Correction to QFT PS4 Q12

$\int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2}$ has both IR and UV.

divergences, dimensional regularisation can't deal with it, we need to use momentum cut-off.

Many thanks to Suvajit Majumder for pointing this out!

If $K \to 0$, $\Lambda \to \infty$.

this integral

$$\to \frac{2}{(4\pi)^{D/2}\Gamma(D/2)} \frac{1}{(D-2)}(\Lambda^{D-2} - K^{D-2})$$

Divergent.

But this doesn't affect the rest of the problem as δZ_ϕ is still 0

$$\delta m^2 = (\ ) \times \frac{2}{4\pi^{D/2}\Gamma(D/2)} \frac{1}{D-2}(\Lambda^{D-2} - K^{D-2})$$

Infrared Divergence

$\frac{1}{a} = \int du\, e^{-ua}$

$\cancel{\int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2}}$ $\quad \frac{1}{a^n} = \frac{1}{\Gamma(n)} \int du\, u^{n-1} e^{-ua}$

$$\boxed{\begin{aligned}\int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2} &= \frac{1}{(2\pi)^D} \int d\Omega_d \int_0^\infty dp \cdot p^{D-1} \frac{1}{p^2} \\ &= \frac{1}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^{\infty \leftarrow \Lambda} dp\, p^{D-3} \\ &= \frac{2}{(4\pi)^{D/2}} \frac{1}{\Gamma(D/2)} \left[\frac{p^{D-2}}{D-2} \right]_{0 \leftarrow K}^{\infty \leftarrow \Lambda} \\ &= \frac{2}{(4\pi)^{D/2}\, \Gamma(D/2)} \left(\frac{1}{D-2} \right) \times \left(\Lambda^{D-2} - K^{D-2} \right)\end{aligned}}$$

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2} = \int \frac{d^D p}{(2\pi)^D} \int_0^\infty du\, e^{-up^2}$$

$$= \int_0^\infty du \int_0^\infty \frac{d^D p}{(2\pi)^D} e^{-up^2}$$

$$= \frac{1}{(2\pi)^D} \int_0^\infty du \left(\frac{\pi}{u} \right)^{D/2}$$

$$= \frac{1}{(2\pi)^D} \cdots \quad \text{infinite.}$$

$$\beta_\lambda = -2\varepsilon\lambda_0\mu^{-2\varepsilon} + \frac{2\lambda_0^2}{32\pi^2}\mu^{-4\varepsilon}$$

$$+ \frac{4\kappa_0^2}{3(32\pi^2)}\mu^{-4\varepsilon}.$$

$$= -\cancel{g} \cancel{2\varepsilon}\lambda_0$$

$$-2\varepsilon g_{\lambda_0} + \frac{12}{32\pi^2} g_{\lambda_0}^2.$$

$$+ \frac{4}{3(32\pi^2)} g_{\kappa_0}^2$$

$$\cancel{g_{\lambda_0} = g}$$

$$\cancel{g_{\lambda_0}}$$

$$g_\lambda = g_{\lambda_0} - \frac{3 g_{\lambda_0}^2}{32\pi^2\varepsilon} - \frac{1}{3(32\pi^2)\varepsilon} g_{\kappa_0}^2 + \dots$$

$\therefore$ invert the perturbation.

$$g_{\lambda_0}^2 = g_\lambda^2 + \dots$$

$$g_{\lambda_0} = g_\lambda + \frac{3 g_\lambda^2}{32\pi^2\varepsilon} + \frac{g_\kappa^2}{3(32\pi^2)\varepsilon} + \dots$$

$$\Rightarrow \beta_\lambda = -2\varepsilon\left(g_\lambda + \frac{3g_\lambda^2}{32\pi^2\varepsilon} + \frac{g_\kappa^2}{3(32\pi^2)\varepsilon} + \dots\right)$$

$$+ \frac{12}{32\pi^2} g_\lambda^2 + \frac{4}{3(32\pi^2)} g_\kappa^2.$$

$$\beta_\lambda = -2\varepsilon g_\lambda - \frac{6 g_\lambda^2}{32\pi^2} - \frac{2 g_x^2}{3(32\pi^2)}$$

$$+ \frac{12}{32\pi^2} g_\lambda^2 + \frac{4}{3(32\pi^2)} g_x^2 + \dots$$

$$= -2\varepsilon g_\lambda + \frac{6}{32(\pi^2)} g_\lambda^2 + \frac{2 g_x^2}{3(32\pi^2)} + \dots$$

when $\varepsilon \to 0$.

$$\beta_\lambda = \frac{6 g_\lambda^2 + \frac{3}{2} g_x^2}{32\pi^2} = \frac{1}{16\pi^2}\left(3 g_\lambda^2 + \frac{1}{3} g_x^2\right)$$

$\frac{1}{16\pi^2}$ instead of $\frac{1}{8\pi^2}$

Similarly $\Rightarrow$ $\beta_x = \frac{1}{16\pi^2}\left(2 g_\lambda g_x + \frac{4}{3} g_x^2\right)$

Also, correction to Q13 of PS4 are in the Q4 of 2017 QFT exam (same question appears).