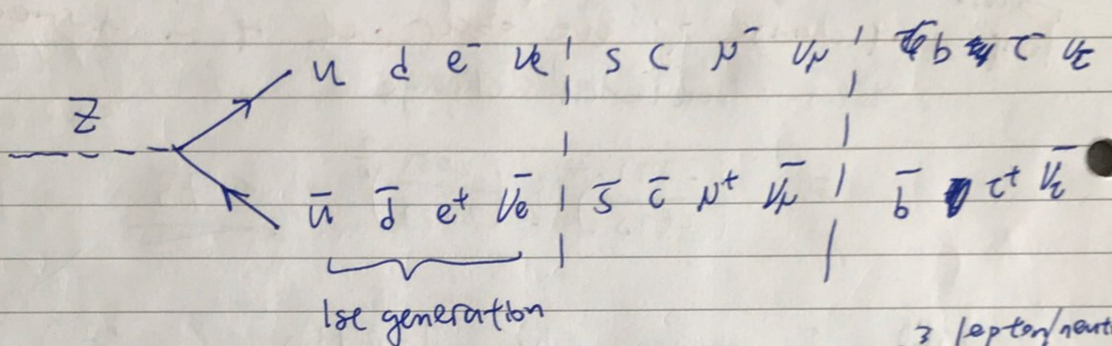
- very low probability to observe the effect of 4-th generation fermions since # of σ is very large.

- 4-th generation fermions cannot contribute to the ~~W~~ W and Z width, ⇒ they either <u>don't exist</u> or they are <u>massive</u>

Corrections:

4. (a)





branching ratio: $Br(W \to l\bar{l}) = \dfrac{3}{3 + \underbrace{2\times 3}} = \dfrac{1}{3}$

3 lepton/neutrino

3 lepton/neutrino ~~pair~~

$3 \to \begin{pmatrix} \bar{u} & \bar{u} \\ d & s \end{pmatrix}$

$3 \to \begin{pmatrix} \bar{c} & \bar{c} \\ d & s \end{pmatrix}$

cabbibo mixing

$$Br(W \to q\bar{q}) = \frac{2\times 3}{3+2\times 3} = \frac{2}{3}$$

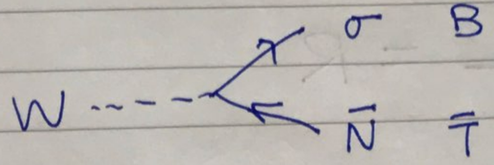
(b) $\because \dfrac{\Gamma_{tot}}{\Gamma_{W\to ee}} = \dfrac{2085}{232} \simeq 9$

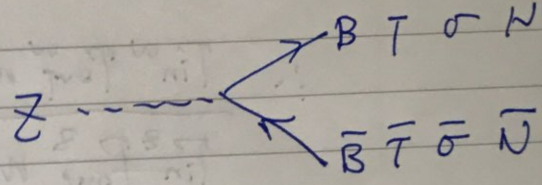
$\therefore$ there are <u>9</u> distinguishable fermion pairs.

Corrections:

(c)

If W and Z can decay into 4th generation ~~qu~~ quark / leptons





But we never observe these decays

$\therefore$ need $m_\sigma + m_N > m_W$

$m_B + m_T > m_W$

$m_B > \frac{m_Z}{2}$, $m_T > \frac{m_Z}{2}$, $m_\sigma > \frac{m_Z}{2}$, $m_N > \frac{m_Z}{2}$

(d) We measured $R = \frac{\sigma(W \to l\nu)}{\sigma(Z \to ll)}$

Breit-Wigner distribution ($B_i = \frac{\cancel{\Gamma_i}}{\cancel{\Gamma_{tot}}} \frac{\Gamma_i}{\Gamma_{tot}}$)

$$\sigma(E) \propto 2J+1 \frac{B_{in} B_{out} \Gamma_{tot}^2}{(E^2-m^2)^2 + m^2\Gamma_{tot}^2} = 2J+1 \frac{\Gamma_{in}\Gamma_{out}}{(E^2-m^2)^2+m^2\Gamma_{tot}^2}$$

~~$\sigma(W \to l\nu) = 2J+1$~~

$\because$ collision of energy = mass of product

$\therefore E = m$

$$\therefore \sigma(W \to l\nu) = 2J+1 \frac{\Gamma_{in}^W \Gamma_{out}^W}{m_W^2 \Gamma_{tot}^2} = 3 \frac{\Gamma_{in}^W \Gamma_{out}^W}{m_W^2 \Gamma_{tot}^2}$$

$J=1$ for W

Similarly $\sigma(Z \to ll) = 3 \dfrac{\Gamma_{in}^{Z} \Gamma_{out}^{Z}}{m_Z^2 \Gamma_Z^2}$

$J_Z = J_W = 1$

$$\therefore \frac{\Gamma_{in}^{W} \Gamma_{out}^{W} m_Z^2 \Gamma_Z^2}{\Gamma_{in}^{Z} \Gamma_{out}^{Z} m_W^2 \Gamma_W^2} = R$$

