To Alfons Weber

B4 Problem Set 3 Ziyan Li

1. a)

Γ is the width of energy distribution, but it is also the decay rate

This is because energy distribution (Lorentzian) being a fourier transform of exponential decay

$$\therefore \frac{\Gamma_i}{\Gamma_f} = \frac{\text{Probability for } \Delta \text{ to decay back to initial state}}{\text{Probability for } \Delta \text{ to decay into final state}}$$

$$\pi^- + p \rightarrow \Delta \rightarrow \underbrace{\pi^0 + n}_{\Gamma_f}$$

$$\pi^- + p \rightarrow \Delta \rightarrow \underbrace{\pi^- + p}_{\Gamma_i}$$

$$\therefore \frac{\Gamma_i}{\Gamma_f} = \frac{P(\Delta \rightarrow \pi^- + p)}{P(\Delta \rightarrow \pi^0 + n)}$$

Use isospin

$\pi^0 \; |1,0\rangle \qquad \pi^+ \; |1,1\rangle \qquad \pi^- \; |1,-1\rangle$

$p \; |\frac{1}{2},\frac{1}{2}\rangle \qquad n \; |\frac{1}{2},-\frac{1}{2}\rangle$

the pion-nucleon combination has basis states

$|I^1 I_3^1\rangle |I^2 I_3^2\rangle$. there are $3 \times 2 = 6$ basis states

When the pion and nucleon combines ~~they~~ their ~~next~~ ~~final~~ product follows the math of addition of angular momenta in quantum mechanics

$\therefore$ Δ is described by the state

$$|I^1 I^2 I^{tot} I_3^{tot}\rangle = |I^1 I^2 I \, I_3\rangle$$

The problem gives: Δ resonance has $I = \frac{3}{2}$

$\because$ Δ is obtained by combining π^- and p

$$\therefore I_3 = I_3^1 + I_3^2 = -1 + \frac{1}{2} = -\frac{1}{2}$$

$\therefore$ Δ has state $|1 \tfrac{1}{2} \tfrac{3}{2} -\tfrac{1}{2}\rangle$

to find the probability for Δ to decay back to $\pi^- + p$ and that for Δ to decay into $\pi^0 + n$

We need to expand the state of Δ in terms of states $|I^1 I_3^1\rangle |I^2 I_3^2\rangle$ (their superposition)

For complete aligned isospin, we know that

$$|1 \tfrac{1}{2} \tfrac{3}{2} -\tfrac{3}{2}\rangle = |1 \; -1\rangle |\tfrac{1}{2} \; -\tfrac{1}{2}\rangle$$ is the only possibility

consider the raising operator $\hat{I}_+ = \hat{I}_+^1 + \hat{I}_+^2$ acting on the above equation

$$\hat{I}_+ |1 \tfrac{1}{2} \tfrac{3}{2} -\tfrac{3}{2}\rangle = (\hat{I}_+^1 + \hat{I}_+^2) |1 \; -1\rangle |\tfrac{1}{2} \; -\tfrac{1}{2}\rangle$$

$\to \sqrt{3}$

$\therefore \sqrt{(\frac{3}{2})(\frac{5}{2}) - (\frac{-3}{2})(-\frac{1}{2})} \; |1 \; \frac{1}{2} \; \frac{3}{2} \; -\frac{1}{2}\rangle$

$= \sqrt{(1)(2) - (0)(-1)} \; |1, 0\rangle |\frac{1}{2}, -\frac{1}{2}\rangle$

$\sqrt{2}$

$+ \sqrt{(\frac{1}{2})(\frac{3}{2}) - (-\frac{1}{2})(\frac{1}{2})} \; |1, -1\rangle |\frac{1}{2}, \frac{1}{2}\rangle$

