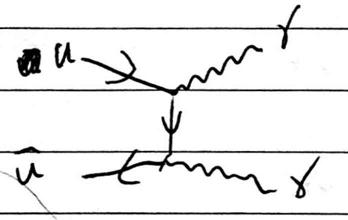
B4 PS 4 Ziyan Li

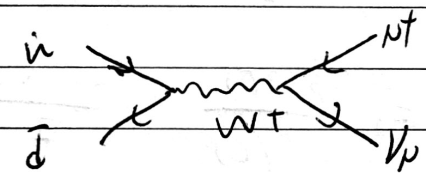
1.

a) $\pi^0 \to \gamma + \gamma$



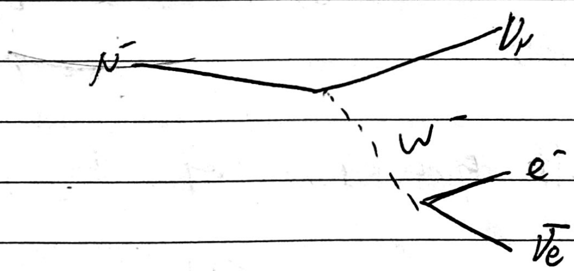
electromagnetic

b) $\pi^+ \to \mu^+ + \nu_\mu$



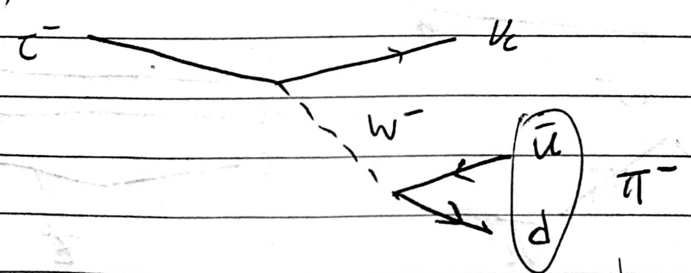
weak

c) $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$



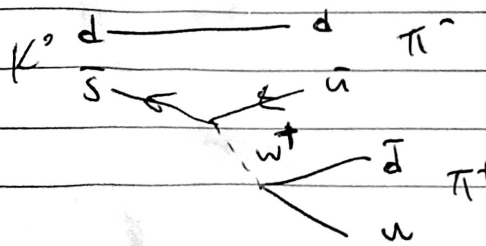
weak

d) $\tau^- \to \pi^- + \nu_\tau$



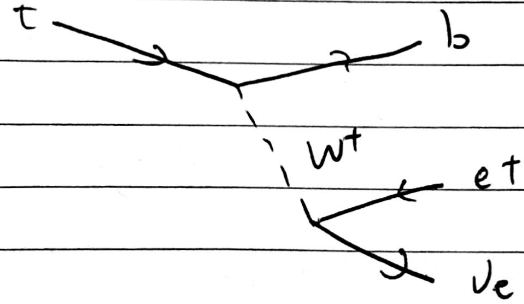
weak

e) $K^0 \to \pi^+ + \pi^-$



weak

f) $t \longrightarrow b + e^+ + \nu_e$



weak.

2, a)

De broglie wavelength of positron is λ_{e^+}

$$\therefore \lambda_{e^+} = \frac{h}{p_{e^+}} 2\pi = \frac{\hbar(2\pi)}{(E/c)} = \frac{2\pi \hbar c}{E}$$