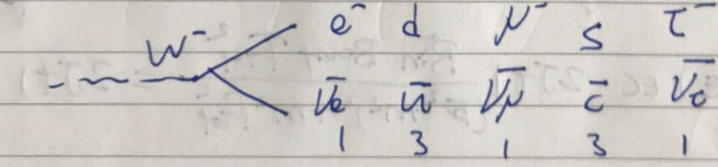
$$\Rightarrow \Gamma_W = \left[\frac{\Gamma_{in}^{W} \Gamma_{out}^{W} m_Z^2 \Gamma_Z^2}{\Gamma_{in}^{Z} \Gamma_{out}^{Z} m_W^2 R} \right]^{\frac{1}{2}}$$

(e) the measured values, which only allow 3 generations, are

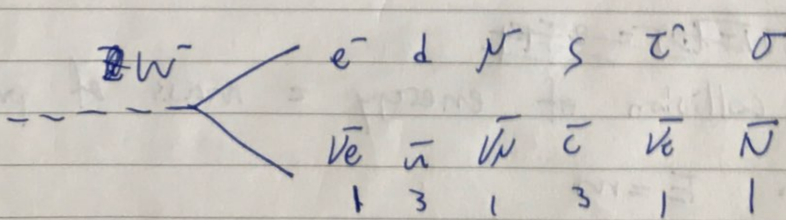
$\Gamma_W^{(3)} = 2085\,\text{MeV}$, $\Gamma_Z^{(3)} = 2495\,\text{MeV}$

Now if decay to leptons N, σ are possible

- For W:



(3 generation)

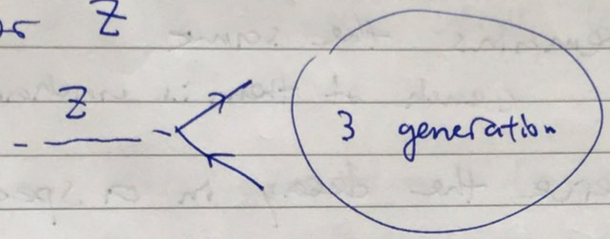


(we treat $\bar{u}, d, \bar{c}, s$ as weak eigenstates. color degeneracy of $\bar{u}.d$ is $3 \times \cos^2\theta_c$ for $\bar{u}, d$ + $3 \times \sin^2\theta_c$ for $\bar{u}, s$. $= 3$).

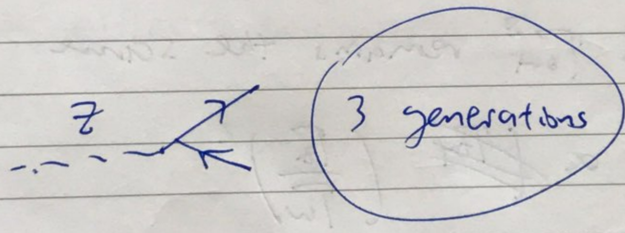
$$\therefore \Gamma_W^{(4)} = \Gamma_W^{(3)} \frac{1+3+1+3+1+1}{1+3+1+3+1} = \frac{10}{9} \times 2085$$

$$= \underline{\underline{2317 \text{ MeV}}}$$

For Z



(3 generation)



σ N

$\bar{\sigma}$ $\bar{N}$

$\Gamma_{Z\to N\bar{N}} = \Gamma_{Z\to\nu\bar{\nu}} = 501 \text{ MeV}$

($\because$ N is a neutrino)

Assume $\Gamma_{Z\to\sigma\bar{\sigma}} = \Gamma_{Z\to ee} = 84 \text{ MeV}$

then $\Gamma_Z^{(4)} = \Gamma_Z^{(3)} + \Gamma_{Z\to\nu\bar{\nu}} + \Gamma_Z$

$= 2495 + 501 + 84$

$= \underline{\underline{3080 \text{ MeV}}}$

– The ratio $R = \dfrac{\Gamma_{in}^W \Gamma_{out}^W \Gamma_Z^2 M_Z^2}{\Gamma_{in}^Z \Gamma_{out}^Z \Gamma_W^2 M_W^2}$

Scanned by Photo Scanner

- $\left(\frac{m_Z}{m_W}\right)^2$ remains the same

$\because$ $p\bar{p}$ collision $\therefore$ colliding quarks remain the same

$\therefore$ $\frac{\Gamma_{in}^{W}}{\Gamma_{in}^{Z}}$ remains the same (each of them is unchanged

$\because$ we observe the decay in a specific channel.

$\therefore$ Γ_{out}^{W}, Γ_{out}^{Z} remains the same

$\therefore$ ~~R~~ $R \propto \cancel{\Gamma_{tot}} \left(\frac{\Gamma_Z}{\Gamma_W}\right)^2$

$\therefore$ fractional change of R:

is $\left(\frac{3080}{2495}\right)^2 \times \left(\frac{2085}{2317}\right)^2 = \underline{1.23}$

(f)

R changes by ~~#~~ $(1.23-1) \times 100\%$
$= 23\%$

$\# \text{ of } \sigma = \frac{23\%}{1.9\%} = \underline{12}$

Γ_Z changes by $3080 - 2495 = 585$ MeV

of σ = ~~585~~ $\frac{585}{2.3} = 254$

- very low probability to ~~observe~~ confirm the 4th generation fermions since # of σ is very large

- 4th generation fermions do not contribute to the ~~Z~~ W and Z width, they are either too massive, or ~~do~~ do not exist