1

$$\therefore \underbrace{|1, \tfrac{1}{2}, \tfrac{3}{2}, -\tfrac{1}{2}\rangle}_{\Delta} = \sqrt{\tfrac{2}{3}} \underbrace{|1, 0\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle}_{\pi^0 + n} + \sqrt{\tfrac{1}{3}} \underbrace{|1, -1\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle}_{\pi^- + p}$$

$$\therefore \frac{\Gamma_i}{\Gamma_f} = \frac{P(\Delta \to \pi^- + p)}{P(\Delta \to \pi^0 + n)} = \frac{|\sqrt{\frac{1}{3}}|^2}{|\sqrt{\frac{2}{3}}|^2} = \frac{1}{2}$$

b) Breit-Wigner equation

$$\sigma = \frac{4\pi}{k^2} \frac{2j+1}{(2s_1+1)(2s_2+1)} \frac{\frac{\Gamma_i \Gamma_f}{4}}{(E - E_R)^2 + \frac{\Gamma^2}{4}}$$

maximum cross-section happens at $E = E_R$

$E_R = 1232 \text{ MeV}$ = rest mass of Δ

$\because$ E_R represents a rest mass (rest energy)

$\therefore$ E must be a energy of 2 incoming particles in the centre of mass frame

$$\therefore E^2 = E_R^2 = -P \cdot P = -\begin{pmatrix} E_{\pi^-} + E_p \\ p_{\pi^-} + p_p \end{pmatrix} \cdot \begin{pmatrix} E_{\pi^-} + E_p \\ p_{\pi^-} + p_p \end{pmatrix}$$

$$= -\begin{pmatrix} E_{\pi^-} + m_p \\ p_{\pi^-} \end{pmatrix} \cdot \begin{pmatrix} E_{\pi^-} + m_p \\ p_{\pi^-} \end{pmatrix}$$

$p_p = 0$

$$= (E_{\pi^-} + m_p)^2 - p_{\pi^-}^2$$

$$= \underbrace{(E_{\pi^-}^2 - p_{\pi^-}^2)}_{m_{\pi^-}^2} + 2E_{\pi^-} m_p + m_p^2$$

$$= m_{\pi^-}^2 + m_p^2 + 2E_{\pi^-} m_p$$

$$\therefore E_{\pi^-} = \frac{E_R^2 - m_{\pi^-}^2 - m_p^2}{2m_p}$$

$$= \frac{(1232)^2 - (139.6)^2 - (938.3)^2}{2 \times 938.3}$$

$$= 329.3 \text{ MeV}$$

The kinetic energy of the beam is

$$T_{\pi^-} = E_{\pi^-} - m_{\pi^-} = 329.3 - 139.6$$

$$= \underline{\underline{189.7 \text{ MeV}}}$$

c) From Breit-Wigner formula

possible final states k

$$\sigma \propto \Gamma_i \Gamma_f \text{ for } \cancel{\text{constant}} \text{ constant } (\Gamma = \sum_k \Gamma_k)$$

For $\pi^- + p \to \Delta \to \pi^- + p$

$\Gamma_i = \Gamma(\Delta \to \pi^- + p) = 40$ MeV

$\Gamma_f = \Gamma(\Delta \to \pi^- + p) = 40$ MeV

In both cases $\Gamma = \sum \Gamma$ of all decay channels

$= 40 \text{ MeV} + 80 \text{ MeV} = 120 \text{ MeV}$

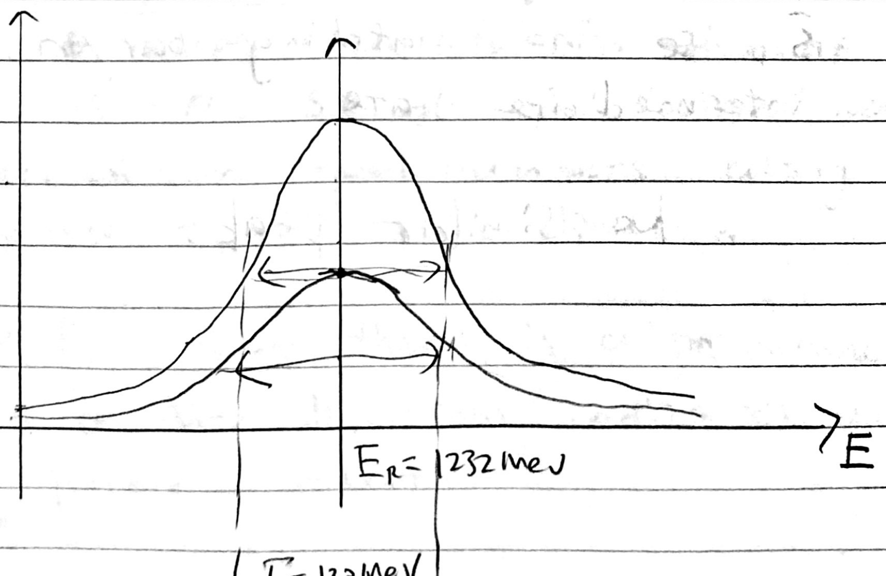
For $\pi^- + p \to \Delta \to \pi^0 + n$