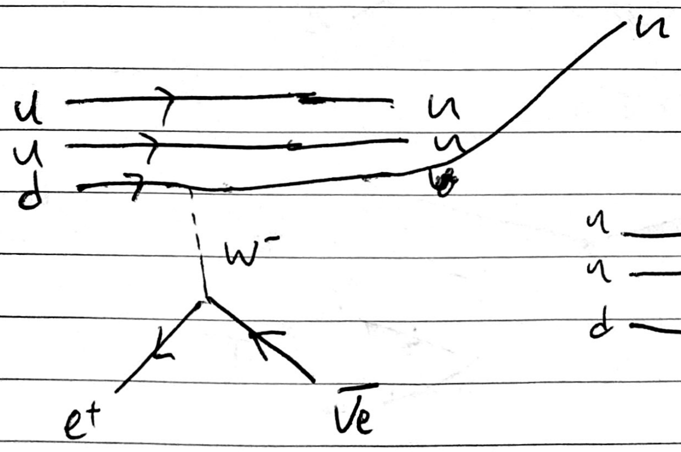
↳ ultrarelativistic

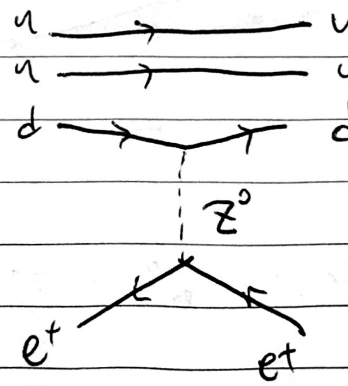
$$= \frac{2\pi \cdot 197\,\text{MeV}\cdot\text{fm}}{27\,\text{GeV}} = 0.05\,\text{fm} < 0.87\,\text{fm}$$

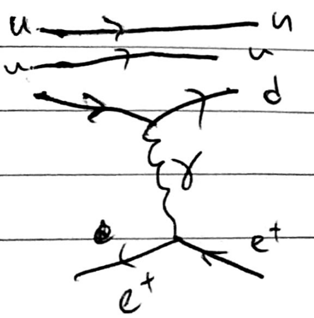
= radius of proton

$\therefore$ scattering off quarks, not the proton itself.

b)







(c) $P_q = f P_p = f\begin{pmatrix} E_p \\ E_p \\ 0 \\ 0 \end{pmatrix}$

$P_{e^+} = \begin{pmatrix} E_e \\ -E_e \\ 0 \\ 0 \end{pmatrix}$

$E_{cm}^2 = -(P_q + P_{e^+})^2 = +(E_p f + E_e)^2 \cancel{+} (E_p f - E_e)^2$

$= 4 f E_p E_e$

$\therefore E_{cm} = \sqrt{4f E_p E_{e^+}}$

(d) $E_{cm} = 2\sqrt{\frac{1}{3} \cdot 27 \cdot 920} = 182 \text{ GeV} > 170 \text{ GeV}$

$\Rightarrow$ can produce top quark.

(e) weak force : $\dfrac{1}{P_w^2 - m_w^2 c^2}$

EM force : $\dfrac{1}{P_\gamma^2}$

At low P $\dfrac{1}{P_w^2 - m_w^2 c^2} \simeq \dfrac{1}{-m_w c^2}$ = finite

$\dfrac{1}{P_\gamma^2} \rightarrow$ very large

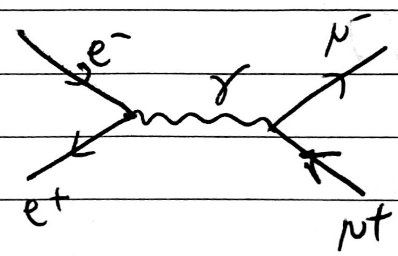
$\therefore \dfrac{\text{weak}}{\text{EM}}$ is very small

large p $m_w c^2$ is small $\ll P_w$

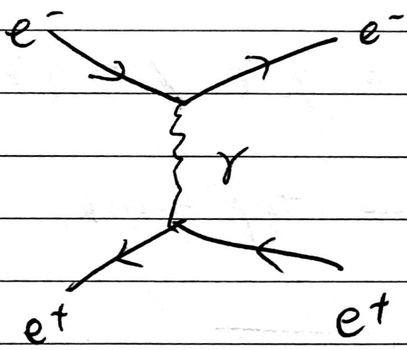
$\therefore$ weak : $\dfrac{1}{P_w^2}$, EM: $\dfrac{1}{P_\gamma^2}$

$\dfrac{\text{weak}}{\text{EM}} \sim 1$

3. (a)



The same is valid for $\tau^+\tau^-$ but not e^+e^- because for the latter there is an additional Feynman diagram (first order



(b) For $E > 10\,GeV$, all quark modes except top quark is excited

$$\therefore R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \times \left[\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 \right]$$

$$= \frac{11}{3}$$

$$\therefore \sigma(e^+e^- \to hadrons) = \frac{11}{3} \times \frac{4\pi}{3} \frac{(\alpha\hbar c)^2}{E^2} = \frac{44\pi}{9} \frac{(\alpha\hbar c)^2}{S^2}$$

