

Corrections to Groups PS2 Q5.

J.C) consider the matrix $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1_n & i\mathbb{1}_n \\ i\mathbb{1}_n & 1_n \end{pmatrix}$.
 Define:

$$\tilde{f}(U) = P f(U) P^{-1}.$$

easy to see that $P^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1_n & -i\mathbb{1}_n \\ -i\mathbb{1}_n & 1_n \end{pmatrix}$.

$$\tilde{f} = PfP^{-1} = \frac{1}{2} \begin{pmatrix} 1_n & i\mathbb{1}_n \\ i\mathbb{1}_n & 1_n \end{pmatrix} \begin{pmatrix} U_R & -U_I \\ U_I & U_R \end{pmatrix} \begin{pmatrix} 1_n & -i\mathbb{1}_n \\ -i\mathbb{1}_n & 1_n \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1_n & i\mathbb{1}_n \\ i\mathbb{1}_n & 1_n \end{pmatrix} \begin{pmatrix} U_R + iU_I & -iU_R - U_I \\ U_I - iU_R & -iU_R - iU_I \end{pmatrix}$$

$$= \begin{pmatrix} U_R + iU_I & 0 \\ 0 & U_R - iU_I \end{pmatrix}$$

$$= \begin{pmatrix} U & 0 \\ 0 & U^* \end{pmatrix}$$

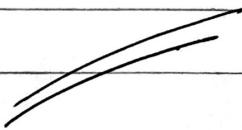
$\tilde{f} \cong f$ is a ~~representation of~~ 2^n dimensional representation of $SO(2n)$

and f is an embedding of $SU(n)$ into $SO(2n)$

U is a \bar{n} representation of $SU(n)$

U^* is a \bar{n} representation of $SU(n)$

$$\therefore \underset{\sim_{SO(2n)}}{8} \underset{\sim_{SO(2n)}}{2n} \rightarrow \underset{\sim_{SU(n)}}{n} \oplus \underset{\sim_{SU(n)}}{\bar{n}}$$



d) The dimension of adjoint representation
= dimension of Lie algebra.

For $SO(10)$, this is $\frac{10(10-1)}{2} = 45$.

\therefore adjoint rep of $SO(10)$ is $\underset{\sim}{45}$

$$\underset{\sim}{SO(10)} \quad \underset{\sim}{SO(10)}$$
$$\underset{\sim}{45} = (\underset{\sim}{10} \otimes \underset{\sim}{10})_A \quad (A \text{ is antisymmetrized})$$

This is because explicit in:

$$\underset{\sim}{10} \otimes \underset{\sim}{10} = (\underset{\sim}{10} \otimes \underset{\sim}{10})_A \oplus (\underset{\sim}{10} \otimes \underset{\sim}{10})_S$$
$$= \underset{\sim}{45} \oplus \underset{\sim}{55} \quad \text{for } SO(10).$$

$$\begin{array}{ccc} \therefore SO(10) & & SU(5) \\ \Downarrow & & \Downarrow \\ \underset{\sim}{45} = (\underset{\sim}{10} \times \underset{\sim}{10})_A & \xrightarrow{\otimes} & ((\underset{\sim}{5} \oplus \underset{\sim}{\bar{5}}) \otimes (\underset{\sim}{5} \oplus \underset{\sim}{\bar{5}}))_A \\ & & \\ & & = (\underset{\sim}{5} \otimes \underset{\sim}{5} \oplus \underset{\sim}{\bar{5}} \otimes \underset{\sim}{\bar{5}} \oplus \underset{\sim}{5} \otimes \underset{\sim}{\bar{5}} \oplus \underset{\sim}{\bar{5}} \otimes \underset{\sim}{5})_A \end{array}$$

$$= (\underline{J} \otimes \underline{J})_A \oplus (\bar{J} \otimes \bar{J})_A \oplus \cancel{\bar{J}(\underline{J} \otimes \bar{J})}$$

$$= \underline{10} \oplus \bar{10} \oplus (\underline{J} \otimes \bar{J})$$

$$= \underline{10} \oplus \bar{10} \oplus \cancel{\frac{1}{2}} \oplus \cancel{\frac{2}{2}}$$

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(This method of calculation follows from
Georgi pp. 286-287).