$\Gamma_i = \Gamma(\Delta \to \pi^- + p) = 40$ MeV

$\Gamma_f = \Gamma(\Delta \to \pi^0 + n) = 80$ MeV

$$\therefore \frac{\sigma_{max}(\Delta \to \pi^0 + n)}{\sigma_{max}(\Delta \to \pi^- + p)} = \frac{40 \times 80}{40 \times 40} = 2$$

$\rightarrow$



d). process of $\pi^- + p \rightarrow \pi^0 + n$

quark content is

$$d\bar{u} + uud \rightarrow udd \rightarrow d\bar{d} + udd$$
$$\pi^- \quad p \qquad \Delta^0 \qquad \pi^0 \quad n$$

mass (n) < mass (π^- + p) ∴ not possible →

reactions

(i) $K^- + p \rightarrow$ products

what products

$K\pi + p \rightarrow n$?

quark flow $s\bar{u} + uud \rightarrow uds$

will have similar peaks in cross-sections

expect to have Σ^0 or Λ^0

(ii) $K^+ + p \rightarrow$ products

Quark flow : $u\bar{s} + uud$

No anti quark of $\bar{s}$ (i.e. s) can annihilate $\bar{s}$, so no matching baryon for the intermediate state.

No similar peak.

2.

~~the~~ In this experiment the spins of the ^{60}Co were aligned at low temperatures in a strong magnetic field and the direction of the emitted electron was observed.

^{60}Co decays into $^{60}Ni^*$, which subsequently de-excites by emitting gamma radiation. The direction of these photons is used as a measure of the polarisation of the sample.

A clear preference for emission of the β electron opposite to the spin direction ~~is~~ was observed

This indicates that the angular distribution ~~to~~ of β-emission would be different in a parity-mirrored world.

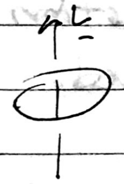
If parity is conserved, then the decay distribution for θ must equal to ~~the d~~ that for $\theta' = 180° - \theta$, so the β- electrons would be observed in equal numbers along and opposite to the spin direction

The fact that there is a preference in the opposite direction indicates violation of parity conservation.

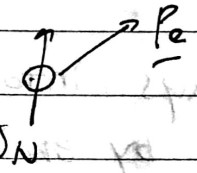
~~PΞ~~ $P\underline{x} \to -\underline{x}$

$\underline{p} \to -\underline{p}$ } vectors
Polar vectors

$\underline{L} \to \underline{L}$ $(\underline{L} = \underline{r} \times \underline{p})$. } axial vectors.
pseudovectors



$\underline{S_N} \cdot \underline{P_e}$



$\hat{P}$ $\underline{S_N} \cdot \underline{P_e} \to -\underline{S_N} \cdot \underline{P_e}$

$\hat{p}$ $\langle \underline{S_N} \cdot \underline{P_e} \rangle \to -\langle \underline{S_N} \cdot \underline{P_e} \rangle$

physics is invariant under $\hat{P}$???

$(\Leftrightarrow) \to \hat{P} \langle \underline{S_N} \cdot \underline{P_e} \rangle = \langle \underline{S_N} \cdot \underline{P_e} \rangle$

$(\Leftrightarrow)$ $\langle \underline{S_N} \cdot \underline{P_e} \rangle = -\langle \underline{S_N} \cdot \underline{P_e} \rangle$

$\Leftrightarrow$ $\langle \underline{S_N} \cdot \underline{P_e} \rangle = 0$.