(c) Rate $= L \times \sigma$

$$= 3 \times 10^{35}\, m^{-2} s^{-1} \times \frac{44\pi}{9} \left(\frac{\frac{1}{137}\, 197\, MeV\, fm}{3 \times 10^4\, MeV} \right)^2$$

$$= 0.011\ \frac{events}{s}$$

4) $\tau^- \to e^- + \nu_\tau + \bar{\nu}_e$

$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

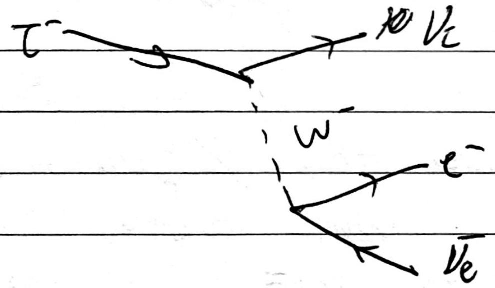
$\because$ ~~M_e~~ $m_\tau, m_\mu \gg m_e$

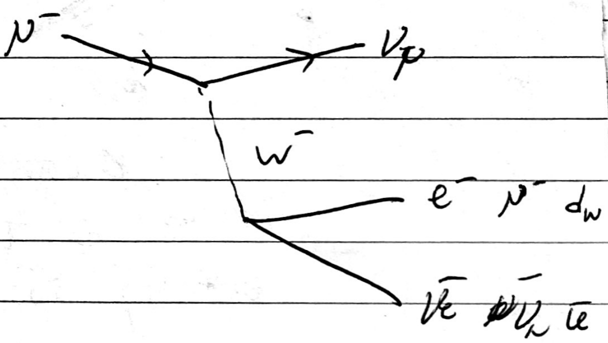
$\therefore$ the Q values $Q_\tau \simeq m_\tau$

$Q_\mu \simeq m_\mu$

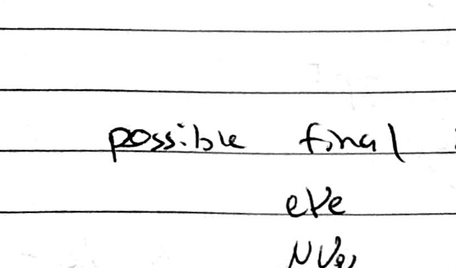
By sargent rule

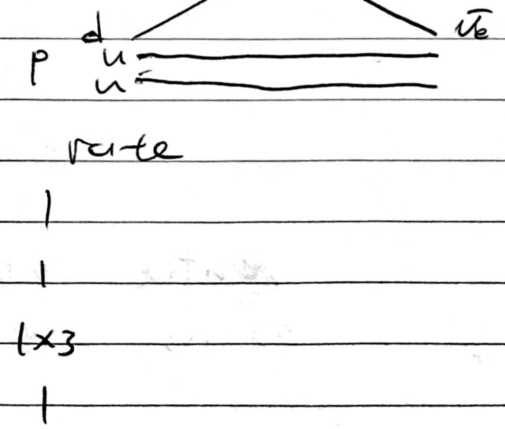
$$\frac{\Gamma(\tau^-)}{\Gamma(\mu^-)} = \frac{Q_\tau^5}{Q_\mu^5} \simeq \left(\frac{m_\tau}{m_\mu}\right)^5$$





5)





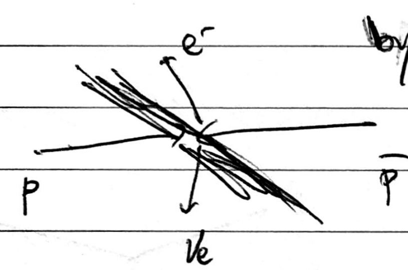
possible final states	rate
$e\nu_e$	1
$\mu\nu_\mu$	1
$u\bar{d}$, $u\bar{s}$	1×3
$\tau\nu_e$	1
$c\bar{d}$, $c\bar{s}$	1×3

lepton universality $\Gamma(e^+e^-) = \frac{1}{1+1+1+3+3} = \frac{1}{9}$

(c) W was discovered in leptonic channel because the hadronic channel has too much background
→ hard to distinguish decay

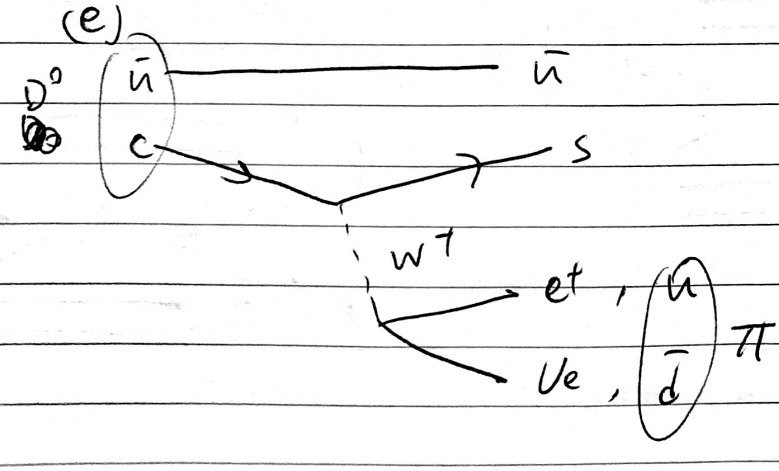
(d) - measure the electron momentum by putting it in a magnetic field $\underline{p = qBr}$

