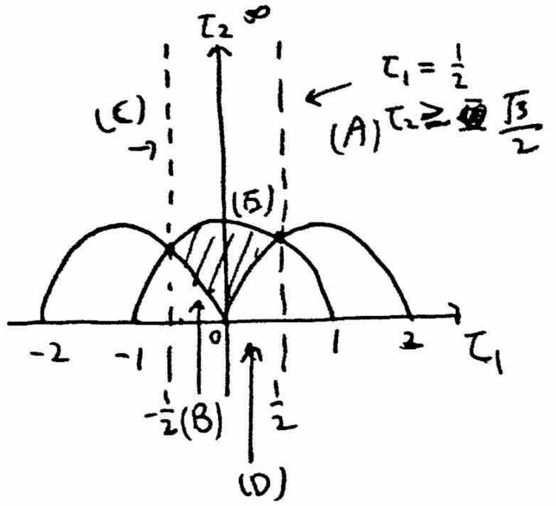
Added Notes to $\boxed{Q1}$ (PS2)

#1 For the mapping of D under S



the boundary $\begin{cases} \tau_1 = 1 \\ \tau_2 \geq \frac{\sqrt{3}}{2} \end{cases}$ (A) mapps to (B)

$\because$ on A :, $\tau = \frac{1}{2} + i\tau_2$

the map S: $\tau \to -\frac{1}{\tau}$ $\cancel{: \frac{1}{2} + i\tau_2}$

$$\therefore \tau \to \frac{-1}{\frac{1}{2}+i\tau_2} = \frac{-2}{1+2i\tau_2} = \frac{-2(1-2i\tau_2)}{1+4\tau_2^2} = \frac{(-2+4i\tau_2)}{1+4\tau_2^2}$$

$$= \underbrace{\left(-\frac{2}{1+4\tau_2^2}\right)}_{\tau_1'} + i\underbrace{\left(\frac{4\tau_2}{1+4\tau_2^2}\right)}_{\tau_2'} \Rightarrow \text{curve (B)}$$

$\therefore$ Note that $(\tau_1'+1)^2 + (\tau_2')^2$

$$= \left(\frac{4\tau_2^2-1}{4\tau_2^2+1}\right)^2 + \left(\frac{4\tau_2}{4\tau_2^2+1}\right)^2 = \frac{1}{(4\tau_2^2+1)^2}\left(16\tau_2^4 - 8\tau_2^2 + 1 + 16\tau_2^2\right)$$

$$= \frac{1}{(4\tau_2^2+1)^2}\left(16\tau_2^4 + 8\tau_2^2 + 1\right) = \left(\frac{4\tau_2+1}{4\tau_2+1}\right)^2 = 1$$

$\therefore\ (\tau_1' + 1)^2 + (\tau_2')^2 = 1$ this describes a circle of radius 1 and centre $\tau_1 = -1$, ~~$\tau_2 = 0$~~ $\tau_2 = 0$.

$\because\ \tau_1' = -\dfrac{2}{1+4\tau_2^2}$, $\tau_2' = \dfrac{4\tau_2}{1+4\tau_2^2}$, $\tau_2 \geq \dfrac{\sqrt{3}}{2}$

when $\tau_2 \to \infty$ $\quad \tau_1' \to 0$

$\tau_2 = \dfrac{\sqrt{3}}{2}$ $\quad \tau_1' = -\dfrac{2}{1+4\cdot\frac{3}{4}} = -\dfrac{1}{2}$

$$\therefore\ \underline{-\frac{1}{2} \leq \tau_1' \leq 0}$$

when $\tau_2 = \dfrac{\sqrt{3}}{2}$, $\tau_2' = \dfrac{\sqrt{3}}{2}$

$\tau_2 \to \infty$ $\quad \tau_2' = 0$

$$\therefore\ \underline{0 \leq \tau_2' \leq \frac{\sqrt{3}}{2}}$$

so (B) is the curve shown in the figure,

- which is $\frac{1}{6}$ of a full circle.

Similarly, (C) ~~maps~~ maps to (D).

- (E) maps to itself

- point at infinity maps to origin.

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we ~~see~~ can turn $\alpha_{-n}^{\mu}\alpha_{n}^{\mu}$ into occupation number operator $nN_{n\mu}$, this is because α_{-n}, α_n ~~resember~~ resembles the creation / annihilation operators $a^\dagger$, a in the Quantum Harmonic Osillator (QHO). ~~This occupat~~ In (QHO) we have $\hat{N} = \hat{a}^\dagger\hat{a}$, and ~~to~~ this occupational number operator commutes with the Hamiltonian.

- In the trace, we sum all ~~poss.~~ quantum ~~n~~ numbers, including ~~the~~ the occupation number which is the eigenvalue of $N_{\mu n}$, so this is where the $\sum_{N_{n\mu}=0}^{\infty}$ sum comes from.