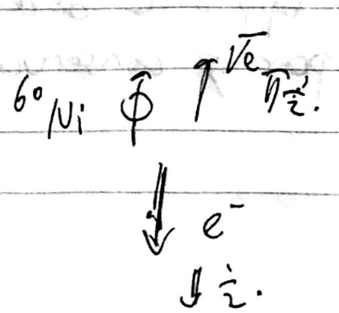
∴ if a preferable direction exists.. then parity is violated..

→ weak interaction:

$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}_e$ (all spin points to direction of ^{60}Co).

5-spin ↑ 4-spin ↑ $\frac{1}{2}$↑ $\frac{1}{2}$↑

⇉ spin
→ $\bar{\nu}_e, e^+$
direction of light
→ ~~$\bar{\nu}_e$~~ ν_e, e^-
⇇ spin.



3.

(1) $\pi^- + p \rightarrow K^0 + \Lambda$

$\pi^- \downarrow d\bar{u}$, $p \downarrow uud$, $K^0 \downarrow d\bar{s}$, $\Lambda \downarrow uds$

Conservation of charge (Q):

$(-1) + (+1) \rightarrow (0) + (0)$

Conservation of flavour (F):

~~$d\bar{u}$~~ $d\bar{u} + uud \rightarrow$ ~~uud~~ $udd \rightarrow d\bar{s} + uds$

Conservation of strangeness (S):

$(0) + (0) = (1) + (-1)$

Conservation of baryon number (B):

$(0) + (1) = (0) + (1)$

(2) $K^- + p \rightarrow K^0 + \Xi^0$

F: $s\bar{u} + uud \rightarrow sud \rightarrow d\bar{s} +$ ~~$ss\ dss$~~ uss

Q: $(-1) + (+1) = 0 = 0 + 0$

S: $(-1) + (0) = -1 = (1) + (-2)$

B: $(0) + (1) = 1 = (0) + (1)$

(3) $\Xi^- + p \rightarrow \Lambda + \Lambda$

F: $dss + uud \rightarrow uds + uds$

Q: $(-1) + (1) = 0 + 0$

S: $(-2) + (0) = -2 = (-1) + (-1)$

B: $1 + 1 = 1 + 1$

(4) $K^- + p \rightarrow K^+ + K^0 + \Omega^-$

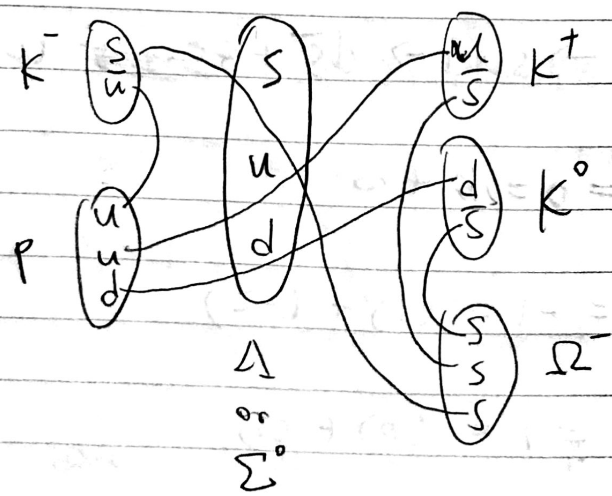
F: $s\bar{u} + uud \rightarrow uds \rightarrow u\bar{s} + d\bar{s} + sss$