- measure the ⊥ momentum of neutrino by measuring all momenta in that direction and ~~to take~~ ~~minus it~~ reverse it.



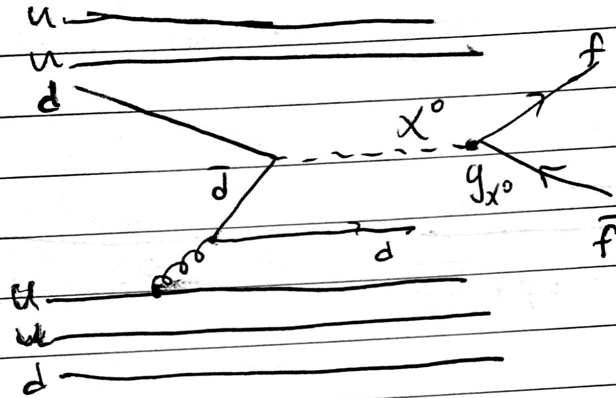
- can't measure the ∥ component because we don't know what fraction of proton energy is given to $W^\pm$

(e)



relative rate is 3 times higher for hadrons due to the color factor.

6.



$\because$ p-p cm energy is $\sqrt{s}$

$\therefore$ momentum ~~p~~ of proton in cm is $\frac{\sqrt{s}}{2}$

(assume $p \gg mc^2$)

$P_1 = \begin{pmatrix} E_1 \\ -P_1 \\ 0 \\ 0 \end{pmatrix}$ $\quad P_2 = \begin{pmatrix} E_2 \\ P_2 \\ 0 \\ 0 \end{pmatrix}$ $\quad P_1 = x_1 \frac{\sqrt{s}}{2}$ $\quad P_2 = x_2 \frac{\sqrt{s}}{2}$

$(P_1 + P_2)^2 = P_{X_0}^2$

$m_1^2 + m_2^2 + 2(E_1E_2 + P_1P_2) = \cancel{m_{X_0}^2}\ m_{X_0}^2$

set $m_1, m_2 = 0$ and $P_1 \simeq E_1$ $\quad P_2 \simeq E_2$

$\Rightarrow \quad 4P_1P_2 = m_{X_0}^2 = x_1x_2(\sqrt{s})^2$

$\therefore \quad m_{X_0}^2 = x_1x_2 s \quad \Rightarrow \underline{m_{X_0} = \sqrt{x_1x_2 s}}$

jet-jet decay can be in all possible quark products and colors

$\therefore \quad \frac{N_{\mu^+\mu^-}}{N_{jj}} = \frac{1}{3\times 6} = \frac{1}{18}$

- The shape of N_{peW} and N_{jj} are the same, both are Breit-Wigner distribution

$$n_{jj} \propto \frac{\frac{\Gamma_{q\bar{q}}\Gamma_{q\bar{q}}}{4}}{(m_{inv}-m_{X_0})^2+\frac{\Gamma^2}{4}} \quad \frac{4\pi}{k^2} \frac{2J_{X_0}+1}{(\cancel{2s}\,2s_q+1)(2s_{\bar{q}}+1)} L$$

$m_{X_0} \sim \cancel{2e}\ 2\,\text{TeV}$ (peak)

$$\tau = \frac{\hbar}{\Gamma} = \frac{6.6\times10^{-22}\ \text{MeV}\cdot s}{2.5\times10^{4}\ \text{MeV}} = \underline{2.6\times10^{-26}\,s}$$

(FWHM)

quarks + color

$$\sigma L \frac{\Gamma_{jj}}{\Gamma} = N_{jj} \Rightarrow \sigma(2\,fb^{-1}) \frac{3\times6}{3\times6+3+3} = 177$$

3 leptons 3 neutrinos

$\Rightarrow \sigma = 118\,fb.$

Conservation of Baryon number

$B(X_0) = 1-1 = 0$