Q: $(-1) + (1) = 0 = 1 + 0 + (-1)$

S: $(-1) + (0) = -1 = 1 + 1 + (-3)$

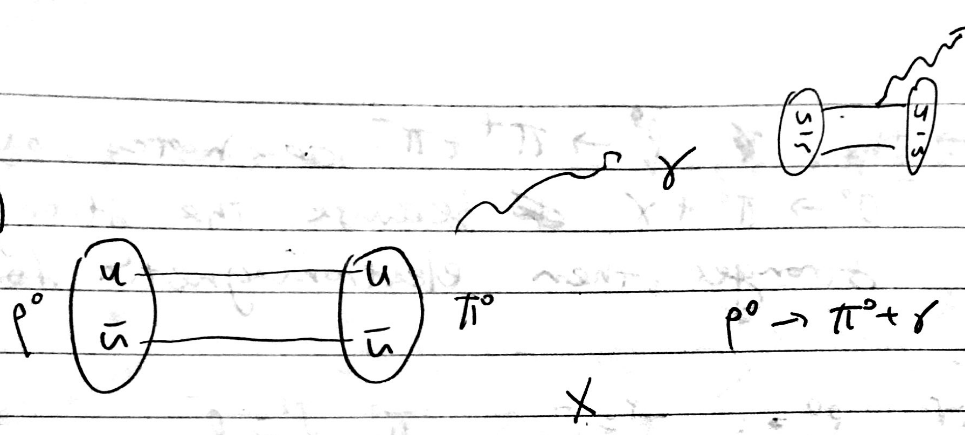
B: $0 + 1 = 0 + 0 + 1$

Quark flow diagram:



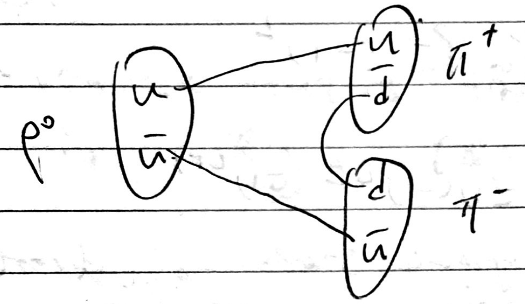
4.

a)



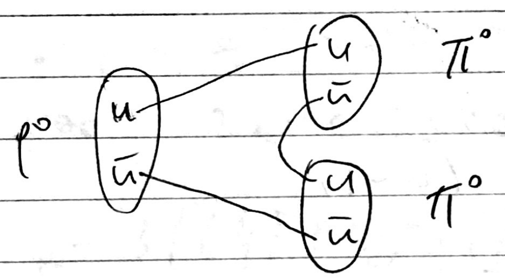
$\rho^0 \to \pi^0 + \gamma$

b)



$\rho^0 \to \pi^+ + \pi^-$

c)



$\rho^0 \to \pi^0 + \pi^0$

For $\rho^0 \to \pi^0 + \pi^0$, $\because$ ρ^0 has $J^P = 1^-$, π^0 has $J^P = 0^-$

$\therefore$ To conserve angular moment need pions to be in $L = 1$ state

$\because$ $\pi^0 + \pi^0$ are two identical bosons

$\therefore$ Their wavefunction needs to be symmetric

But $L = 1$ states are antisymmetric because the parity of wavefunctions is given by $(-1)^L$

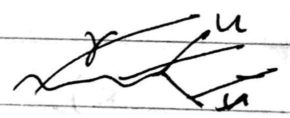
$\Rightarrow$ contradiction, ~~trom~~ decay forbidden.

→ the ~~ρ~~ $\rho^0 \rightarrow \pi^+ + \pi^-$ dominates over the $\rho^0 \rightarrow \pi^0 + \gamma$ ~~ok~~ because the strong force is stronger then electromagnetic force.

force $\sim \frac{1}{137}$ EM

strong force $\alpha \sim 1$.

for ρ^0 : $J^P = 1^-$ $\quad \pi^0 \; J^P = 0^-$



$$P_\rho = P_u P_{\bar{u}} (-1)^L$$

$u, \bar{u}$, opposite parity.

$$= (1)(-1)^L = (-1)^{L+1}$$

$\because L = 0$ ground state (almost always the case

$\therefore$ $1, 0^-$.

$\rho^0 \rightarrow \pi^0 + \pi^0$

$J \; 1 \rightarrow 0 + 0 + (L=1)$

$\because J_\pi = 0$.

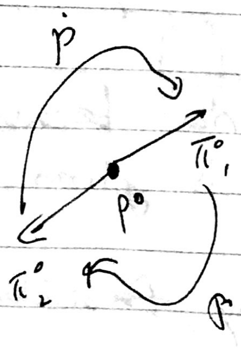
Parity $P_J = 1$ $\quad P_{\pi_0}^2 (-1)^1 = -1$.

two identical bosons ⇒ symmetric wavefunction.

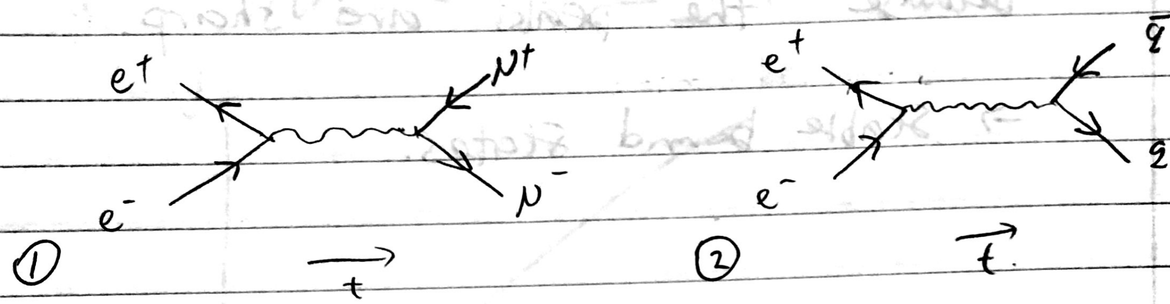
In the CM system, parity operation = exchange of particles.

← →

symmetric in exchange ⇒ $P = +1$.



5. $e^+ + e^- \to \mu^+ + \mu^-$ $\qquad$ $e^+ + e^- \to q + \bar{q}$



The vertex factors for ① is g_{EM}, for ② is $Q g_{EM}$ ($Q = +\frac{2}{3}$ or $-\frac{1}{3}$)

$$R = \frac{\sum_i q_i^2}{1}$$

color degree of freedom of (rgb) for quark

For $2\,GeV \lesssim \sqrt{s} \lesssim 3.5\,GeV$

$$R = 3\left[\left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{2}{3}\right)^2\right] = 2$$

For $3.5\,GeV \lesssim \sqrt{s} \lesssim 11\,GeV$

$$R = 2 + 3\left(\tfrac{2}{3}\right)^2 = \frac{10}{3}$$

For ~~$\sqrt{s} \gtrsim 11\,GeV$~~ $11\,GeV \lesssim \sqrt{s} \lesssim 20\,GeV$

$$R = \frac{10}{3} + 3\left(\tfrac{1}{3}\right)^2 = \frac{11}{3}$$

3 GeV → charm quark
10 GeV → bottom quark
100 GeV → Z^0

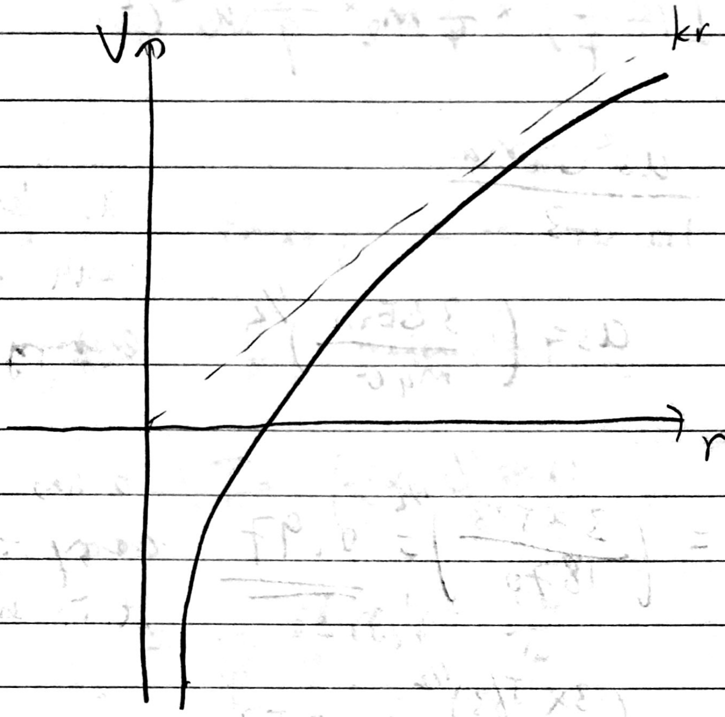
Quarks are point like
because the peaks are sharp.

→ Stable bound states.

why

6.

a) $V(r) = -\frac{4}{3}\frac{\hbar c \alpha_s}{r} + kr$



$$V(r) = -\frac{4}{3}\frac{\hbar c\alpha_s}{r}\left[1 - \frac{3kr^2}{4\hbar c\alpha_s}\right]$$

If $r \ll r_0 = \sqrt{\frac{\hbar c\alpha_s}{k}}$, $\frac{3kr^2}{4\hbar c\alpha_s} \ll 1$

$$V(r) \simeq -\frac{4}{3}\frac{\hbar c\alpha_s}{r}$$

$\Rightarrow$ the $\frac{1}{r}$ term dominates.

b) ignore linear term $V(r) = -\left(\frac{4}{3}\hbar c\alpha_s\right)\frac{1}{r}$

Compare with ~~at~~ Coulomb potential $V(r) = -\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{r}$

we get $\left(\frac{e^2}{4\pi\varepsilon_0}\right) = \left(\frac{4}{3}\hbar c\alpha_s\right)$

use gross structure energy model for Hydrogen

$$E = -\frac{1}{n^2}\left(\frac{1}{2}\mu\right)\left(\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{\hbar}\right)^2 = -\frac{1}{n^2}\left(\frac{1}{2}\mu\right)\left(\frac{4}{3}\alpha_s c\right)^2$$

$$= -\frac{1}{n^2}\,\frac{1}{2}\mu\,\frac{16}{9}\alpha_s^2 c^2 \qquad \mu = \frac{m_q}{2} \text{ for equal masses}$$

$$\Delta E_{2,1} = \left(1-\frac{1}{4}\right) \times \frac{1}{4} m_q \times \frac{16}{9} \alpha_s^2 c^2$$

$$= \frac{\alpha_s^2 c^2 m_q}{3}$$

α_s decreases with increasing energy

$$\therefore \quad \alpha_s = \left(\frac{3\,\Delta E_{2,1}}{m_q c^2}\right)^{1/2}$$

i)

$$\alpha_s(\psi) = \left(\frac{3 \times 588}{1870}\right)^{1/2} = \underline{\underline{0.97}}$$

b has little energy typically $\because$ it is ~~high~~ heavier.

ii) $$\alpha_s(\Upsilon) = \left(\frac{3 \times 563}{5280}\right)^{1/2} = \underline{\underline{0.57}}$$

they are different because α_s depends on the quark content.

c) $$a_0 = \frac{4\pi\varepsilon_0}{e^2}\frac{\hbar^2}{\mu}$$

$$= \frac{3\hbar^2}{4\alpha_s \hbar c} \times \frac{2}{m_q} = \frac{3\hbar c}{\cancel{4}\,2 m_q c^2 \alpha_s}$$

$$a_0(\psi) = 2.95 \times 10^{-13} \frac{1}{1870 \times 0.97} = \underline{1.63 \times 10^{-16}\,m}$$

$$a_0(\Upsilon) = 2.95 \times 10^{-13} \frac{1}{5280 \times 0.57} = \underline{9.80 \times 10^{-17}\,m}$$

$$\alpha_s \sim 1$$

$$r_0 \sim \sqrt{\frac{\hbar c}{k}} = 0.5 \times 10^{-15}\,m > a_0(\psi),\ a_0(\Upsilon)$$

$\rightarrow$ can ignore kr term

d) For $n=1$

Total energy $E = -\left(\frac{4}{3}\alpha_s c\right)^2 \left(\frac{1}{2}\mu\right)$

kinetic energy $= -$ total energy

for $\frac{1}{r}$ potential

$\therefore \quad T = \left(\frac{4}{3}\alpha_s c\right)^2 \frac{\mu}{2}$

$= \frac{16}{9}\alpha_s^2 c^2 \frac{m_q}{4}$

$= \frac{4}{9}\alpha_s^2 m_q c^2$

$T(\psi) = \underline{\underline{782 \text{ MeV}}}$

$T(\Upsilon) = \underline{\underline{762 \text{ MeV}}}$

They are less than rest mass

$\rightarrow$ can use non-rel limit

$\rightarrow$ relativistic correction are larger than positronium $\because$ stronger force has larger coupling than electromagnetic force.